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Study on Nonlinear Vibration and Crack Fault of Rotor-bearing-seal Coupling System

Yuegang LUO, Songhe ZHANG, Bin WU, Wanlei WANG
College of Electromechanical & Information Engineering, Dalian Nationalities University,
Dalian, 116600, China
E-mail: luoyg@dlnu.edu.cn

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Abstract: The nonlinear dynamic model of rotor-bearing-seal system with crack in shaft is set up based on the coupling model of nonlinear oil-film force and Muszyska's nonlinear seal fluid force. The dynamic vibration characteristics of the rotor-bearing-seal system and the effects of physical and structural parameters of labyrinth seal and crack fault on movement character of the rotor were analyzed. The increases of seal length, seal pressure differential, seal radius and axial velocity are in favor of the stability of the system, and it of seal gap and crack depth are not in favor of the stability of the system. *Copyright © 2014 IFSA Publishing, S. L.*

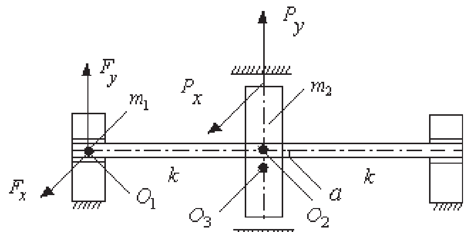
Keywords: Rotor system, Oil-film force, Seal force, Shaft crack, Nonlinear vibration.

1. Introduction

It has important influences of nonlinear oil-film force, nonlinear seal force and shaft crack et al to the rotor system in safety running. Many researchers studied the nonlinear dynamic characteristics of the rotor system. Zhang Yanmei et al. [1] analyzed a bearing-rotor system by using a new analytic model of the unsteady oil film force. The bifurcation and chaos behavior of the rotor system varying with the rotation angular frequency were obtained through numerical simulations. Based on the analysis of the oil film force of a journal bearing, four characteristic coefficients of the journal bearing are derived by Yang Jinfu et al. [2]. Moreover, according to fluid-structure-interaction equilibrium equations, they presented a stability criterion equation of the oil film-rotor system, which can explain the acting mechanism of the oil film force on the rotor. The dynamic destabilization characteristics of the journal bearing were given by testing on the work set. Chen Yushu [3] investigated the relation of the linearized stability of a single span rotor-seal system between its parameters using the Muszynska's model of seal fluid dynamical forces. According to the theory of Hopf bifurcation, the instability leads to self-excited whirling motion of the rotor generated from its equilibrium position. The direction of Hopf bifurcation and the stability of the bifurcated periodic orbit are determined by Poore's algebraic criterion. The theoretical results are verified by the numerical results. Yang Xiaoming and Li Songtao et al. [4, 5] analyzed the nonlinear dynamic characteristics of labyrinth seal-rotating wheel system in hydraulic turbines, and investigated the nonlinear dynamic stability of hydraulic turbines with labyrinth seal using the Muszynska's model of seal fluid dynamic forces. The results show that there is occurrence of Hopf bifurcation after threshold speed is exceeded and the rotating wheel comes in to bifurcated periodic orbit. While the amplitude of bifurcated motion will rise according to improvement of rotor speed, which may cause the rotating wheel impact seal at certain speed. Lyapunov's first approximate theory was used to analyze nonlinear system stability, and Runge-Kutta method was used to analyze numerical simulation computation. Finally, the influence of physical and structural parameter of seal on rotor was investigated. Sensitivity analysis of nonlinear dynamic characteristics was performed by changing the physical and structural parameters of seal-rotating wheel system. By comparing with eight-parameter model, it was tested that the variation tendency of Muszynska's model is identical. Luo Yuegang et al. [6] set up a dynamic model of two-span rotor-bearing system with crack fault. Using the continuation-shooting algorithm for periodic solution of nonlinear non-autonomous system, the stability of the system periodic motion was studied by the Floquet theory. The periodical, quasi-periodical and chaos motions were found in the system responses. The unstable form of the rotor system with crack fault was period-doubling bifurcation. There are unstable forms of period-doubling bifurcation and Hopf bifurcation in different rotate speed. There were many harmonic element s of 1/3, 1/2, 2/3, 1, 2 and so on within the sub-critical speed range. But the 2-harmonic element decreased within the super-critical speed range. Gao Chongren et al. [7] studied the seal behaviors taking cracked rotor system as object. The Muszynska's model of the nonlinear seal force was applied to building the coupled dynamic equation and analyzing the movement character of cracked rotor with airflow induced force. The effect of physical and structural parameters of labyrinth seal on movement character of cracked rotor was emphatically discussed. The research results shown that the nonlinear dynamics character were very plentiful in the cracked rotor seal system, the airflow induced force can reduce the periodic response of the cracked rotor, the main parameters of the sealed structure had important impact on the stability of cracked rotor system, and the dynamic stability can be improved by adjusting seal parameters. Darpe [8] studied transient response of a simple Jeffcott rotor with two transverse surface cracks during its passage through first bending critical speed, and the response of the rotor during its passage through 1/3rd and 1/2 subharmonic resonances was also investigated. In this paper, the nonlinear dynamic model of rotor-bearing-seal system with crack in shaft is set up based on the coupling model of nonlinear oil-film force and Muszyska's nonlinear seal fluid force. The dynamic vibration characteristics of the rotor-bearing-seal system were analyzed by numeral integrated method, and the effects of physical and structural parameters of labyrinth seal and crack fault on movement character of the rotor were investigated. The results provide theoretical references for safe operation and fault diagnosis of the coupling systems.

