Sensors & Transducers
Published by IFSA Publishing, S. L., 2021
http://www.sensorsportal.com

Parallel, Multi-Channel Frequency & Time Measurements, with Picosecond Resolution

S. Johansson
Pendulum Instruments, Lotnicza 37, 80-297 Banino, Poland
Tel.: +48 (58) 681 89 01
E-mail: staffan.johansson@pendulum-instruments.com

Received: 27 October 2021 /Accepted: 6 December 2021 /Published: 30 December 2021

Abstract: A new approach to combine the performance of traditional Universal Frequency Counters (UFC) and high performance Time Interval Analyzers (TIA) in a single instrument in a portable benchtop format. For the first time in the industry, a UFC with all traditional timer/counter functions and full input signal dynamic range to 100 Vp-p has up to 5 parallel timestamping channels for frequency, period, time, or phase measurements. Advanced TIA functionality is included, like continuous timestamping with 7 ps resolution, acquisition speed of 20 MSa/s of complete measurement results, multi-stop Time Interval measurements, and continuous streaming of time stamped trigger events. Resolution enhancement techniques improve basic resolution up to 0.8 ps/Gate_Time for Frequency using linear regression, and up to 3.5 ps/Timestamp (Single-shot) using parallel multi-channel timestamping on a single input signal. This project was developed by Pendulum Instruments, Banino, Poland in close cooperation with the Military University of Technology, Warsaw, Poland.

Keywords: Zero dead-time, Multi-channel, Parallel timestamping, TIA, Frequency Analyzer, Resolution enhancement.

1. Introduction

Universal Frequency Counters and Time Interval Analyzers are two classes of instruments that have been around for many years, but they have had different typical characteristics. See Table 1.

Traditional Benchtop Universal Frequency Counters have been used in the past 50+ years as a standard tool on the R&D bench, as a component in test systems, as a precision frequency meter in metrology labs, and as a portable tool for field maintenance and calibration of frequency sources.

In the latest 20 years a new class of very-high resolution continuous time-stamping Time Interval Analyzers (TIA) in various board-level formats (PXI, PCI, PCIe) have emerged, intended for specialized applications, like semiconductor test, fast serial bus test, time-of-flight in mass spectrometry and photonic research, with a limited input signal range of typically <5 V.

A few projects have resulted in TIA performance also in stand-alone bench-top cabinets. For example, [1]).

The project described in this paper is the industry's first combination of a high-performance 7 ps TIA, and a full-featured Multi-Channel Universal 400 MHz / 24 GHz Frequency Counter/Analyzer.

Special resolution enhancement techniques enable an up to 12-fold improvement of frequency resolution and an up to 2-fold improvement of single-shot time resolution.

Table 1. *Typical* characteristics for traditional high-performance Universal Frequency Counters (UFC) and Time Interval Analyzers (TIA).

Item	UFC	TIA
Cabinet	Benchtop	Card or 19" card cabinet
Usage	Bench, Field, or ATE	Lab or ATE (special card cases)
Stand-alone operation	Yes	No
Computer controlled operation	Yes	Yes
Input Voltage	-50 V to +50 V	-5 V to +5 V
Suited for Sine signals	Yes	No
Suited for Pulse signals	Yes	Yes
Resolution	20-100 ps	1-20 ps
Multiple parallel inputs	No	Yes
Fast zero-dead-time timestamping	No	Yes
Multi-stop Time Interval	No	No

This article is based on the presented paper (ID: 6) at the 3rd IFSA Frequency & Time Conference (IFTC'2021), 22-24 September 2021, Palma de Mallorca, Spain, with some added contents, by the same author, [8].

1.1. Key Characteristics

The project called CNT-100 (Fig. 1) has the following performances:

- 5 parallel input channels with independent timestamping of input events, using common time scale;
- Housed in ½ 19" Benchtop cabinet;
- 7 ps resolution per timestamp;
- 20 MSa/s of data capture to internal memory;
- Wide input voltage range from 15 mVrms sine to ± 50 V pulses;
- All traditional Universal Frequency Counter/Analyzer measurements plus all common TIA measurements;
- Multi-stop Time Interval measurements;
- Frequency measurements to 24 GHz, other measurements from 0.001 Hz to 300 or 400 MHz.

