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Cosine-circular Partially Coherent Sources with Spiral Polarization Pattern

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Abstract: A type of partially coherent and non-uniformly totally polarized sources is proposed. This type of sources belongs to those with cosine-circular coherence, that is, they present a cosine-type dependence on the square of the distances of the points from the source center. On the other hand, these sources are endowed with a non-uniform polarization pattern across their transverse section. In particular, the electric field is linearly polarized at each point of the transverse section and symmetrically oriented with respect to the source center, so that the electric field lines form spirals. These sources will be denoted as “Spirally polarized sources with cosine-circular coherence”. Simple analytical expression for the generated field upon free propagation under paraxial conditions are derived. The intensity and polarization at a typical plane z=const. is studied. It is demonstrated that, under particular conditions, these type of fields remains spirally polarized and with a donut intensity profile at each plane z=const. A simple procedure of synthesis of such sources is also schematized.

Keywords: Polarization, Coherence, Propagation, Physical optics, Statistical optics.

1. Introduction

One of the topics that is receiving a lot of interest in recent years is the proposal of new physically realizable sources with special coherence and polarization characteristics. Several models of scalar partially coherent sources with peculiar coherence characteristics have recently been proposed [1-10]. Among them, sources with circular coherence are those described by a cross spectral density (CSD) function [11] that only depends on the difference of the moduli of the position vectors and have been shown to present self-focusing phenomena [4, 5, 7]. A particular case of these sources are those with cosine-circular coherence, that is, sources whose CSD has a functional form of cosine-type and a dependence on the square of the distances of the points from the source center [6]. In the scalar case, it has been demonstrated that the field generated from these sources presents interesting properties in propagation under paraxial conditions. For instance, the intensity of the propagated field changes and shows a near self-focusing behavior at a certain plane that, in general, does not coincide with the waist plane [4, 5, 7].

On the other hand, it has been shown that spirally polarized beams (SPBs) [12-17], which include radially and azimuthally polarized light as particular cases, are useful in some application fields, such as microscopy, particle manipulation, polarimetry, etc. (see for example, [18-29]). SPBs present linear polarization across the transverse beam section but the electric field forms a fixed angle with the radial direction at each point, so that the azimuth changes from one point to another. They belong to the so-called non-uniformly totally polarized beams [28-33].

Usually, a circularly symmetric intensity is considered for totally coherent SPBs, that is, their electric field amplitude depends only on the radial distance to the beam axis. A typical example of amplitude dependence is a Laguerre-Gauss function, in which case not only the polarization but also the intensity remains invariant in propagation, up to a scale factor. On the other hand, the case of partially coherent SPBs have been theoretically studied and experimentally synthesized for some particular examples (see for example [9, 13, 17]).

In this work, sources presenting cosine-circular coherence properties and spiral polarization pattern across the transverse plane will be considered. For these kind of sources, analytical expression for the cross spectral density (CSD) matrix at each transverse plane are derived. Special attention will be paid to the evolution of the intensity and the polarization pattern in free space propagation of the beam radiated from the source. It will be shown that, under certain conditions, the beam remains spirally polarized at each transverse plane but the intensity reaches a maximum value in a plane that will be different from that of the beam waist. Both the maximum intensity value and the position of the plane at which this maximum is reached depend on the coherence characteristics of the source.

The paper is structured as follows. This section constitutes the Introduction. The main characteristics of the source are presented in Section 2. In Section 3 the beam generated from the previous source is analyzed at different planes. A simple scheme of the experimental synthesis is given in Section 4 and finally the main conclusions are briefly exposed in Section 5.

