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Matrix Decomposition Approach for Pre Estimation of Number and Coherence of Waves

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Abstract: At first, necessity of pre estimation of number and coherence of waves in direction finding of multiple emitters was noted. Secondly, the theory of matrix decomposition approach of pre estimation of number and coherence of waves was studied and the computing steps of matrix decomposition approach were given. Thirdly, some simulation experiments of pre estimation of number and coherence of waves were done based on the simulation environment. The simulation experiments show that the pre estimation results basically reveal the real situation of the waves. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Number of waves, Coherence of waves, Direction finding, Pre estimation, Matrix decomposition approach, Simulation.

1. Introduction

In the past decades, the theory of space spectrum estimation has developed rapidly. Many direction finding approaches have been invented such as MUSIC algorithm, smoothing algorithm, ESPRIT, SVD, etc. Pre estimation of number and coherence of waves is important in the theory research of space spectrum estimation and in the practice of DOA finding.

In this paper, the theoretical basis of matrix decomposition approach of pre estimation of number and coherence of waves was studied and the computing steps of matrix decomposition approach were given. In order to attest the correctness and validity of the approach, the simulation environment of pre estimation of number and coherence of waves was constructed based on .NET platform. Some simulation experiments were done based on the simulation environment.

2. Necessity of Pre Estimation of Number and Coherence of Waves

2.1. Necessity of Pre Estimation of Number of Waves

In MUSIC algorithm to find the directions of multiple independent waves, it is necessary to estimate the number of waves. The estimation result will not only affect the uniform linear array size but affect the direction finding accuracy as well.

1) Array arrangement need to pre-estimate the number of waves.

On one hand, the number of antennas can not be too small. For finding the directions of K independent waves on the same frequency band, MUSIC algorithm will get a *K*-dimensional subspace E_S which signal direction vectors are belong to and a signal zero space E_N with one-dimension at least [1]. Thus, the sum of the dimensions of E_S and E_N should be $K+1$ at least, so the order of the array covariance matrix is $K+1$ at least, therefore the number of antennas must not be less than $K+1$. Simulation results show that, if the number of array elements is less than $K+1$, not only will some waves not be found, but also the measured directions are not the real directions of the waves.

On the other hand, the number of antennas can not be too more. As is known, when the number of antennas gets larger, it becomes more difficult to arrange uniform linear array, because the factors which destruct the uniform of array increase. Non-uniform of array will reduce the accuracy of direction finding. Meanwhile, the key of MUSIC algorithm is eigenvalue decomposition of the array covariance matrix. When the order of a matrix is N, the complexity of matrix eigenvalue decomposition is more than $O(N^3)$. If the order of a matrix doubles, the complexity of matrix eigenvalue decomposition will increase by more than eight times, it will require higher hardware equipment and longer computing time.

Therefore, when we design the antenna array for direction finding, we have to arrange proper number antennae according to the number of waves, so we have to pre estimate the number of waves.

2) Space division need to pre estimate the number of waves.

In the ideal case, the array covariance matrix can be expressed as

$$\begin{aligned} R &= E[X(t)X^H(t)] \\ &= A(\theta)S(t)A^H(\theta)+\sigma_n^2 I \\ &= \lambda_1 e_1 e_1^H + \cdots + \lambda_K e_K e_K^H \\ &+ \lambda_{K+1} e_{K+1} e_{K+1}^H + \cdots + \lambda_N e_N e_N^H , \end{aligned}$$

where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_K > \lambda_{K+1} = \lambda_{K+2} = \cdots = \lambda_N$.

The K larger eigenvalues are corresponding to signals and the corresponding eigenvectors span the subspace E_S which signal direction vectors are belong to, the $N-K$ smaller equal eigenvalues corresponding to the noise, the corresponding eigenvectors span the signal zero space E_N. We can construct the spatial spectrum function according to E_N and gain

$$P(\theta)=\frac{1}{a^H(\theta)E_N E_N^H a(\theta)}=\frac{1}{\left\|a^H(\theta)E_N\right\|^2} .$$

Scan the angle θ and observe the value of $P(\theta)$, the DOA can be found according to the peak positions of $P(\theta)$ [2-6]. K is the number of waves which need not be pre estimated.