The CNT-100 can display the numeric values of up to four separate inputs in parallel. Each input is color coded, see Fig. 2.

A second display view is the statistics parameter screen of all four input signal sources, showing mean value, maximum and minimum values, peak-to peak values, standard deviation, and Allan deviation. See Fig. 3.

Fig. 1. CNT-100 Multi-channel Frequency Analyzer from Pendulum Instruments.

Fig. 2. Simultaneous display of four 10 MHz input signals (Numeric frequency values plus voltage levels).

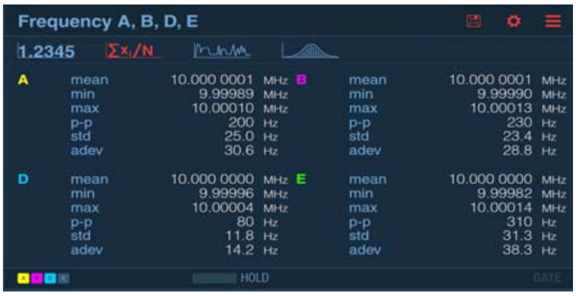

Fig. 3. Simultaneous display of four 10 MHz input frequencies (Statistics parameters).

A third display view is the timeline graphics display of the frequency values over time. Statistics parameters are shown in small text under the graph, Fig. 4.

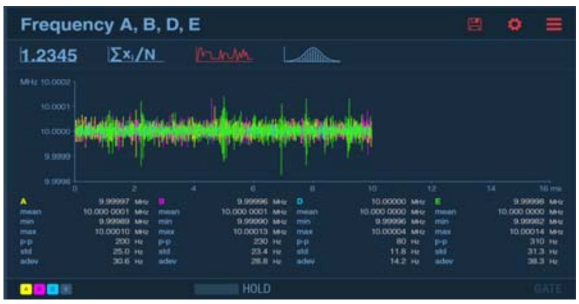

Fig. 4. Simultaneous display of four 10 MHz input frequencies (value vs. time).

The fourth and final display view of the same data set, is the graphics display of value distribution, for each input signal. Statistics parameters are shown in small text under the graph, Fig. 5.

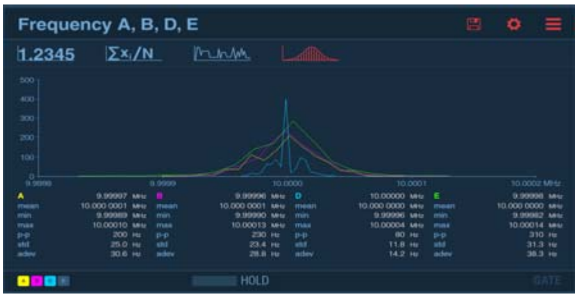

Fig. 5. Simultaneous display of four 10 MHz input frequencies (frequency distribution diagram).

2. Design

2.1. Block Diagram

A simplified block diagram of the design is shown in Fig. 6.

The **Input Signal Conditioning block**, see Fig. 7, contains all necessary analog circuitry to handle all types of input signals, whether clean or noisy, whether sine or pulse shaped, whether low or high amplitude, and to create a well-defined trigger event at the desired trigger level between -50 V and +50 V.

This blocks contains controls for:

- 50 Ω or 1 MΩ input impedance;
- AC or DC coupling;
- Attenuation ×10 or ×1;
- Trigger level settings;
- Noise reduction filters;
- Trigger sensitivity settings.

The outputs of the block are digital signals with well-defined levels, and a fast edge that coincides with the set trigger point.

Every 400 MHz input signal conditioning block is terminated with 2 independent comparators, to allow for standard pulse measurements like pulse width (trigger levels set on opposite edges at the 50 % midpoint of the slopes) or rise time (trigger levels set on rising edge at 10 % and 90 % of the slope amplitude).

2.2. Parallel Frequency Measurements

The **5-channel Digitizer block**, see Fig. 8, contains 5 fully independent time-stamping units that will timestamp incoming trigger events on up to 5 inputs in parallel. The cross-point switch, Fig. 7, selects which input channel(s) to time stamp. Any input signal can be routed to any of the 5 digitizer.