2. Dynamic Models of the Rotor System and Motion Differential Equation

Model of the rotor-bearing-seal system with crack fault is shown as Fig. 1. Shaft coupling connects the motor and rotor. The system mass is equivalently concentrated on the center of every disc and bearing support respectively. The torsional vibration and gyro moment are neglected and only the lateral vibration of system is considered. Both ends of rotor are supported by journal bearings with symmetrical structures. O_1 is geometric centers of bearing, O_2 is geometric centers of rotor, O_3 is center of mass of rotor, m_1 is lumped masses of rotor at bearing, m_2 is equivalently lumped masses at disc. The shaft with zero quality connect disc with bearing, k is the stiffness of elastic shaft. F_x and F_y are nonlinear oil film forces, P_x and P_y are nonlinear seal fluid forces. There is a transversal crack with deepness of a in the middle of shaft.



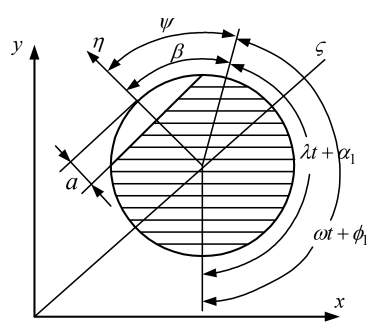


Fig. 1. Models of rotor-bearing-seal coupling system and section of crack.

2.1. Oil-film Force of Short Bearing

Non-dimensional Reynolds equation based on oil-film of short bearing assumption is expressed as

$$\left[\frac{R}{L}\right]^2 \frac{\partial}{\partial z}\left(h^3 \frac{\partial p}{\partial z}\right) = x\sin\theta - y\cos\theta - 2(x'\cos\theta + y'\sin\theta) \quad (1)$$

where R is the radius of bearing, L is the length of bearing. Non-dimensional nonlinear oil-film force can be obtained form above formula:

$$p = \frac{1}{2}\left[\frac{L}{D}\right]^2 \frac{(x-2y')\sin\theta - (y+2x')\cos\theta}{(1-x\cos\theta - y\sin\theta)^3}(4z^2-1) \quad (2)$$