2. Source Characteristics

Let us consider a source defined by a CSD matrix [34-36] of the form

$$\widehat{W}_0(\mathbf{r}_1,\mathbf{r}_2) = W_{SC}(\mathbf{r}_1,\mathbf{r}_2)\hat{P}(\varphi_1,\varphi_2), \qquad (1)$$

being $\mathbf{r}_j = (r_j,\varphi_j)$ with j=1, 2, two arbitrary points across the transverse source plane ($z = 0$) and r_1 and r_2 are the distances to the center of such a points. Here $W_{SC}(\mathbf{r}_1,\mathbf{r}_2)$ is a scalar CSD that for a partially coherent source with cosine-circular coherence is given by [6, 34]

$$W_{SC}(\mathbf{r}_1,\mathbf{r}_2) = f(\mathbf{r}_1)f^*(\mathbf{r}_2)\cos\left(\frac{r_1^2-r_2^2}{\delta^2}\right), \qquad (2)$$

where δ is a length factor related to the coherence. The function $f(\mathbf{r})$ is, in general, a complex-valued function that depends on the radial and angular coordinates (r,φ). In order to be compatible with a spiral polarization state, this function must vanish at origin r=0. In the present work, $f(\mathbf{r})$ functions that only depend on the radial coordinate r will be considered.

On the other hand, the polarization matrix $\hat{P}(\varphi_1,\varphi_2)$ [28, 33] for a spirally polarized source only depends on the angular coordinate and is given by

$$\hat{P}(\varphi_1,\varphi_2) = \mathbf{\Phi}(\varphi_1)\mathbf{\Phi}^\dagger(\varphi_2), \qquad (3)$$

with

$$\mathbf{\Phi}(\varphi) = \begin{pmatrix} -\sin(\varphi+\alpha) \\ \cos(\varphi+\alpha) \end{pmatrix}, \qquad (4)$$

where α is the angle of the electric field vector with respect to the line tangent to the circle of radius r [12-17].

The CSD given in Eq. (1), fulfils the non-negativeness condition, and therefore represents a physically realizable source [34-36]. Here and in the following the explicit dependence on the temporal frequency will be omitted. Sources described by Eq. (2), which are circularly symmetric and exhibit a cosine-type dependence on the square of the distances of the points from the source center will be called spirally polarized sources with cosine-circular coherence.

From the CSD matrix, several relevant spatial characteristics of the source can be derived. For example, the intensity is found as

$$I_0(\mathbf{r}) = \mathrm{tr}\{\widehat{W}_0(\mathbf{r},\mathbf{r})\} \qquad (5)$$

Taking into account that for a spirally polarized beam,

$$\mathrm{tr}\{\widehat{W}_0(\mathbf{r},\mathbf{r})\} = \mathrm{tr}\{\hat{P}(\varphi,\varphi)\} = 1, \qquad (6)$$

the intensity is completely determined by the function $f(\mathbf{r})$ and results

$$I_0(\mathbf{r}) = |f(\mathbf{r})|^2 \qquad (7)$$

On the other hand, the coherence properties of the source can be described by its spectral degree of coherence. For the case of an electromagnetic field, this parameter can be obtained as [38-40]

$$\mu_0^2(\mathbf{r}_1,\mathbf{r}_2) = \frac{\mathrm{tr}\{\widehat{W}_0(\mathbf{r}_1,\mathbf{r}_2)\,\widehat{W}_0(\mathbf{r}_2,\mathbf{r}_1)\}}{\mathrm{tr}\{\widehat{W}_0(\mathbf{r}_1,\mathbf{r}_1)\}\,\mathrm{tr}\{\widehat{W}_0(\mathbf{r}_2,\mathbf{r}_2)\}} \qquad (8)$$

The polarization pattern superimposed over the intensity profile is shown in Fig. 1 for the particular case of $\alpha=\pi/4$ and

$$f(r) = A\, r \exp\left(-\frac{r^2}{\omega_0^2}\right), \tag{9}$$

where ω_0 is the width of the Gaussian function and A is a constant. For this particular choice, the intensity shows a donut shape with a maximum value on a circle of radius:

$$r_M = {}^{\omega_0}/_{\sqrt{2}} \tag{10}$$

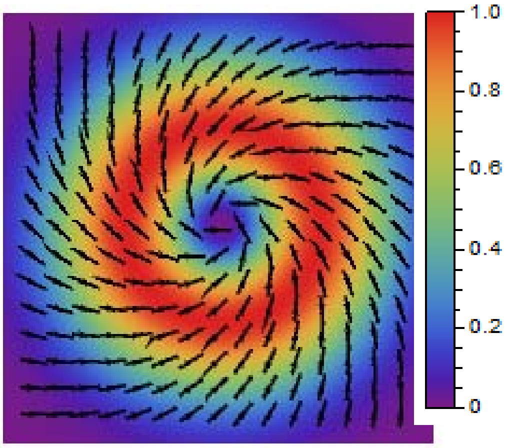

Fig. 1. Intensity profile (color scale) and polarization pattern (straight segments) of a SPB for $\alpha=\pi/4$.