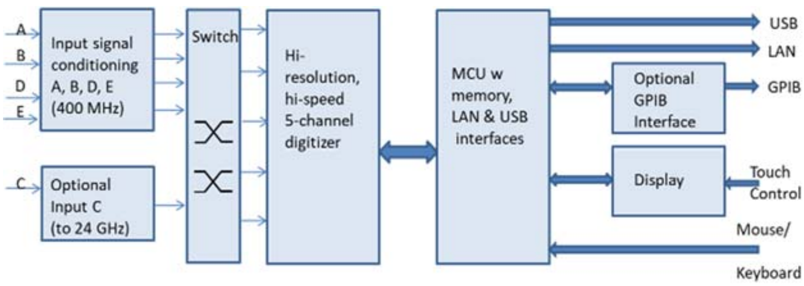

Fig. 6. Block diagram of CNT-100 (simplified).

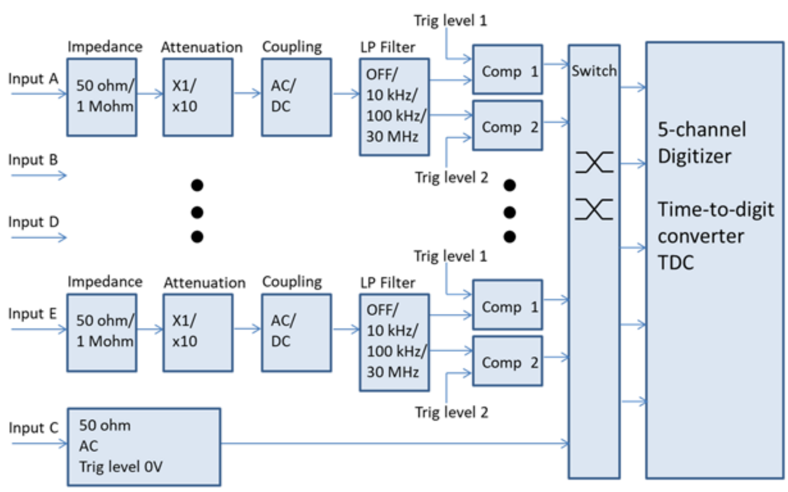

Fig. 7. Input signal conditioning.

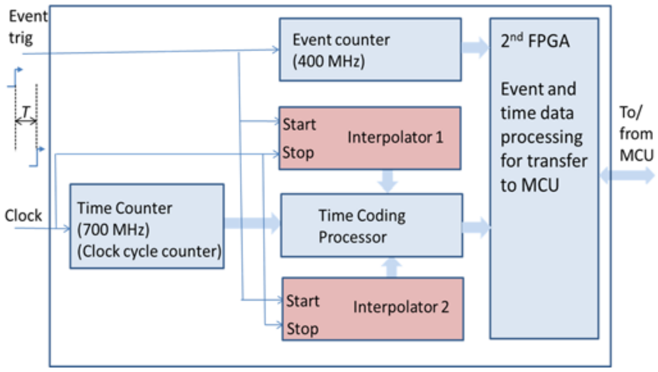

Fig. 8. Simplified block diagram of the Digitizer or Time-to-Digit-Converter block (1 channel).

The time between time stamps, commonly known as "gate time" for frequency measurements, is controlled by a pacing clock (not shown in Fig. 6) that enables capture and storage of the current number of accumulated trigger events E_i (number of input cycles), and the relative time at each capture TS_i.

The actual Gate Time is synchronized with the Input trigger Events, ensuring an exact number of input cycles between time stamps. The actual gate time can therefor differ from the set pacing time.

The Time count registers accumulates the amount of 700 MHz clock cycles, with a cycle time of 1.4 ns. This is the basic quantization resolution of the time stamp of trigger events. The interpolators improve that resolution by a factor of 200 down to 7 ps, by also measuring the fractional clock pulse, via a modified digital interpolation method, with dual Time Coding Lines as described in [1] and [2].

Fig. 9, showing the principle of a Time Coding Line, is copied from [2].

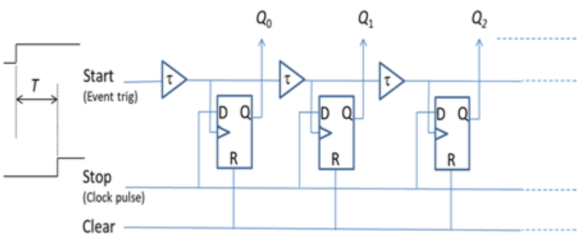

Fig. 9. Principle of the digital interpolating Time Coding Line (TLC).