Non-dimensional nonlinear oil-film force can be finally expressed as

$$\begin{Bmatrix} f_x \\ f_y \end{Bmatrix} = -\frac{[(x-2y')^2+(y+2x')^2]^{1/2}}{1-x^2-y^2} \cdot \begin{Bmatrix} 3xV(x,y,\alpha) - \sin\alpha G(x,y,\alpha) - 2\cos\alpha S(x,y,\alpha) \\ 3yV(x,y,\alpha) + \cos\alpha G(x,y,\alpha) - 2\sin\alpha S(x,y,\alpha) \end{Bmatrix}, \quad (3)$$

where

$$V(x,y,\alpha)=\frac{2+(y\cos\alpha-x\sin\alpha)G(x,y,\alpha)}{1-x^2-y^2} \quad (4)$$

$$S(x,y,\alpha)=\frac{x\cos\alpha+y\sin\alpha}{1-(x\cos\alpha+y\sin\alpha)^2} \quad (5)$$

$$G(x,y,\alpha)=\frac{2}{(1-x^2-y^2)^{1/2}}\left[\frac{\pi}{2}+\mathrm{arctg}\frac{y\cos\alpha-x\sin\alpha}{(1-x^2-y^2)^{1/2}}\right] \quad (6)$$

$$\alpha=\mathrm{arctg}\frac{y+2x'}{x-2y'}-\frac{\pi}{2}\mathrm{sign}\left[\frac{y+2x'}{x-2y'}\right]-\frac{\pi}{2}\mathrm{sign}(y+2x') \quad (7)$$

2.2. Nonlinear Seal Fluid Force

The Muszynska's model is applied to investigate the nonlinear behaviors of seal fluid force, it can be expressed as [9, 10]

$$\begin{Bmatrix}P_x\\P_y\end{Bmatrix}=-\begin{bmatrix}K-m_f\tau_f^2\omega^2 & \tau_f\omega D\\ -\tau_f\omega D & K-m_f\tau_f^2\omega^2\end{bmatrix}\begin{Bmatrix}x\\y\end{Bmatrix} -\begin{bmatrix}D & 2m_f\tau_f\omega\\ -2m_f\tau_f\omega & D\end{bmatrix}\begin{Bmatrix}\dot{x}\\\dot{y}\end{Bmatrix}-\begin{bmatrix}m_f & 0\\ 0 & m_f\end{bmatrix}\begin{Bmatrix}\ddot{x}\\\ddot{y}\end{Bmatrix} \quad (8)$$

where ω is the rotational angular velocity, K, D and m_f are the represent equivalent stiffness, equivalent damping and equivalent mass respectively. τ_f is the average velocity ratio of fluids in axially. K, D and τ_f are the nonlinear functions of the displacements x and y, which can be expressed as

$$\begin{cases}K=K_0(1-e^2)^{-n}\\ D=D_0(1-e^2)^{-n}, \quad n=0.5\sim 3\\ \tau_f=\tau_0(1-e)^b, \quad 0<b<1\end{cases} \quad (9)$$

where $e=\sqrt{x^2+y^2}/c_s$ is the relative eccentricity of rotor, c_s is the seal gap, n, b and τ_0 are the seal parameters, general $\tau_0<0.5$. K_0, D_0 and m_f can be calculated by Black-Childs formulas [11].

2.3. Motion differential equations

The radial displacements of axes center of rotor system in left are assumed as (x_1, y_1) respectively, the radial displacements of disc are (x_2, y_2) respectively, then non-dimensional motion differential equation of system can be expressed as