However, we can't get $X(t)$ continuously in practice of direction finding, we can only sample $X(t)$ at a certain rate in a certain period, and then calculate the mean array covariance matrix according to M samples of $X(t)$: $X(t_m)\,(m=1,2,\cdots,M)$. Because the number M of samples is limited, there must be differences between the mean array covariance matrix and the actual array covariance matrix, so the small eigenvalues are not equal any longer.

On the other hand, in order to process array signals, we always amplify and transform the signals received to make the amplitude large enough. Because the amplification and transformation of each antenna unit may lead in the channel noise, and the noise of one channel is not consensus with another channel, which makes the noise outputs of the array not be the stationary white noise, so the array covariance matrix is $R=AR_S A^H+\sigma_n^2\sum$, the diagonal elements of the diagonal matrix $\sum$ are not equal, then we can not get $N-K$ small equal eigenvalues after eigenvalue decomposition. If we do not know the number of waves in advance, we can not decide which eigenvalue corresponding to the noise and which eigenvalues corresponding to the signals from the eigenvalues directly, thus, we can neither divide E_S and E_N, nor construct the spatial spectrum function.

3) High resolution direction finding need to pre-estimate the number of waves. If there are two independent waves which have same powers and almost same directions, the array covariance matrix pair $(R,\sigma_n^2\Sigma)$ will have two eigenvalues $\lambda_k = NP(1\pm|\rho|)\,(k=1,2)$. Because the directions of two waves are almost same, that is $|\rho|\approx 1$, one eigenvalue is close to 0, so the eigenvalue of the array covariance matrix corresponding to one signal may be smaller than the largest eigenvalue corresponding to noise [4]. In this case, even if we know the number of waves, we can not divide the signals and the noise by the eigenvalues; if we do not know the number of waves, it is more difficult to divide the signals and the noise. If we still divide the signals and the noise according to the eigenvalues, we shall make a mistake.

Hypothesize there are two independent waves into the antenna array, which have same powers and almost same directions, and we have already known the number of waves is 2 through pre estimation. Although we can not divide the signals and the noise according to the eigenvalues, we still can find the directions of two waves. According to the literature [7], by selection of appropriate number $K\,(K>2)$, let $K=3$, MUSIC algorithm is still effective. There will be a false peak on the spectrum curve, but two real signals can be separated, and the false peak can be removed with posterior source number decision algorithm.

4) The effect of mistake pre estimation of the number of waves on direction finding. If the number

of waves K decided to construct the space spectral function is not same with the actual number of waves K_0, the peak number on the spatial spectrum curve will not equate to the actual number of waves.

If $K < K_0$, $K_0 - K$ eigenvectors which ought to belong to E_S will be put into E_N. In this case, $a(\theta_i)\,(i = K_0 - K + 1, \cdots, K_0)$ are not orthogonal with E_N, there are no peaks appearing at $\theta_i\,(i = K_0 - K + 1, \cdots, K_0)$, leading to omission of DOA. Meanwhile, the other direction vectors $a(\theta_i)$ $(i = 1, \cdots, K)$ are not completely orthogonal with E_N, the peaks deviate from the true directions of arrival, so the accuracy of direction finding is decreased.

If $K > K_0$, the number of eigenvectors is expanded, peaks may appear at the direction without waves, resulting in false alarm. Here, we need to remove the false peaks with posterior source number decision algorithm.

The above analysis indicates that, to improve the accuracy of direction finding, it is necessary to estimate the number of waves.

2.2. Necessity of Pre Estimation of Coherence of Waves

The MUSIC algorithm can find the directions of multiple independent waves with high precision and high resolution. However, it can not find the directions of coherent waves. In order to find the direction of coherent waves, we must implement processing to remove the coherence of waves. This shows that the approach used for independent waves and coherent waves are different among approaches of spatial spectrum estimation, therefore, we need to pre estimate coherence of waves, namely, we need to judge whether there are coherent waves in the presence.

If there are coherent waves in the presence, we even need to estimate the distribution of the independent waves and the coherent waves [8].

