

Medical Image Non-rigid Registration Based on Improved Finite Element Method

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Abstract: Non-rigid registration has become a very important and popular topic in medical image processing field. In this paper, an improved non-registration method based on finite element (FEM) was proposed. First, the calculation process of the non-registration was derivate from the principle of finite element, and the characteristic of the modulus of elasticity was analyzed. And then, an adaptive method which could accurate the speed of the registration and improve the effect of registration was proposed to set up the elasticity modulus dynamically. The experimental results verified the effectiveness of the algorithm. *Copyright © 2013 IFSA.*

Keywords: Medical image, Finite element, Non-rigid registration.

1. Introduction

Within the current clinical setting, medical imaging is a vital component of a large number of applications. Since information gained from different medical images is usually of a complementary nature, proper integration of useful data obtained from the separate images is often desired. A first step in this integration process is to bring two images into spatial alignment, a procedure referred to as registration [1]. Registration plays a central role in medical image processing: it is the foundation of image fusion and reconstruction.

Image registration requires finding an optimal transformation between an image pair, the source and the target. Based on the transformation, registration can be classified as being “rigid” or “non-rigid”. Due to the flexibility, the deformation of the human organs is complex in medical image, and rigid registration in many cases cannot meet the needs of

the practical application. Therefore, more accurate non-rigid registration is particularly important. Many non-rigid registration methods have been proposed in recent years, and can be categorized into pixel-based or feature-based approaches [2]. The methods of pixel-based registration can make use of the image directly to avoid the feature extraction process of the error, and become more and more important. The finite element method (FEM) is one of the fully automatic registration algorithms based on gray-scale information of the pixels [3-6]. This method makes use of the displacement method in the finite element method to calculate the displacement in the process of registration, based on the principle of minimum potential energy. This method can adapt to a variety of complex deformation, and thus become an effective means of registration. However, the process of this method is relatively time-consuming, and the process of setting the elastic modulus for different images is inconvenience. In order to expedite the

registration rate and improve registration results, a novel method of dynamic adjustment of the elastic modulus has been presented in this paper.

2. Finite Element Model

The basic idea of FEM is replacing the complex problems by relatively simple problems, and then solving. It assumes that the solving domain is composed of some interconnect sub-domains called finite element. For each sub-domain, a (relatively simple) approximate solution is assumed, and then the conditions for solving the full field are deduced, at last, the solution of the problem is obtained. The finite element method is not only high accuracy and can adapt to a variety of complex deformation, and thus become an effective means of registration.

2.1. The Relationship Between Strain and Displacement

Considering there is a displacement $(u(x, y), v(x, y))$ in a two-dimensional space for the infinitesimal element, let ε be a representative of strain, and γ be a representative of the shear strain. Because the horizontal displacement $u(x, y)$ and vertical displacement $v(x, y)$ are independent of each other, we can get the relationship between strain and displacement as follows.

$$\varepsilon_x = \varepsilon_x^h + \varepsilon_x^v = \frac{\partial u}{\partial x} dx + 0 = \frac{\partial u}{\partial x} \quad (1)$$

$$\varepsilon_y = \varepsilon_y^h + \varepsilon_y^v = 0 + \frac{\partial v}{\partial y} dy = \frac{\partial v}{\partial y} \quad (2)$$

$$\gamma_{xy} = \gamma_{xy}^h + \gamma_{xy}^v = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \quad (3)$$

That is,

$$\varepsilon = \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \vec{\mu} = \vec{B} \vec{\mu}, \quad (4)$$

where the superscript h and v represents the horizontal and vertical displacement respectively.