The trigger event (start) triggers a chain of delay elements and D-flip-flops, with approx. 10 ps delay per delay element. The flip-flops are triggered one-by-one, and as long as the clock pulse (the stop) is in the zero state the corresponding outputs will be zero. At the clock pulse edge, the state changes from zero to one, and all subsequent flip-flops will show ones at the outputs. When all flipflops in the chain have been triggered, you could read the fractional clock pulse on the D-flipflops' outputs as a thermometer scale, all zeroes followed by all ones.

The nominal delay per delay element is 10 ps, which is also the digital interpolator's resolution step, but with quite wide variations from element to element. By using two Time Coding Lines, you could treat them as uncorrelated interpolators, thereby doubling the number of interpolation steps. The effective interpolator resolution by using dual Time Coding Lines is 7 ps.

Since the spread in the delay elements when implemented in an FPGA is wide, this interpolator design needs to be carefully calibrated before use. The CNT-100 uses the calibration principle of applying a multitude of asynchronous pulse edges to the inputs, with a random distribution within a clock pulse period, to define the transfer function, see [2].

All measurements are back-to-back, or gap-free, since the trigger event that marks the stop of one gate time, automatically is the start of the following gate time. See Fig. 10. The pacing clock has a maximum speed of 20 MHz, which means that the minimum gate time is 50 ns.

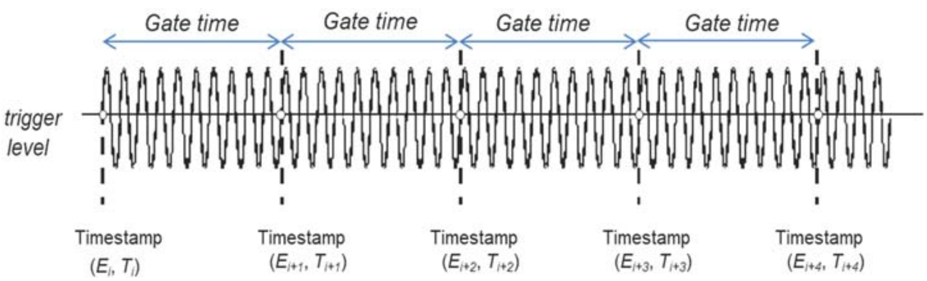

Fig. 10. Continuous zero-dead-time, gap-free, frequency measurement.

The Event data (E), with zero uncertainty (an exact number of input cycles) and Time Stamp data (TS), with 7 ps uncertainty, are transferred to the MCU which calculates frequency on-the-fly for each Gate Time period as:

$$Freq(i, i-1) = \frac{E_i - E_{i-1}}{TS_i - TS_{i-1}} \quad (1)$$

2.3. HW Configurations for Various Measurements

The CNT-100 internal HW configuration can be re-programmed to handle various types of measurement functions, either involving multiple independent parallel one-input-signal measurements, like 4x Frequency/Period, or four-input-signal inter-dependent measurements like Time Interval with one common start event and three stop events.

The basic configuration is for Frequency and Period Average, where each digitizer is connected to one of four input signals. See Fig. 11.

The comparators in the input signal conditioning stage are set to 50 % of the input signal amplitude with a positive trigger slope setting. The frequency is calculated as in Eq. (1), and the average period is calculated as the reciprocal value; $Freq^{-1}$.

The same configuration is also used for single Period measurements >50 ns.

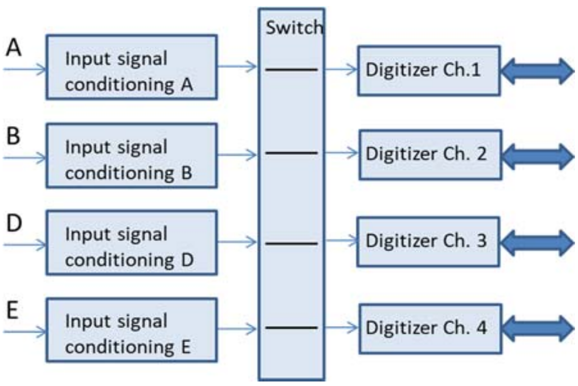

Fig. 11. Configuration for Frequency and Period Average.