3. Matrix Decomposition Approach

3.1. Theory of Matrix Decomposition Approach

The matrix decomposition approach to decide number and coherence of waves is mainly to calculate the rank of the array covariance matrix pair $(R, \sigma_n^2\Sigma)$.

It can obtain not only the number of independent waves, but also the number of independent waves and the number of coherent waves when there are coherent waves.

Let the array covariance matrix be $R = AR_SA^H + \sigma_n^2\Sigma$, the array covariance matrix pair is

$$R_0 = (R, \sigma_n^2\Sigma) = AR_SA^H, \tag{1}$$

Generally, we get the matrix R rather than R_0. However, in the following cases, we can get R_0:

1) The noise covariance matrix $\sigma_n^2\Sigma$ is known, then $R_0 = R - \sigma_n^2\Sigma = AR_SA^H$;

2) SNR is high, $\sigma_n^2\Sigma$ is relatively small, R can directly replace R_0.

Usually, we assume that one of above two cases occurs, that is R_0 has already been known. In this paper, we assume that the second case occurs. Now, let us pre estimate the status of waves by calculating the ranks of matrixes which are related to R_0.

Write the formula (1) as

$$R_0 = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1N} \\ r_{21} & r_{22} & \cdots & r_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ r_{N1} & r_{N2} & \cdots & r_{NN} \end{bmatrix},$$

and mark $r = rank(R_0)$, $p = \lfloor (N + r)/2 \rfloor$.

For an integer $l\,(l = 0, 1, \cdots, N - p)$, row $l + 1$ to row $l + p$ of R_0 constitute a new matrix

$$R_0^{(l)} = \begin{bmatrix} r_{l+1,1} & r_{l+1,2} & \cdots & r_{l+1,N} \\ \cdots & \cdots & \cdots & \cdots \\ r_{l+p,1} & r_{l+p,2} & \cdots & r_{l+p,N} \end{bmatrix}, \tag{2}$$

There are $N - p + 1$ matrixes $R_0^{(l)}$ with order $p \times N$ in all. Construct $N - p + 1$ judgment matrixes according to $R_0^{(l)}$

$$W^{(l)} = \left(R_0^{(0)}, R_0^{(1)}, \cdots, R_0^{(l)}\right), (l = 0, 1, \cdots, N - p). \tag{3}$$

Theorem If there is an integer l_0 ($l_0 < N - p$) satisfying the following conditions:

$$\begin{aligned} & rank\left\{\left(R_0^{(0)}, R_0^{(1)}, \cdots, R_0^{(l_0)}\right)\right\} \\ &= rank\left\{\left(R_0^{(0)}, \cdots, R_0^{(l_0)}, R_0^{(l_0+1)}, \cdots, R_0^{(l_0+m)}\right)\right\} \\ &= K < p \quad (m = 1, 2, \cdots, N - p - l_0), \end{aligned} \tag{4}$$

the number of waves is K [4].

The theorem implies that for an integer l_0, $rank\left\{W^{(l_0)}\right\} = rank\left\{\left(R_0^{(0)}, R_0^{(1)}, \cdots, R_0^{(l_0)}\right)\right\} = K < p$, if $rank\left\{W^{(l)}\right\} = rank\left\{\left(R_0^{(0)}, \cdots, R_0^{(l_0)}, R_0^{(l_0+1)}, \cdots, R_0^{(l)}\right)\right\}$ does not increase with the integer l increasing from l_0, the number of waves is K.

3.2. The Steps of the Matrix Decomposition Approach

According to the theorem, the steps of the matrix decomposition approach are as following.

1) Calculate the array covariance matrix R_0 according to the inputs of the array.

2) Calculate the rank of R_0. Mark $r = rank(R_0)$, $p = \lfloor (N+r)/2 \rfloor$.

3) Make a series of matrix $R_0^{(l)}$ $(l = 0, 1, \cdots, \ N-p)$ according to formula (2).

4) Make a series of matrix $W^{(l)}$ $(l = 0, 1, \cdots, \ N-p)$ according to formula (3).

5) Increase l of formula $W^{(l)} = \left(R_0^{(0)}, \cdots, R_0^{(l)}\right)$ step by step, and calculate $rank\{W^{(l)}\}$.