2.2. The Relationship Between Stress and Strain

For the problem of plane stress, considering the relationship between stress and strain still in two-dimensional space, the formulas obtained by the generalized Hooke's law are as follows.

$$\varepsilon_x = \frac{1}{E} (\sigma_x - \mu \sigma_y) \quad (5)$$

$$\varepsilon_y = \frac{1}{E} (\sigma_y - \mu \sigma_x) \quad (6)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{2(1+\mu)}{E} \tau_{xy} \quad (7)$$

Then, the relationships between stress and strain are as follows.

$$\sigma_x = \frac{E}{1-\mu^2} (\varepsilon_x + \mu \varepsilon_y) \quad (8)$$

$$\sigma_y = \frac{E}{1-\mu^2} (\varepsilon_y + \mu \varepsilon_x) \quad (9)$$

$$\tau_{xy} = \frac{E}{1-\mu^2} \frac{1-\mu}{2} \gamma_{xy} \quad (10)$$

Express the equation in the form of a matrix:

$$\sigma = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} = \frac{E}{1-\mu^2} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix} \varepsilon = D \varepsilon, \quad (11)$$

where $\sigma = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$ is the stress, and $\varepsilon = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$ is the direction of strain, μ is the Poisson's ratio, E is the modulus of elasticity, $D = \frac{E}{1-\mu^2} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix}$ is the elasticity matrix.

2.3. The Solution of Displacement

An elastic energy formula is established as follows.

$$Energy = \frac{1}{2} \int_{\Omega} \sigma^T \varepsilon d\Omega + \int_{\Omega} F \mu d\Omega, \quad (12)$$

where two parts of the left side of the formula (12) are the power of the internal forces and the external forces respectively, F is the force on the elastomer, μ is the displacement of the deformation, ε is the deformation, σ is the stress, Ω is the integral area [7]. It is known by the energy formula that the energy will reach the minimum when the floating image meets the registration requirement. So, registration is the process of solving the displacement field which makes the energy minimum. To calculate the displacement field, the partial derivatives to the displacement are calculated on both side of the formula (12). Through simplification, we can get:

$$\begin{aligned} & \int_{\Omega} \sum_{i=1}^3 (B_i^T D B_i + N_i \nabla I_1^T \nabla I_1 N_i) \mu_i d\Omega \\ &= \int_{\Omega} (I_1 - I_2) \nabla I_1^T N_i d\Omega \end{aligned} \quad (13)$$

The formula (13) is the finite element formulation for each unit, and it can be expressed as the form of $K\mu = F$, where K is the stiffness matrix for each unit (it is a symmetric matrix, and the value of elements on the main diagonal is constant positive), F is the equivalent load matrix. All the problems of solving the finite element will eventually be simplified to the form of the above equation which is about the displacement. Now, the final displacement, which enables the energy minimum, can be obtained by solving this equation, and it shall be the final results of the registration.

3. Adaptive Elastic Modulus

In material mechanics, the elastic modulus is a reflection of the difficulty of material to produce the elastic deformation. It can be seen from the above formula that, the role of the elastic modulus in the registration process is to decide the step size of the iteration. The greater the elastic modulus, the smaller the step, the better the convergence effect; on the contrary, the smaller the elastic modulus, the greater the step size, the faster the convergence. For the same image, there are significant differences for the elastic modulus of each part because of their complexity. In the process of the registration based on finite element, each image is generally specified a larger modulus of elasticity which can guarantee that the various parts of the image can be convergent. However, it may lead to the iteration step of the entire image relatively small, and a waste of computing time. On the contrary, too small elastic modulus will lead to the iteration cannot converge. An improved method has been proposed in this paper to accelerate the convergence speed and does not

affect the convergence of the algorithm. The basic idea of our method is to adaptively change the elastic modulus in the process of registration. In the initial stages of the iteration, in order to speed up the convergence, a small value is set to the elastic modulus. And then increases the modulus of elasticity gradually, reducing the step size to ensure convergence effect.

4. Process of the Algorithm Implementation

The process of the non-rigid registration based on finite element is shown in Fig. 1.