For short single-shot periods from 2.5 to 50 ns, another architecture is used. The input signal is routed to two digitizers, one that timestamps the start and the other that timestamps the stop of the period. One or two input signals could be measured in parallel. See Fig. 12.

To make sure that the two digitizers are not timestamping the same trigger event, the start edge is enabling the stop edge, as shown in Fig. 12.

The comparators in the input signal conditioning stage are set to 50 % of the input signal amplitude with a positive trigger slope setting. The single period is calculated as the difference of Timestamp values between Digitizer *n* and Digitizer *n*-1, or

TS(Digitizer 2) – TS(Digitizer 1)
TS(Digitizer 4) – TS(Digitizer 3)

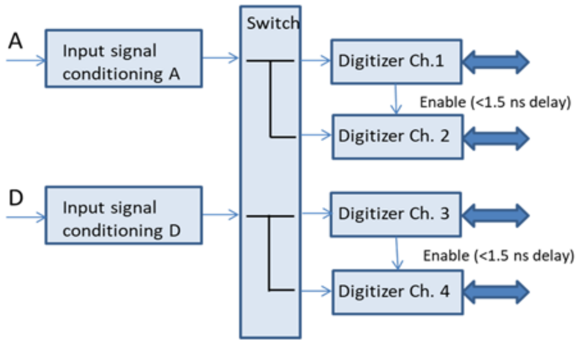

Fig. 12. Configuration for Single-shot Period measurements <50 ns.

Single-shot Time Interval measurements has yet another configuration, see Fig. 13, where input A defines the start trigger event, and inputs B, D and E defines the stop trigger events.

Each input signal is connected to its own digitizer for timestamping of the respective trigger event. To make sure that the input A signal defines the start, the start channel digitizer will enable timestamping of the other signals, the stop signals.

The comparators in the input signal conditioning stage can be set to any trigger level as required, with either positive or negative trigger slope setting. The three stop time intervals since the start trigger event, are calculated as:

TS(Digitizer 2) – TS(Digitizer 1)
TS(Digitizer 3) – TS(Digitizer 1)
TS(Digitizer 4) – TS(Digitizer 1)

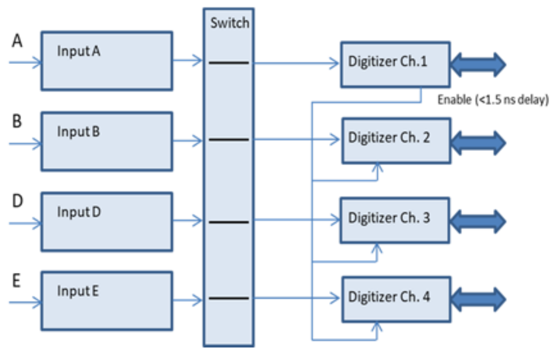

Fig. 13. Configuration for Single-shot Multi-stop Time Interval measurements.

3. Basic Measurement Uncertainty

3.1. Frequency Measurements

There are several contributing uncertainty elements in a frequency measurement. The most important being:

- Quantization error including interpolators, which is normally referred to as “resolution”;
- Noise processes in the electronics, both internal and external;
- Temperature, which can cause offsets in the timebase oscillator, and the calibration of interpolators;
- Ageing of the timebase reference oscillator.

By tradition, the first two error factors are called random errors, and the last two are called systematic errors.

The *random errors* results in stochastic variations of the measured value, with the assumption that you could approximate the distribution as Gaussian. The resolution and noise trigger errors are therefore normally expressed as “rms-values”.

By making N measurements (observations) and calculate the mean value you can reduce the effect of these errors with $\sqrt{N}$.

The effect of systematic errors cannot be reduced by averaging N measurements, hence the name “systematic”. These factors are normally expressed as being inside a given boundary. For example: temperature effect on a timebase oscillator is ±1E-10 (0 to +50 degrees), or ageing per month is <5E-11.

The resolution and noise trigger are Type A components, and the Timebase oscillator offset due to temperature and ageing are Type B components according to [4].