6) Judgment criterion is as following:

(i) If there is an integer l_0 $(0 < l_0 < N-p)$ satisfying the following conditions:

$$rank\{W^{(l_0)}\} = rank\{W^{(l_0+m)}\} = K < p$$
$$(m = 1, 2, \cdots, N-p-l_0),$$

i.e., if $rank\{W^{(l)}\}$ does not increase with the integer l increasing from l_0 $(l_0 \neq 0)$, the number of waves is K, and there are coherent waves.

(ii) If $l_0 = 0, K = r$, and the waves are independent.

(iii) If $rank\{W^{(l)}\}$ increases until $rank\{W^{(l)}\} = p$ when the integer l increases from l_0, we can not decide the number K of waves by the array with N antennas, and $K > p-1$.

3.3. Notes

1) At the case (i) of the judgment criterion, the number of waves is K, whereas the rank of R_0 is $r = rank(R_0)$, so there are $K - r$ waves to be coherent with other waves.

2) At the case (ii) of the judgment criterion, K waves are independent each other.

3) At the case (iii) of the judgment criterion, the number and coherence of waves can not be decided by the array with N antennas. The number N of antennas must be increased to get the accurate estimation.

4. Simulation of Matrix Decomposition Approach

The following simulation experiments are based on the communication countermeasure simulation software we developed [9]. The channel is additive white Gauss noise (AWGN), the array is uniform linear with 8 antennae, and the rank of matrix is gotten by Gauss elimination method with threshold.

In the figures of simulation results, the parameters' meaning are as following: N is the number of antennas, $r = rank(R_0)$, $p = \lfloor (N+r)/2 \rfloor$, $ri = rank(W^{(i)})$ $(i = 0, 1, 2, 3)$, $ri = 0$ $(i = 0, 1, 2, 3)$ for program initialization. It is easy to get the judgment result in figures from these judgment parameters' values according to the judgment criterion.

Experiment 1. Pre estimation of two independent signals.

Two simulated signals are pulse modulated signals of equal amplitudes. One signal is 2ASK modulated, and another signal is BPSK modulated. Two signals are independent because of different modulation modes. The threshold value to find the rank of a matrix is 0.00001. The result of pre estimation is shown in Fig. 1. The pre estimation result completely conforms to the actual situation.

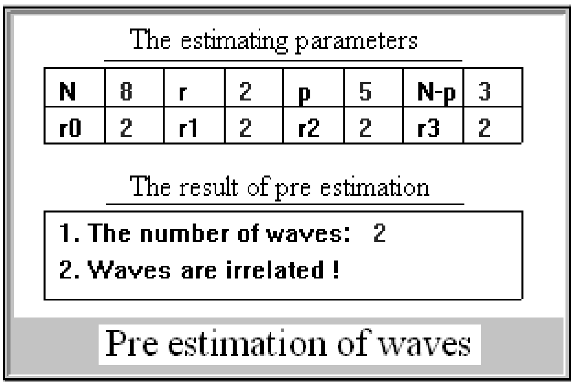

Fig. 1. The result of pre estimation of two independent signals.

Experiment 2. Pre estimation of three independent signals.

Three simulated signals are pulse modulated signals of unequal amplitudes. The first signal is 2ASK modulated, the second is BPSK modulated and the third is QPSK modulated. Because of different modulation modes, the three signals are independent. The relative amplitude of the first signal is 1.0, the relative amplitude of the second is 0.5, and the relative amplitude of the third is 0.35. The threshold value to find the rank of a matrix is 0.00001. The result of pre estimation is shown in Fig. 2. The pre estimation result is correct.

Experiment 3. Pre estimation of an independent signal and two coherent signals.

Three simulated signals are pulse modulated signals of unequal amplitudes. The first signal is 2ASK modulated, the second and the third are all BPSK modulated. Because of different modulation modes, the first signal is independent with the other two signals. Because of same modulation modes, the second signal is coherent with the third. The relative amplitude of the first signal is 1.0, the relative amplitude of the second is 0.9, and the relative amplitude of the third is 0.2. The phase of the third signal delays 15° with respect to the second. The threshold value to find the rank of a matrix is 0.001. The result of pre estimation is shown in Fig. 3. The pre estimation result is correct.