In Fig. 2, the absolute value of the spectral degree of coherence for $\delta/\omega_0=1$ is represented across the (r_1, r_2) plane. It can be observed that, up to a possible phase factor, the degree of coherence is determined by the function

$$\mu_0(\mathbf{r}_1, \mathbf{r}_2) = \cos\left(\frac{r_1^2 - r_2^2}{\delta^2}\right) \tag{11}$$

Then, two points in the source plane are completely coherent if they are on the same circle centered at the source center. Moreover, all points belonging to concentric circles that satisfy:

$$\frac{r_1^2 - r_2^2}{\delta^2} = n\pi, \tag{12}$$

with n integer, are completely coherent. On the other hand, points laying on the set of concentric circles to the source center where the following condition is fulfilled

$$\frac{r_1^2 - r_2^2}{\delta^2} = \left(n + \frac{1}{2}\right)\pi, \tag{13}$$

are completely incoherent.

These facts can be appreciated in Fig. 2. The area close to the diagonal ($r_1=r_2$) and the regions around points that satisfy the condition given by Eq. (12), present a high absolute value of the spectral degree of coherence, while for points satisfying the condition given by Eq. (13), the spectral degree of coherence is zero.

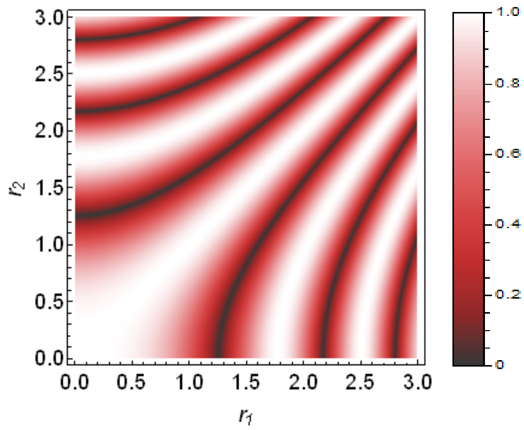

Fig. 2. Absolute value of the spectral degree of coherence at the source plane for $\delta=\omega_0=1$.

Fig. 3 shows the absolute value of the spectral degree of coherence across the source plane relative to the ring where the intensity reaches its maximum (the circle with radius r_M). The degree of coherence is high, not only for points with r_2 close to r_M, but also for points around concentric circles with radii satisfying Eq. (12). Then, two points can be very far away, but they will remain completely coherent if they satisfy such a condition.

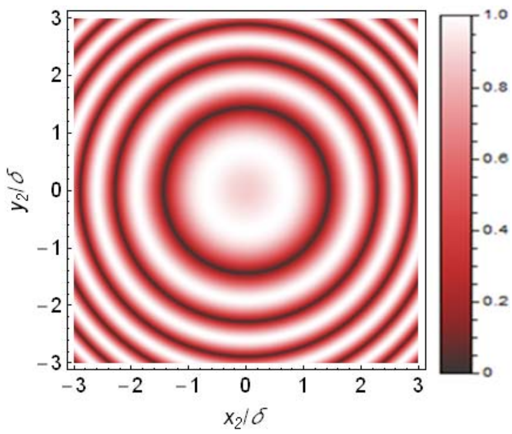

Fig. 3. Absolute value of the spectral degree of coherence across the source plane for $x_1 = \omega_0/\sqrt{2}$ and $\delta=\omega_0=1$.

On the other hand, it can be appreciated that there is second set of concentric circles for which the degree of coherence is zero. These are the circles with radii r_2 that fulfill the condition in Eq. (13). As a result, two points can be as close as a dark circle is to the next bright circle, and they are completely incoherent, whereas diametrically opposite points are completely coherent.