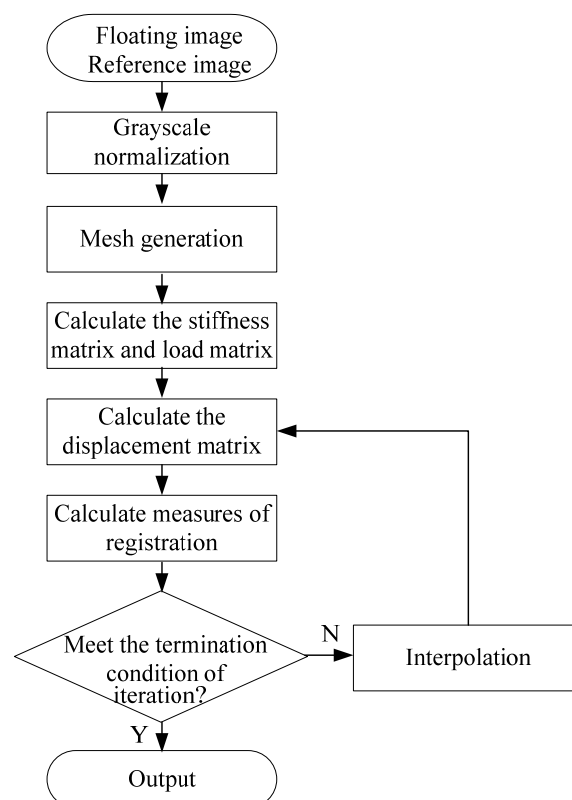


Fig. 1. The flowchart of the non-rigid registration algorithm based on finite element.

4.1. Grayscale Normalization

In the process of registration, grayscale information is used as a similarity measure function between the reference image and the floating image. It assumes that the gray value is consistent with the corresponding points between two images. However, in fact, even if the two images from the same type of imaging device, the gray of the same anatomical structure may also be inconsistent because of the process of image acquisition, so the grayscale normalization is necessary.

In this paper, a normalization method based on a polynomial least squares fitting [8] is proposed to match the grayscale of the two images.

- 1) Calculate the histograms of the floating image and the reference image, denoted as $\{f_i\}$ and $\{r_j\}$, respectively, where $i = 1, 2, \dots, L$, $j = 1, 2, \dots, M$, L and M are the grey level of the floating image and the reference image.
- 2) Calculate the N quintiles of the histogram of the floating [9], the two endpoints of the histogram of the floating image, and the $2(N+1)$ maximum and minimum intensities of the $N+1$ intervals constituted by these points, denoted as $q_{N+2}^f, q_{N+3}^f, \dots, q_{3N+3}^f$. Repeat the above procedure on the histogram of the reference image, and denote the corresponding intensity points as $q_{N+2}^r, q_{N+3}^r, \dots, q_{3N+3}^r$.
- 3) Calculate the mapping function between the intensities of the floating image and the reference image by applying the least square fitting on (q^f, q^r) . Denote the function is

$$f' = \alpha_0 + \alpha_1 f + \alpha_2 f^2 + \dots + \alpha_p f^p, \quad (14)$$

where α is the coefficient, and p is the maximum power of the polynomial, f and f' are the intensities of the floating image before and after mapping.

- 4) Normalize the intensities of the floating image by applying the mapping function calculated above.

4.2. Mesh Generation

Grid (mesh) generation is the discretization of original image. As shown in Fig. 2, the mesh is composed of many elements, and the elements are again a lot of nodes, the node formed by the plurality of pixels. Uniform triangular mesh division is used in this study.

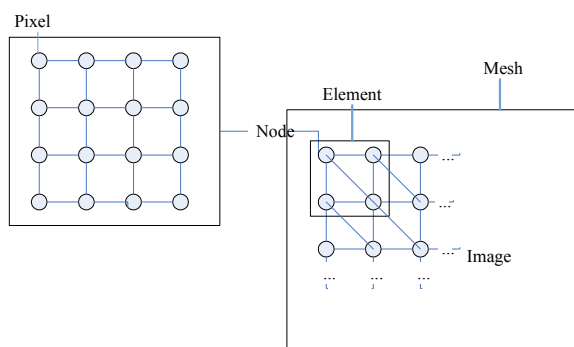


Fig. 2. Mesh generation.