$$\begin{cases}\ddot{x}_1+\xi_1\dot{x}_1+(\eta_{11}x_1-\eta_{12}x_2)[1-\frac{\varepsilon\delta}{2}F(\psi)]+\frac{\varepsilon}{2}F(\psi)[(\eta_{11}x_1-\eta_{12}x_2)\cos 2t+(\eta_{11}y_1-\eta_{12}y_2)\sin 2t]=\frac{1}{M_1}f_x(x_1,y_1,\dot{x}_1,\dot{y}_1)\\ \ddot{y}_1+\xi_1\dot{y}_1+(\eta_{11}y_1-\eta_{12}y_2)[1-\frac{\varepsilon\delta}{2}F(\psi)]+\frac{\varepsilon}{2}F(\psi)[(\eta_{11}x_1-\eta_{12}x_2)\sin 2t-(\eta_{11}y_1-\eta_{12}y_2)\cos 2t]=\frac{1}{M_1}f_y(x_1,y_1,\dot{x}_1,\dot{y}_1)-G_1\\ \ddot{x}_2+\xi_2\dot{x}_2+2(\eta_{22}x_2-\eta_{21}x_1)[1-\frac{\varepsilon\delta}{2}F(\psi)]+\varepsilon F(\psi)[(\eta_{22}x_2-\eta_{21}x_1)\cos 2t+(\eta_{22}y_2-\eta_{21}y_1)\sin 2t]=\frac{\bar{P}_x}{N}+\frac{1}{M_2}\cos\tau\\ \ddot{y}_2+\xi_2\dot{y}_2+2(\eta_{22}y_2-\eta_{21}y_1)[1-\frac{\varepsilon\delta}{2}F(\psi)]+\varepsilon F(\psi)[(\eta_{22}x_2-\eta_{21}x_1)\sin 2t-(\eta_{22}y_2-\eta_{21}y_1)\cos 2t]=\frac{\bar{P}_y}{N}+\frac{1}{M_2}\sin\tau-G_2,\end{cases} \quad (10)$$

where ε and δ are the relative stiffness parameters about crack deepness a [6]; $F(\psi)$ is the open and close function of crack, $F(\psi)=\frac{1+\cos\psi}{2}$, $\psi=t-\phi_0+\beta-\arctan\frac{y}{x}$, β is the contained angle between crack direction and eccentricity; ϕ_0 is the initial phase place. $\xi_1=\frac{c_1}{m_1\omega}$, $\xi_2=\frac{c_2}{(m_2+m_f)\omega}$, c_1 is the damping coefficient of bearing; c_2 is the damping coefficient of disc, $\eta_{11}=\frac{k}{m_1\omega^2}$, $\eta_{12}=\frac{kc_s}{m_1c\omega^2}$, $\eta_{21}=\frac{kc}{(m_2+m_f)c_s\omega^2}$, $\eta_{22}=\frac{k}{(m_2+m_f)\omega^2}$. The parameter c is the bearing gap. $M_1=\frac{m_1c\omega^2}{\delta_1}$, $M_2=\frac{(m_2+m_f)c_s}{m_2e}$, $N=(m_2+m_f)\omega^2$, $G_1=\frac{g}{c\omega^2}$, $G_2=\frac{m_2g}{(m_2+m_f)c_s\omega^2}$, $x_1=\frac{X_1}{c}$, $y_1=\frac{Y_1}{c}$, $x_2=\frac{X_2}{c_s}$, $y_2=\frac{Y_2}{c_s}$. τ is the non-dimensional time, $\tau=\omega t$. $f_x=\frac{F_x}{\delta}$, $f_y=\frac{F_y}{\delta}$ are the non-dimensional nonlinear oil-film force components, which are shown in formula (3). δ is the Sommerfeld correction coefficient, $\delta=\frac{\mu\omega RL}{m_ig}\left(\frac{R}{c}\right)^2\left(\frac{L}{2R}\right)^2$, μ is the viscidity of lubricant. $\bar{P}_x$ and $\bar{P}_y$ are the non-dimensional nonlinear seal fluid forces

$$\begin{Bmatrix} \overline{P}_x \\ \overline{P}_y \end{Bmatrix} = -\begin{bmatrix} K - m_f \tau_f^2 \omega^2 & \tau_f \omega D \\ -\tau_f \omega D & K - m_f \tau_f^2 \omega^2 \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} - \omega \begin{bmatrix} D & 2m_f \tau_f \omega \\ -2m_f \tau_f \omega & D \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} \tag{11}$$