The behavior of the absolute value of the spectral degree of coherence with the distance r_2 to the source center has been represented in Fig. 4 for several values of r_1.

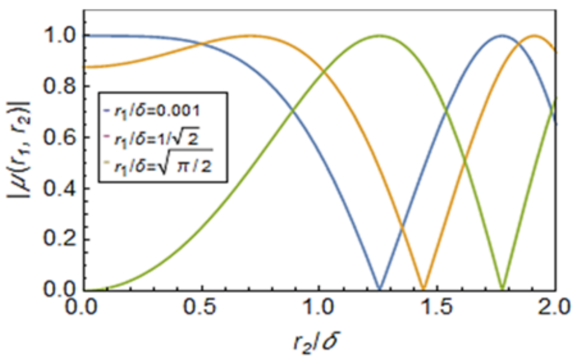

Fig. 4. Absolute value of the spectral degree of coherence with $\boldsymbol{r_2}$ for several values of $\boldsymbol{r_1}$ at the source plane with $\delta=\omega_0=1$.

The first value correspond to a point very close to the source center (blue line); the second one to the distance where the intensity reaches its maximum (orange line); and the last one to the smallest distance from the source center that satisfy Eq. (13) (green line). The first and the last cases can be considered as complementary in the sense that the sum of their squared values result the unity.

From Fig. 3 and Fig. 4, it is clear that the spectral degree of coherence experiences a continuous and repetitive variation from +1 to –1 and then again to +1, across the source plane.

3. Propagation

Let us assume that the radiated field from the source propagates mainly along the *z*-axis perpendicular to the source plane. The CSD matrix of the propagated beam under paraxial approximation at a distance *z* from the source can be obtained by means of the Fresnel diffraction integral [11] of every element of the matrix in Eq. (1).

$$\widehat{W}(\mathbf{R}_1,\mathbf{R}_2,z)=\frac{1}{\lambda^2 z^2}\iint \widehat{W}_0(\mathbf{r}_1,\mathbf{r}_2)\exp\left[\frac{\mathrm{i}k}{2z}\left(|\mathbf{R}_2-\mathbf{r}_2|^2-|\mathbf{R}_1-\mathbf{r}_1|^2\right)\right]\mathrm{d}\mathbf{r}_1\mathrm{d}\mathbf{r}_2, \tag{14}$$

where $\mathbf{R}_j=(R_j,\ \phi_j)$, with j=1,2, are the position vectors of two points across the plane z = constant and $k=2\pi/\lambda$ being λ the wavelength.

It has been demonstrated that when the function $f(\mathbf{r})$ only depends on the radial coordinate, which implies circular symmetry for $\widehat{W}_0(\mathbf{r}_1,\mathbf{r}_2)$ the propagation integral can be expressed in the simple form [13]

$$\widehat{W}(\mathbf{R}_1,\mathbf{R}_2,z)=W_{SC}(\mathbf{R}_1,\mathbf{R}_2,z)\hat{P}(\phi_1,\phi_2), \tag{15}$$

where the scalar factor of the CSD matrix is

$$\begin{aligned} W_{SC}(\mathbf{R}_1,\mathbf{R}_2,z)=&\frac{1}{\lambda^2 z^2}\exp\left[\frac{\mathrm{i}\,k}{2z}\left(R_2^2-R_1^2\right)\right]\iint f(r_1)f^*(r_2)\cos\left(\frac{r_1^2-r_2^2}{\delta^2}\right)\exp\left[\frac{\mathrm{i}\,k}{2z}\left(r_2^2-r_1^2\right)\right]\\ &\times J_1\left(\frac{kr_1R_1}{z}\right)J_1\left(\frac{kr_2R_2}{z}\right)r_1r_2\mathrm{d}r_1\mathrm{d}r_2, \end{aligned} \tag{16}$$

being $J_1(\cdot)$ the Bessel function of the first kind and first order [41]. The matrix $\hat{P}(\phi_1,\phi_2)$, is given by Eqs. (3) and (4). This means that the propagated beam remains spirally polarized but the irradiance changes at each plane. In order to show the intensity upon free propagation let us consider as an example the function $f(r)$ given by Eq. (7). In this case, Eq. (16) can be solved analytically resulting the following expression for the scalar cross spectral density function,