4.3. Calculate the Matrixes

First, calculate the stiffness matrix and load matrix by calculating the equation coefficient K and

F in $K\mu = F$. Then calculate the displacement matrix by solving equation (13). That is to obtain a displacement field between the floating image and the reference image.

4.5. Calculate Measure of Registration

The deformed floating image can be obtained by applying the displacement field to the floating image. And, the mutual information (MI) between the reference image and deformed floating image is calculated. At the same time, whether the results meet the termination conditions of iteration is determined.

4.6. The Terminate Condition of Iteration

An automatic terminate condition of iteration is designed in this paper. It is composed of the following three parts.

- 1) Algorithm automatically terminated when the difference of the measure between two adjacent iterations is less than user-specified threshold.
- 2) If the maximum displacement in the displacement matrix is less than a certain threshold set by user, stop the entire iterative process.
- 3) The times of iterations exceed a set maximum number of iteration.

5. Experiments and Results

In the experiments of comparing the effect of using single modulus of elasticity and adaptive multi-modulus of elasticity, triangular grid is used. The floating image and reference image used in the experiment are shown as Fig. 3.

When we use a single elastic modulus, the trend of mutual information between the result image and reference image in the process of registration is shown as Fig. 4.

It can clearly be seen from Fig. 4 that with the increasing of the modulus of elasticity, mutual information has a mutation, indicating the existence of the lower limit of elastic modulus. Algorithm cannot achieve convergence when the elastic modulus is less than the lower limit. That's to say, only when the elastic modulus greater than or equal to the lower limit, the algorithm can achieve convergence. However, for different images, the value of the lower limit is different. What's more, the value of the lower limit is difficult to know before registration. In this case, if a large value has been set to the elastic modulus at the beginning of the registration, the step size will be very small, and the convergence rate is very slow. Conversely, if a small value has been set to the elastic modulus at the beginning of the registration, the step will be too large; leading to the algorithm not converge.

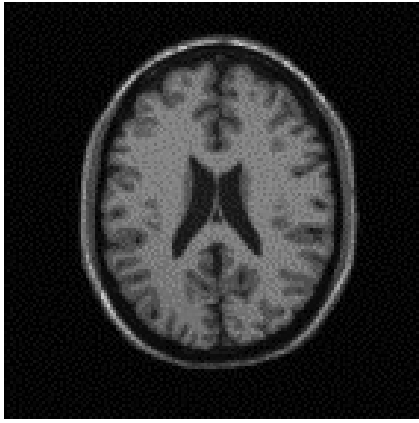


Fig. 3 (a). The floating image.

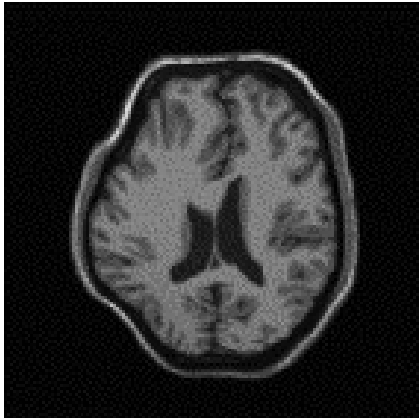


Fig. 3 (b). The reference image.

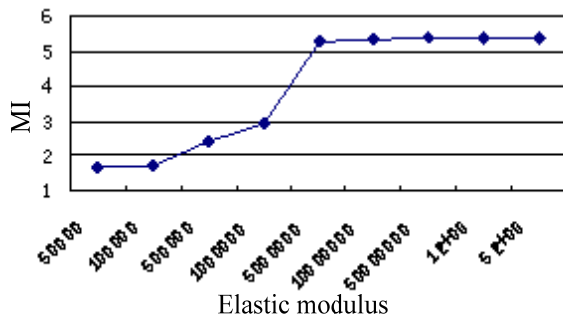


Fig. 4. Changes of MI value.

In order to ensure that the registration finally is able to achieve convergence, while ensuring the speed of convergence is not too slow, a method of dynamically adjusting the elastic modulus has been proposed in this paper.