3. Nonlinear Dynamic Characteristics Analysis of the System

3.1. Dynamic Characteristics of the System in Nonlinear Coupling Fluid Forces

The parameters of the system are: m_1=4.0 kg, m_2=32.1 kg, R=25 mm, L=12 mm, c=0.11 mm, c_s=0.5 mm, μ=0.018 $\mathrm{P_a \cdot s}$, c_1=1050 N·s/m, c_2=2100 N·s/m, k=2.5×10^7 N/m, e=0.05 mm. The physical and structural parameters of seal are: n=2.5, b=0.45, $\tau_0 = 0.4$, $m_0 = -0.25$. The imported loss coefficient of seal is z=0.1, and the axial velocity is 5 m/s. The first critical rotate speed of the system (with no fault) is ω_0=882.5 rad/s.

Fig. 2 is the bifurcation diagrams of rotor-bearing-seal response with rotational speed. When the rotational speed is low, the response of rotor system is periodic-1 motion. Along with the increase of the rotational speed, at about ω=620rad/s, the system loses stability of the periodic motion, and bifurcation from periodic-1 to periodic-2. The main motion in the region of critical rotational speed is periodic-2, and it in the supercritical rotational speed is periodic-2, periodic-4 and quasi-periodic motion. The main motion of the system in the ultra-supercritical rotational speed is quasi-periodic. For investigating the influences of physical and structural parameter of seal on rotor system, the seal length, seal pressure differential, seal radius, axial velocity and seal gap are respectively changed. The results are shown in Fig. 3. The instability critical speed of the system shows nonlinear improvement along with the increase of the seal length, seal pressure differential, seal radius and axial velocity. It testifies the increases of these parameters are in favor of the stability of the system. The instability critical speed of the system shows nonlinear decreased along with the increase of the seal gap, which testifies the increase of the seal gap is not in favor of the stability of the system. It can be seen comparing references [4, 5, 7] that the influences of the physical and structural parameters of seal on rotor system in the coupling actions of nonlinear oil-film force and nonlinear seal force are significantly different of it in action of nonlinear seal force only.

3.2. Influences of Crack to the Motion Character of System

Fig. 4(a)-(d) are the bifurcation diagrams of the rotor system with the rotating speed ω as the control parameters at different non-dimensional crack depths of a=0.4, 0.7, 1.0, 1.3. When the crack is less (a=0.4), the effects of crack fault to the motion characteristics of rotor system is little.

Along with the increases of crack depth, the periodic-4 motion translates to chaotic motion around four regions in the region of critical rotational speed, and there appear periodic-3 motions in the regions of supercritical rotational speed and ultra-supercritical rotational speed, which can be the basis for judging the crack fault and degree. Fig. 3(f) shows when $a \le 0.4$, the instability critical speed is about ω=620 rad/s. The instability critical speed of the system shows nonlinear decreased along with the increase of the crack depth, which testifies the increase of the crack depth will reduce the stability of the system.