The combined measurement uncertainty U of the relative random uncertainty (u_{rnd}) and the relative systematic uncertainty (u_{syst}) is shown in Eq. (2), under the assumption that the effect of all contributing error factors are uncorrelated [4], p. 19 eq. 11). If they are correlated, the expression is more complex ([4], p. 21).

$$U = \sqrt{u_{rnd}^2 + u_{syst}^2} \tag{2}$$

Let’s assume we use a “perfect” external timebase oscillator, at a precisely controlled temperature, as frequency reference, e.g. a Cs oscillator or a Hydrogen Maser. Then we can neglect the influence of the timebase used on the measurement uncertainty, and only focus on the random uncertainty.

The relative random uncertainty (u_{rnd}) rms of a basic frequency measurement involving two timestamps (start and stop of Gate Time), is found in Eq. (3) below.

$$u_{rnd} = \frac{\sqrt{2 * TS_{res}^2 + 2 * noise\_trig\_err^2}}{Gate\_Time} \tag{3}$$

The noise trigger error contribution (in seconds rms) is caused by external and internal noise, and calculated as:

$$\frac{\sqrt{(ext.noise\ voltage)^2 + (int.noise\ voltage)^2}}{input\ signal\ slewrate\ (\frac{V}{s})at\ trig.point} \tag{4}$$

With a specified internal noise of <500 µVrms, and typical internal noise of 200 µVrms, the typical noise trigger error contribution is neglectable for pulses with slew rate >0.1 V/ns, or high frequency sinewaves >20 MHz @1Vrms, where both cases give <2 ps rms noise error contribution. But it could be quite substantial for low frequency, and/or low level, sine waves, with consequently low slew rates.

Conclusion: For high-slew rate input signals, and a very high-performance Cesium clock or Hydrogen Maser (timebase error 2E-12 or less) used as external timebase reference at a constant ambient temperature, the relative uncertainty for frequency measurements is solely dependent on the Timestamp resolution.

With a TS_{res} of 7 ps rms in CNT-100, the relative frequency measurement uncertainty (Eq. (3), or relative frequency error, is:

$$u_{rnd} = \frac{10\ ps}{Gate\_Time} \tag{5}$$

Note that the *relative* error, Δf/f, is independent of actual frequency, as long as a high slew rate input signal is applied.

The displayed resolution will normally be 11-12 result digits, for a 1 s gate time for a basic frequency measurement, with one Timestamp at the Gate time start event, and one Timestamp at the Gate time stop event.

3.2. Frequency Resolution Improvement via Regression Line Fitting

However, this basic resolution can be significantly improved by taking 1000 intermediate time stamp values during the gate time, instead of just two timestamps at the start and stop of the gate time and fitting all points $TS[E(k)]$ to a regression line, using least squares method. The input data is the continuous phase change vs. time during the gate time.

The slope of the regression line is a more accurate estimate of average period (=frequency^{-1}), than the basic 2-timestamp method. The uncertainty is not only determined by the two timestamps at the start and stop of the gate time, but also on the 1000 timestamps during the measurement.

This resolution enhancement technique in a counter is described in detail in [3] as the first industrial implementation in a commercial counter, and in [6] with a solid statistics analysis of this so called “Ω-type” of frequency counting technique.

The relative random uncertainty (resolution), assuming neglectable noise trigger errors, is enhanced to:

$$u_{rnd} = \frac{\sqrt{12} \cdot TS_{res}}{Gate\_Time \cdot \sqrt{N-2}} \tag{6}$$

See [3] and [6].

In CNT-100, N is set to 1000, which means an improvement of resolution, compared to the 2-timestamp frequency measurement of $\frac{\sqrt{6}}{\sqrt{998}}$ or approx. 12.5 times. If we still neglect the noise trigger error, the resolution for frequency measurements on fast pulses is improved to:

$$u_{rnd} = \frac{0.8\,ps}{Gate\_Time} \quad (7)$$

Note that these enhanced resolution frequency results cannot be used as input values for standard Allan Variance (AVAR) calculation. Instead, they can be used for Parabolic Variance (PVAR) calculation, see [7].

This resolution enhancement mode can be applied for set gate times from 80 µs (= 80 ns minimum Sample Interval * 1000 timestamps) and up.

Note that this resolution enhancement mode improves the resolution of the measuring instrument itself, but the full improvement effect on the displayed values is also dependent on the test signal. The method is based on *linear* regression, which will improve the resolution as intended on measurements of stable frequency signal sources with a white noise.