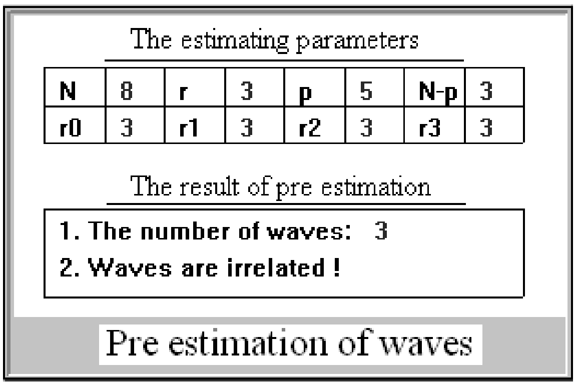

Fig. 2. The result of pre estimation of three independent signals.

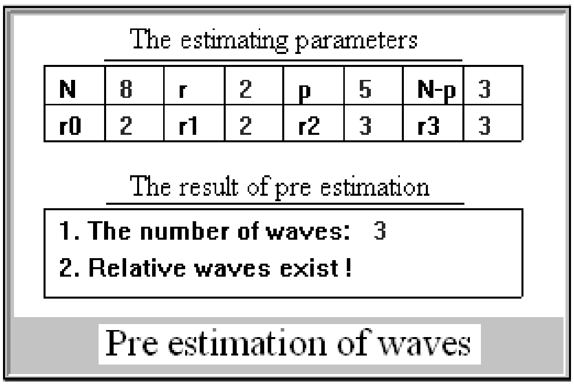

Fig. 3. The result of pre estimation of an independent signal and two coherent signals.

Experiment 4. Pre estimation of two groups of coherent signals.

Four simulated signals are pulse modulated signals of unequal amplitudes. The first and the second signal are all BPSK modulated, the third and the fourth signal are all QPSK modulated. The first signal is coherent with the second, and the third signal is coherent with the fourth. The relative amplitude of the first signal is 1.0, the relative amplitude of the second is 0.2, the relative amplitude of the third signal is 0.9, and the relative amplitude of the fourth is 0.3. The phase of the second signal delays 15° with respect to the first, and the phase of the fourth signal delays 37° with respect to the third. The threshold value to find the rank of a matrix is 0.001. The result of pre estimation is shown in Fig. 4. The pre estimation result is nearly consistent with the actual situation; however, the number of waves estimated is more than the actual number of waves.

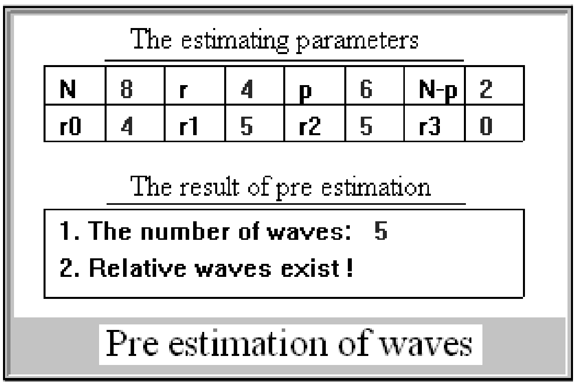

Fig. 4. The result of pre estimation of two groups of coherent signals.

5. Conclusions

In the process of pre estimation of waves, our intention is to estimate number and coherence of waves fast, simply and roughly, to have a preliminary understanding of waves, in order to determine the size of the array and the corresponding algorithm. The estimation precision need not be high. We can increase the number of array antennas appropriately when we design the array. The number of waves can be estimated a little more, experiment 4 is such a case. In the process of direction finding, we can remove spurious peaks later by posterior source number decision algorithm. From this point of view, the result of experiment 4 is significative.

There are many judging methods of number and coherence of waves, for example, AIC and MDL to determine number based on information theoretic criterion, smooth order column method, decision feedback method, and Gerschgorin radii method, etc. However, those methods need generally to estimate the angles jointly, i.e. we must pre estimate the angles of waves, and then estimate the number and the coherence of waves. If we use these complex methods to pre estimate the status of electromagnetic wave field, we will deviate from our intention of pre estimation. Therefore, we are not going to use those methods.

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