$$\begin{aligned} W_{SC}(\mathbf{R}_1,\mathbf{R}_2,z)=&\frac{A^2R_1R_2}{2}\exp\left[\frac{\mathrm{i}}{\omega_0^2 z_R}\left(R_2^2-R_1^2\right)\right]\\ &\times\left\{\frac{1}{B_-^2}\exp\left[\frac{\mathrm{i}(R_2^2-R_1^2)}{B_-\,\omega_0^2 z_R}\left(1+z_R\frac{\omega_0^2}{\delta^2}\right)-\frac{R_1^2+R_2^2}{B_-\omega_0^2}\right]\right.\\ &\left.+\frac{1}{B_+^2}\exp\left[\frac{\mathrm{i}(R_2^2-R_1^2)}{B_+\,\omega_0^2 z_R}\left(1+z_R\frac{\omega_0^2}{\delta^2}\right)-\frac{R_1^2+R_2^2}{B_+\,\omega_0^2}\right]\right\}, \end{aligned} \tag{17}$$

where

$$B_{\pm} = \left(1 \pm z_R \frac{\omega_0^2}{\delta^2}\right)^2 + z_R^2, \tag{18}$$

and

$$z_R = \frac{2z}{k\omega_0^2} \tag{19}$$

Although the CSD preserves its circular symmetry during propagation, irradiance profile changes in general and coherence characteristics present a more complicated dependence on radial distances. For example, in Fig. 5, it can be observed that the absolute value of the spectral degree of coherence presents a similar pattern than at the source plane, but after propagation, the minimum values are no longer zero for all the distances difference that satisfy Eq. (13).

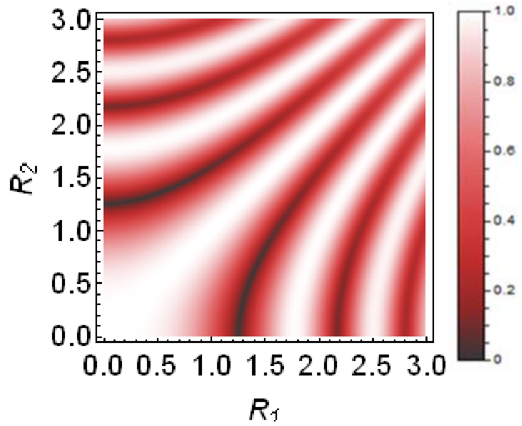

Fig. 5. Absolute value of the spectral degree of coherence at a distance $z = 0.02z_R$ from the source plane for $\delta = \omega_0 = 1$.

The intensity profile of the propagated field can be easily obtained by using Eq. (5). Due to the polarization invariance for the propagated field, Eq. (6) holds at each z-plane, so a simple expression for the intensity can be found.

$$I(\mathbf{R}, z) = \frac{A^2R^2}{2}\left\{\frac{1}{B_-^2}\exp\left(-\frac{2R^2}{B_-\,\omega_0^2}\right) + \frac{1}{B_+^2}\exp\left(-\frac{2R^2}{B_+\,\omega_0^2}\right)\right\} \tag{20}$$

Fig. 6, shows the intensity across the beam transverse section at several propagation distances, z/z_R, in terms of normalized length, z_R, with $\delta/\omega_0=0.25$. As it can be seen from Eq. (20), the propagated intensity remains with a null in the beam center.

It can be observed that the profile is sharper (it has a smaller full width at half maximum) outside the source plane.

In Fig. 7, the intensity profile along the normalized radial coordinate R/ω_0 is shown at different z planes. In this figure we can observe that the beam preserves its donut shape in propagation. However, there is a plane where the beam reach a maximum focus that does not coincide with the source plane (z=0 plane). At that plane, the intensity profile becomes quite sharper and with a quite larger peak value than at the source plane.

Fig. 8 shows the behavior of the intensity at different points across the transverse section of the beam and its evolution with the propagation distance z/z_R. It can be observed that the intensity is zero along the beam axis.