At the initial stage of the iteration, the difference between the floating image and reference image is significant. Therefore, the elastic modulus is set to be a small value to ensure that the step of iteration is large, and the images can quickly reach a preliminary match. With the reducing of the difference between the two images, the algorithm gradually increase the value of elastic modulus. Therefore, the step of the

iteration decreases, which makes the details of image achieve precise alignment step by step.

For the two images in Fig. 3 (a) and Fig. 3 (b), using the single-modulus of elasticity and adaptive elastic modulus for registration respectively. The required time of the process of registration and the mutual information between the resulting image and reference image are shown as Table 1. Obviously, the method of adaptive elastic modulus not only improves the effect of registration, but also speeds up the iterative convergence rate.

Table 1. Comparison of the two results by single elastic modulus and adaptive elastic modulus.

Method	MI	T
Single modulus	2.9	16 (s)
Adaptive modulus	5.1	7 (s)

To further test the validity of the proposed algorithm in this paper, Fig. 5 shows the result (c) of registration by the proposed method on two other brain images (a and b), and the difference between resulting image and reference image (d). Obviously, the proposed algorithm achieves an ideal effect.

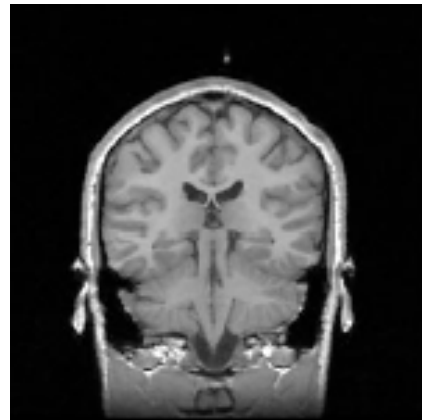


Fig. 5 (a). Result of registration by the method of adaptive elastic modulus: reference image.

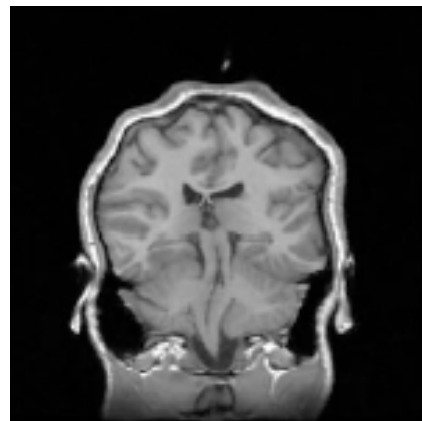


Fig. 5 (b). Result of registration by the method of adaptive elastic modulus: floating image.

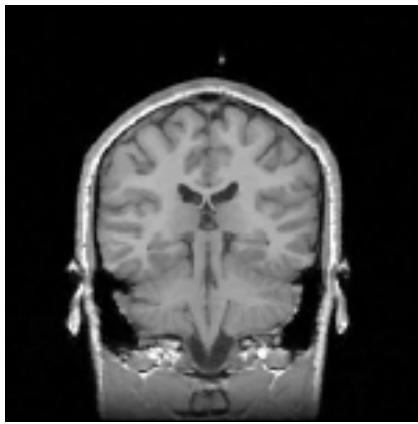


Fig. 5 (c). Result of registration by the method of adaptive elastic modulus: resulting image.



Fig. 5 (d). Result of registration by the method of adaptive elastic modulus: difference image

6. Conclusions

The basic principles and implementation process of the non-rigid registration based on finite element is described in this paper. Through the analysis of the characteristics of the commonly used single modulus, it has been found that there is a lower limit for the elastic modulus, and the iteration process of registration can converge only when the value of elastic modulus is greater than or equal to the lower limit. Based on this, an adaptive algorithm to adjust the modulus of elasticity has been proposed in this paper, which can improve the effect of registration and at the same time speed up the iterative

convergence rate. Experimental results show that the proposed method achieved satisfactory results.

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