A constant and stable frequency value is represented by a straight line in the Events vs. Timestamps graph, which is the perfect signal for improvement via linear regression line fitting. In these cases, we should see the projected 12x resolution improvement.

A non-linear frequency will not be a straight line in the graph, and then the linear regression is less fitted for resolution improvement.

Examples of non-linear signals under test include for example modulated signals, frequency drifting signals, or signals subject to periodic spikes or "outliers".

In extreme cases the calculated average frequency result could even be less accurate with the linear regression enhancement technique, for example with a strong drift during the Measurement Time MT.

A frequency source with a frequency drift, can be described as:

$$f(t) = f_0 + f_d(t), \quad (8)$$

where f_0 is the start frequency value, $f_d(t)$ is the frequency drift over time, with mean value ≠ 0 Hz.

If $f_d(t)$ has a *linear* drift with time, then $f_d(t) = d \cdot t$, where d is the frequency drift rate (Hz/s). A linear drift during the measurement would result in an accumulated phase $\phi(t)$ vs time relation that is expressed as:

$$\Phi(t) = 2\pi f_0 t + \pi d t^2 + \Phi_0 \quad (9)$$

Fig. 14, copied from [3], p. 6, shows a simulation example with an exaggerated frequency drift, according to Eq. (9). The straight line is the regression line, and the dotted line is the accumulated phase, expressed as number of accumulated signal periods (Event counts) vs. time.

The accumulated phase is a 2nd order function and should ideally be approximated with a 2nd order polynomial and not a 1st order straight line.

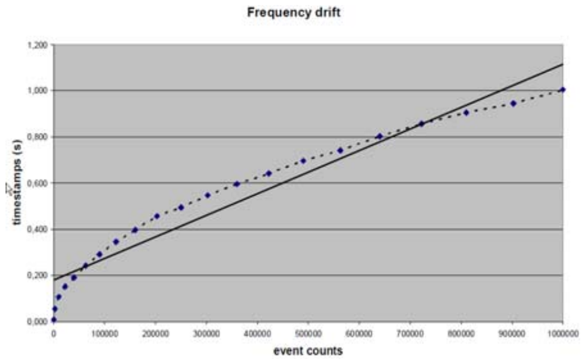

Fig. 14. A regression line can give additional error if there is a strong frequency drift.

It is obvious from the graph that the slope of the regression line is a worse representation of average frequency, compared to the true average frequency represented by the slope of a straight line from first to last timestamp during the Measurement Time MT.

3.3. Single-shot Period Resolution Improvement via Parallel Measurements

A single-shot period is the time for one individual cycle, not the average of a multiple cycles as in frequency or average period measurements.

A normal single-shot period >50 ns is measured as the difference between two consecutive timestamps in each digitizing channel. Up to 4 input signals can be measured in parallel, see Fig. 15.

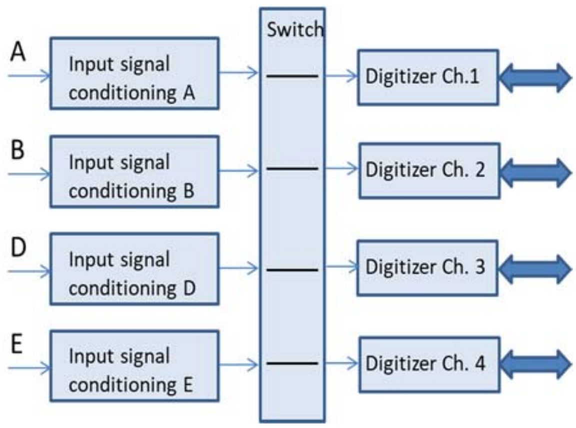

Fig. 15. Configuration for Single-shot Period measurements >50 ns.

The regression line fitting for resolution improvement is based on multiple timestamps of trigger events during the Measurement Time. It works fine for continuous Frequency and *Period Average* measurements on a stable input signal, but cannot be applied for single shot events, like a *single period* measurement.

Then you are back to the basic 10 ps per period (2 timestamps), resolution, or 7 ps per timestamp in each channel.

However, instead of timestamping the trigger events in four separate input signals in parallel, you could route one and the same input signal with a period >50 ns to five independent time digitizing channels and calculate the average value, between the start and stop trigger events in each separate channel.

This technique is described in [1]. See Fig. 16.