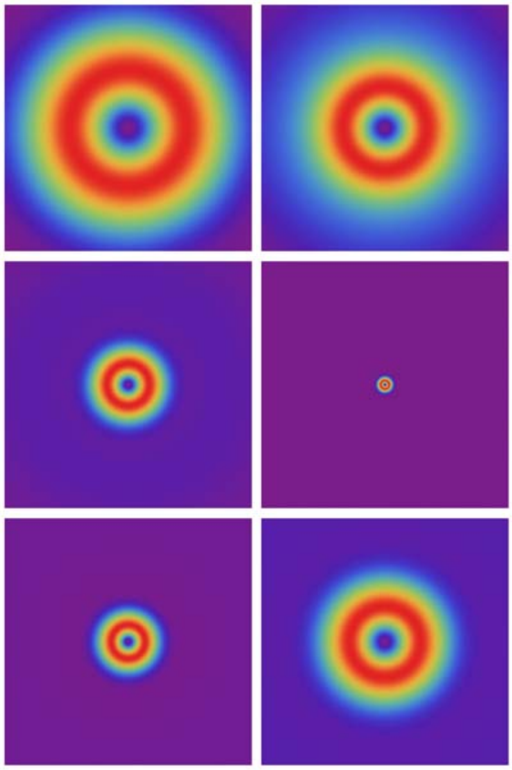

Fig. 6. Evolution of the intensity for $\delta/\omega_0=0.25$ and different propagation distances z/z_R= 0, 0.02, 0.04, 0.06, 0.8 and 0.1 from left to right and top to bottom. For all cases, the square side length is $3\omega_0$.

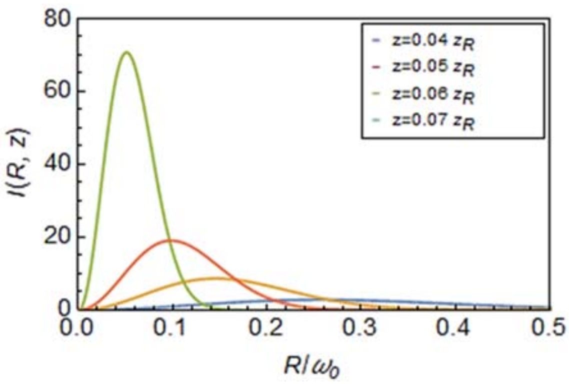

Fig. 7. Intensity profile (in A^2 units) with $\delta/\omega_0=0.5$ versus R/ω_0 for different propagation distances z/z_R.

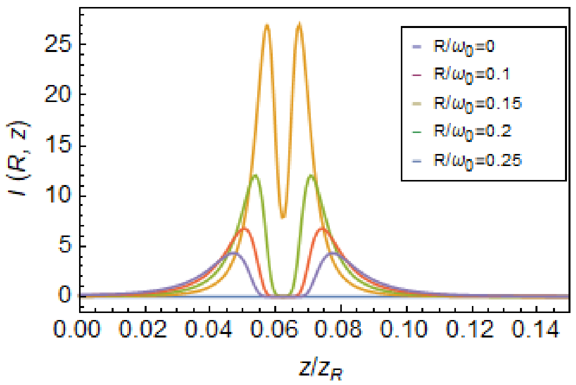

Fig. 8. Intensity (in A^2 units) with δ/ω_0=0.5 versus z/z_R at different distances from the beam center.

On the other hand, the changes of the CSD matrix and the intensity in propagation depend on the parameters of the source. In Fig. 9, the intensity for the particular value R/ω_0=0.1 has been plotted versus the propagation distance, z/z_R, for several values of the coherence parameter δ/ω_0. It can be observed that the intensity shows a maximum that occurs after a given propagation distance and not at z=0 (see [4-7] and references therein). As can be seen from this figure, as δ/ω_0 decreases two relative maxima of the intensity appear in different planes and with higher values for higher values of δ.