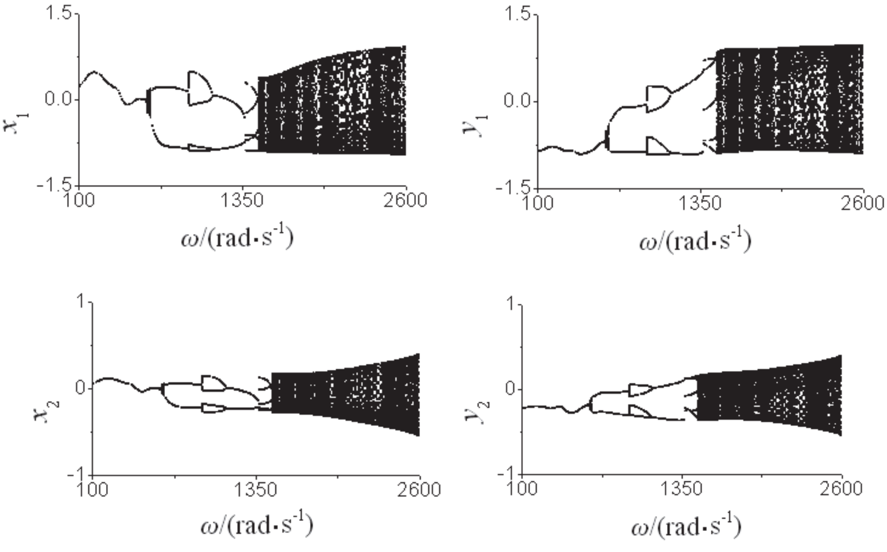
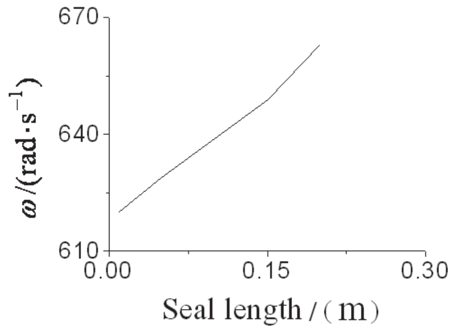
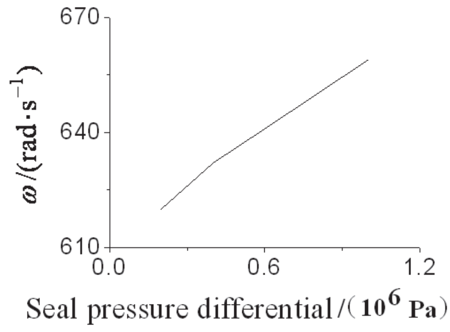


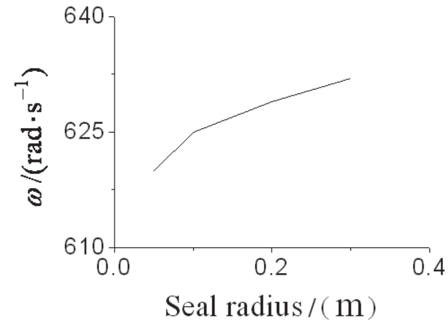
Fig. 2. The bifurcation diagrams of rotor response with ω.



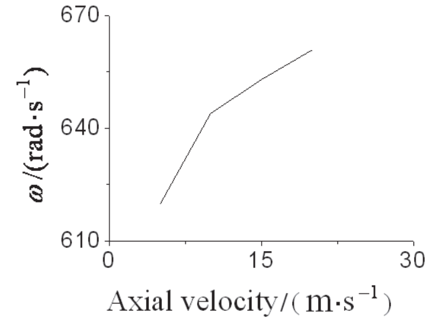
(a) Seal length



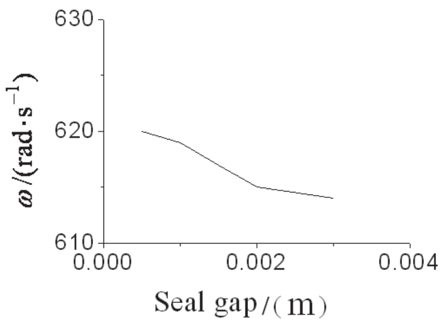
(b) Seal pressure differential



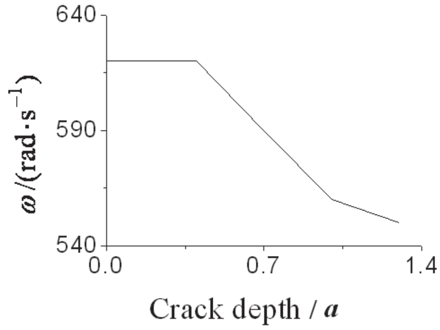
(c) Seal radius



(d) Axial velocity



(e) Seal gap



(f) Crack depth

Fig. 3. Effects of parameters on critical unstable rotating speed.