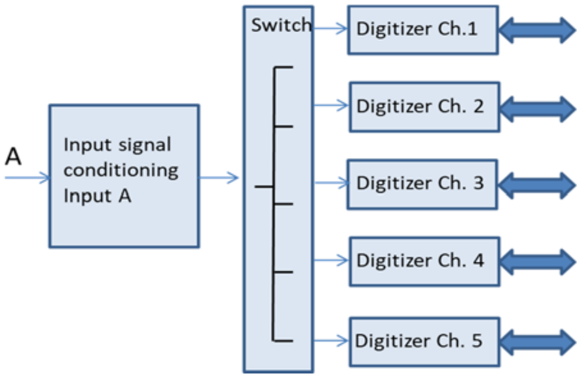

Fig. 16. Single-shot period resolution is improved up to 2 times by averaging parallel timestamping in 5 digitizers.

Note that all five digitizers are used in this special mode, as opposed to the earlier described examples involving four digitizers.

The resolution is improved approximately 2 times (square root of the number of parallel measurements minus one), and the resolution per timestamp for single periods >50 ns is improved from 7 ps to theoretically 3.5 ps.

Note that the actual improvement may be less than the theoretical value, due to the fact that the five channels are not perfectly independent and uncorrelated, which is a prerequisite for this statistical calculation.

Note that this resolution improvement is not applicable to short time intervals <50 ns, since these measurements require a basically different HW configuration, see Fig. 12.

4. Conclusion

A new combined Multi-channel Universal Frequency Counter (UFC) and Time Interval Analyzer (TIA), is presented having a 5-channel parallel continuous time stamping architecture, with a basic measurement resolution for each channel of:

10 ps/Gate_Time for frequency measurements (UFC mode), and

7 ps/Timestamp for single-shot time measurements (TIA mode).

This can be improved via resolution enhancement techniques up to (best case):

0.8 ps/Gate_Time for continuous frequency measurements (UFC mode), and

3.5 ps/Timestamp for single-shot time measurements (TIA mode), for single-period measurements.

Acknowledgements

This project was defined and developed by Pendulum Instruments in close cooperation with the *Military University of Technology, Faculty of Electronics, Warsaw, Poland (MUT).*

The high-resolution measurement kernel is designed by MUT, based on refinement of earlier MUT designs ([1] and [2]).

We would especially acknowledge the MUT team; Ryszard Szplet, Pawel Kwiatkowski, Zbigniew Jachna and Krzysztof Różyc for their dedicated contribution.

References

[1]. R. Szplet, P. Kwiatkowski, K. Różyc, Z. Jachna, T. Sondej, Picosecond-precision multichannel autonomous time and frequency counter, *Review of Scientific Instruments*, Vol. 88, Issue 12, 2017, 125101.

[2]. R. Szplet, P. Kwiatkowski, Z. Jachna, K. Różyc, A 2.9 ps equivalent resolution interpolating time counter based on multiple independent coding lines, *Measurement Science and Technology*, Vol. 24, Issue 3, 2013, 035904.

[3]. S. Johansson, New frequency counting principle improves resolution, in *Proceedings of the IEEE Int. Freq. Control Symp. Expo*, Vancouver, BC, Canada, Aug. 2005, pp. 628-635.

[4]. Guide to the expression of uncertainty in Measurement (GUM), ISO publication, 1993.

[5]. User's manual for CNT-100, *Pendulum Instruments*, document number 4031 601 10001.

[6]. Enrico Rubiola, Michel Lenczner, Pierre-Yves Bourgeois, François Vernotte, The Ω Counter, a Frequency Counter Based on the Linear Regression, *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, Vol. 63, Issue 7, July 2016, pp. 961-969.

[7]. F. Vernotte, M. Lenczner, P.-Y. Bourgeois, E. Rubiola, The parabolic variance (PVAR): A wavelet variance based on the least-square fit, *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, Vol. 63, Issue 4, April 2016, pp. 611-623.

[8]. S. Johansson, A new picosecond Multi-Channel combined Universal Frequency Counter & Time Interval Analyzer in a Benchtop format, in *Proceedings of the 3rd IFSA Frequency & Time Conference (IFTC'2021)*, Palma de Mallorca, Spain, 22-24 September 2021, pp. 25-29.