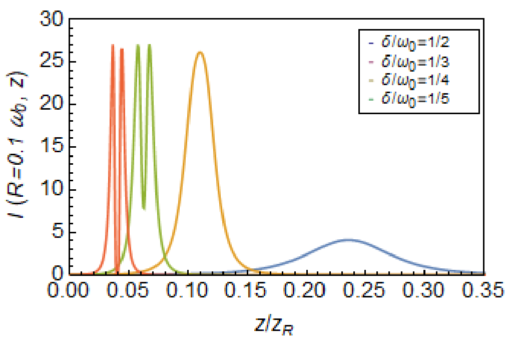

Fig. 9. Intensity (in A^2 units) at R/ω_0=0.1 versus z/z_R. for different values of δ/ω_0. $I(R$=0, z=0)=1.

4. Proposal of Synthesis

In order to synthesize the source given by Eq. (1) and taking into account the separability of the CSD matrix (see Eq. (15)) we can first synthesize the partially coherent source using a linear and uniformly polarized beam (scalar CSD) and after that, we endow the source with a spiral polarization pattern across the transverse plane of the source.

A possible way to synthesize the scalar source given by Eqs. (2) and (9) in a simple way at z=0 plane, can be envisaged if Eq. (2) is written as

$$W_{SC}(\mathbf{r}_1,\mathbf{r}_2) = g(\mathbf{r}_1)g^*(\mathbf{r}_2) + g^*(\mathbf{r}_1)g(\mathbf{r}_2), \quad (21)$$

with the auxiliary function $g(\mathbf{r})$, for the case of circular symmetry, given by

$$g(\mathbf{r}) = Ar\exp\left(-\frac{r^2}{\omega_0^2}\right)\exp\left(-\mathrm{i}\frac{r^2}{\delta^2}\right), \quad (22)$$

where the Euler formula has been used to rewrite Eq. (2).

Eq. (21) represents the superposition of two beams with different phase factors, which means their waists are positioned in different planes. This can be though as the incoherent superposition of two beams with amplitude profile given by Eq. (9) and different radius of curvature at z=0 plane.

Afterwards, to provide the source with the polarization characteristics, both sources must be spirally polarized with the same α angle. This can be done, for example, by means of a polarization converter [16, 17, 23, 25, 28]. Due to the separability of the CSD matrix given in Eq. (15), the propagated beam will remain spirally polarized.

The propagated field for any z plane is given by Eq. (15) where the expression in Eq. (16) can be replaced by

$$W_{SC}(\mathbf{R}_1,\mathbf{R}_2,z) = G(\mathbf{R}_1,z)G^*(\mathbf{R}_2,z) + G^*(\mathbf{R}_1,z)G(\mathbf{R}_2,z), \quad (23)$$

where $G(\mathbf{R}_j,z)$ with j=1,2, takes the simple form

$$G(\mathbf{R}_j,z) = \frac{1}{\lambda^2 z^2}\exp\left(\frac{\mathrm{i}\,k}{2z}R_j^2\right)\iint g(r_j)\exp\left(\frac{\mathrm{i}\,k}{2z}r_j^2\right)J_1\left(\frac{kr_jR_j}{z}\right)r_j\mathrm{d}r_j \quad (24)$$

Eqs. (21) and (22) allow to calculate analytically in a simple way the propagation of the intensity at any z=const. plane and the results are the same as that given in Eq. (20).

5. Conclusions

Spirally polarized and partially coherent sources with cosine-circular coherence have been introduced. The degree of coherence between two points at the source plane depends only on the difference between the squared radial coordinates of the two points. The proposed sources have a non-uniformly polarization pattern across the beam transverse section, they are spirally polarized. Particular cases are radially and azimuthally polarized sources. In order to be compatible with a spiral polarization pattern the amplitude must vanish at r=0, that is, it must be donut-type.

Analytical expression for the CSD matrix of the propagated field has been derived. The synthesized beam remains spirally polarized invariant upon free propagation and with a null at the beam intensity center. Moreover, the intensity reaches a maximum value for a given propagation distance that does not coincide with the beam waist (located at the source plane). At such plane, the intensity profile is sharper than at the source plane. Then, by choosing the plane adequately, it is possible to have a focused spirally polarized beam.

It should be noticed that due the invariance polarization characteristics and the donut-type intensity this kind of beams can be useful in applications such as polarimetry, particle manipulation, free space communications, self-focusing, etc.

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