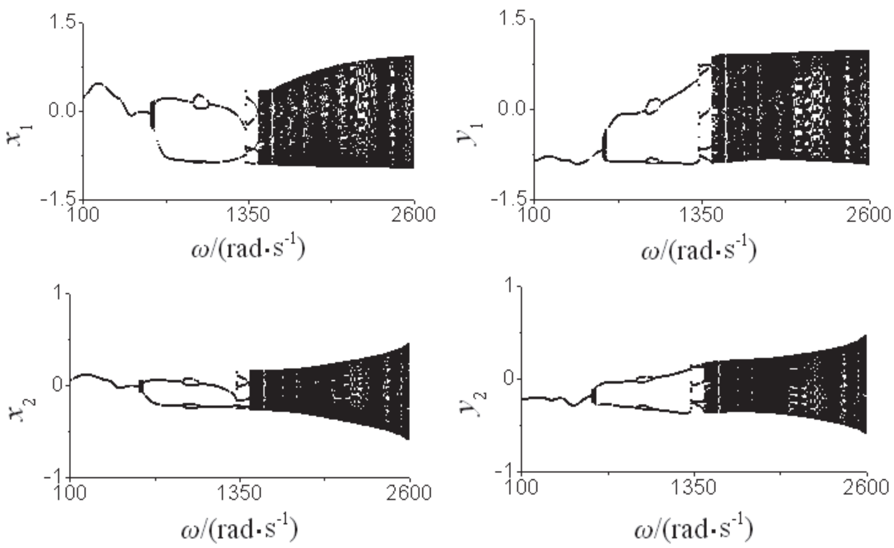


Fig. 4 (a). The bifurcation diagrams of rotor response with ω at different crack depth a=0.4.

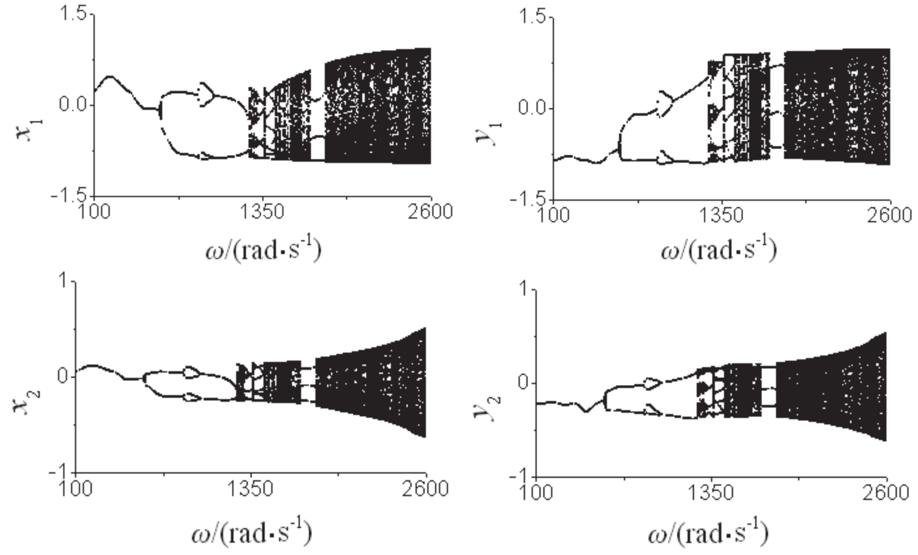


Fig. 4 (b). The bifurcation diagrams of rotor response with ω at different crack depth *a*=0.7.

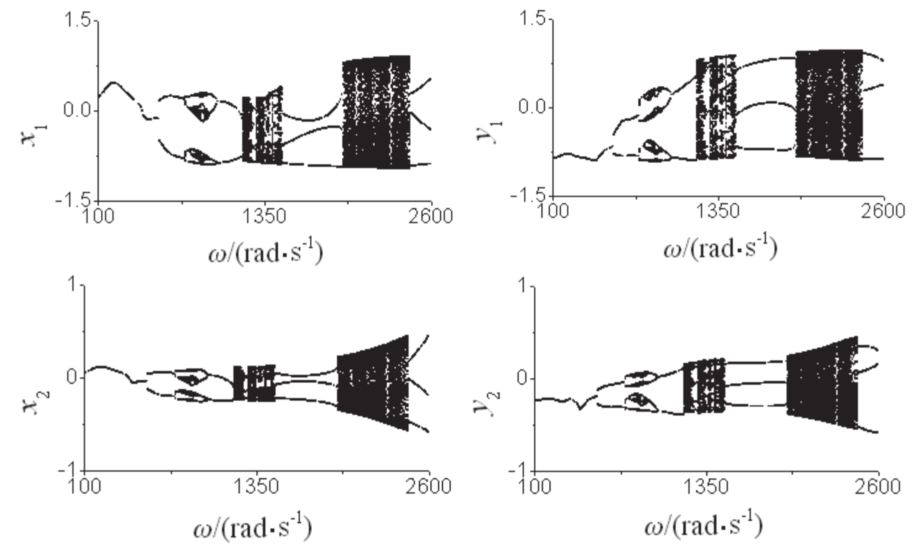


Fig. 4 (c). The bifurcation diagrams of rotor response with ω at different crack depth *a*=1.0.

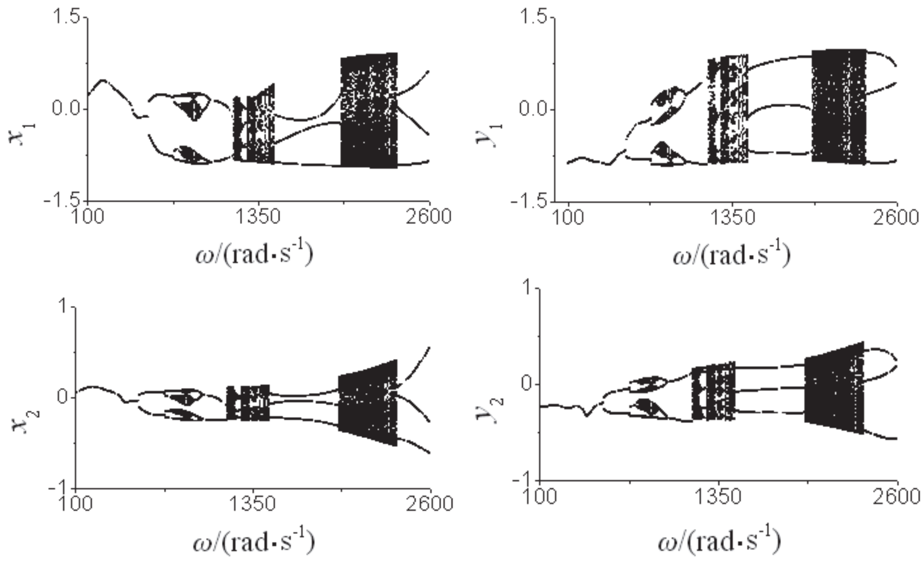


Fig. 4 (d). The bifurcation diagrams of rotor response with ω at different crack depth *a*=1.3.

4. Conclusions

1) The nonlinear dynamic model of rotor-bearing-seal system with crack in shaft is set up based on the coupling model of nonlinear oil-film force and Muszyska's nonlinear seal fluid force. The dynamic vibration characteristics of the rotor-bearing-seal system were analyzed by numeral integrated method, and the effects of physical and structural parameters of labyrinth seal and crack fault on movement character of the rotor were investigated.

2) The increases of seal length, seal pressure differential, seal radius and axial velocity are in favor of the stability of the system, and it of the seal gap is not in favor of the stability of the system. The influences of the physical and structural parameters of seal on rotor system in the coupling actions of nonlinear oil-film force and nonlinear seal force are significantly different of it in action of nonlinear seal force only.

3) The increase of the crack dep will reduce the stability of the system. The periodic-3 motions appearing in the regions of supercritical rotational speed and ultra-supercritical rotational speed can be the basis for judging the crack fault and degree.

Acknowledgements

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