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Analysis on the Errors Caused by the Axial Acceleration of a Single Mass Capacitive Vibratory Micro-machined Gyro and Design of High-order Low-pass Filtering Module

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Abstract: In order to estimate and get the flight attitude, trajectory, launching speed, ejection altitude, landing point, landing speed and displacement, etc. of aircraft, parameters of six degree of freedom inertial measurement unit are usually acquired to establish the settlement data base in the design and study of aircraft. Moreover, the gyro used to measure the angular rate is an important part of the six degree of freedom inertial measurement unit. Belonging to the category of dynamic measurement, the research in this paper, taking a single mass capacitive vibratory micro-machined gyro as the object and combining Coriolis principle with the measurement mechanism of capacitive micro-machined vibrating gyro, made an analysis on the measurement errors of angular rate of this kind of gyro caused by the axial acceleration. Meanwhile, a kind of anti-aliasing filtering module of signal preconditioning circuit of the gyro was designed in this paper by taking advantage of elliptic high-order low-pass filter, which was consonant with the frequency band range in the actual application of angular rate, and preferably restrained the errors caused by the axial acceleration of a single mass capacitive vibrating micro-machined gyro, and provided some important test data and recommendations for its engineering practice as well. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Single mass, Capacitive micro-machined gyro, Axial acceleration, Low-pass filtering.

1. Introduction

Making accurate measurement on critical dynamic parameters of aircraft and other systems can help to pre-design the plan for assessment or provides powerful referential data for the subsequent improvement and service. In order to obtain the attitude of aircraft and further resolve the more real flight trace, inertial measurement unit must be the arranged measuring points. In addition, the inertial measurement unit mentioned above usually consists of a three-axis angular rate gyro and a three-axis overload (plus table) sensor with an external manifestation of a six degree of freedom integrated device [1]. Moreover, in the unit, the measuring axis should be separately parallel or coincidence with the center line of the aircraft structure, the three-axis gyro can provide the angular rate data of the pitch, yaw and tumble in the process of launching the aircraft, and the three-axis overload is mainly used to provide the acceleration data including the axial acceleration and the horizontal radial acceleration. After the data is reclaimed, the angular information of the yaw about the aircraft is obtained by the first integral of the signals of the angular rate, and the speed and displacement information of the aircraft can be acquired respectively by the first integral and second integral of the overload signals [2].

As the experiment conducted in the stage of the design and research of aircraft is usually accompanied by an environment with strong impact and vibration, the sensors used in the experiment should have a better ability to resist the overload and impact, at the same time, the cost of measurement needs to be considered on the premise of assuring the accuracy of measurement. Gyro made of silicon micro technology, because of its better characteristics than that of the other type of gyros, such as, fine adaptability to the environment, reliable accuracy of measurement, low cost and easy mass production, has become the first choice for sensors used in measurement, and got more and more attentions as well as become the focus of the present study [3].

The Coriolis acceleration is proportional to the rotation angular velocity and particle velocity. The angular velocity ω of rotating system can be obtained through measuring Coriolis acceleration and the speed of moving object. Therefore, using Coriolis acceleration to measure rotation velocity is the operating principle for all the vibratory micro-machined gyros. The driving mass of capacitive vibratory micro-machined gyros described in this paper is in single mass style, its process of producing is simple, and the measurement of its angle rate is stable and reliable. Whereas, it is because of its structural features of single mass that the measurement error caused by the axial acceleration cannot be ignored. Therefore, the analysis, resistance and solutions to the measurement error are particularly important in the process of using it to measure the angle rate.

2. Measurement Principle for Vibratory Micro-machined Gyro

When the transport motion of the moving point is rotation, the transport motion will make the direction of the relative acceleration constantly change and the relative movement will make the speed of the transport motion constantly change. These two reasons induce the changing rate of the additional speed in the same direction, and the changing rate of the additional speed is the Coriolis acceleration. In short, Coriolis acceleration is caused by the mutual influence between the relative movement and the transport motion. Coriolis acceleration is the basis for the study of micro-machined gyro. According to the kinetic model of vibratory micro-machined gyro, a general expression of Coriolis acceleration can be drawn as follow:

$$a_k = 2 \cdot \omega \cdot r \tag{1}$$

In formula (1), assuming ω and r is the vertical, a_k is the Coriolis acceleration, ω is the instantaneous angular velocity of moving point, and r is the instantaneous velocity of moving point. It can be seen that the direction of the Coriolis acceleration, the angular velocity ω and the movement velocity are mutually vertical. Therefore, two mutually orthogonal vibratory modals in the direction of vibration are required for the vibratory micro-machined gyro, in which the single mass moves under the driving force along the direction of vibratory modal. In addition, the direction of the Coriolis acceleration is always in the direction of detecting mode perpendicular to the direction of driving modal, thus the direction of detecting angular velocity, driving vibration and detecting vibration are always orthogonal to each other.

When the vibratory micro-machined gyro is driven to vibrate around the head shaft with high frequency, the Coriolis acceleration of each particle in the detecting single mass will form a Coriolis inertia moment to the output shaft, that is, the gyro moment. The direction of the gyro moment is opposite to that of the Coriolis acceleration, and the function of the gyro moment is to support the frame of the detecting single mass. Due to the existence of the Coriolis acceleration and the gyro moment, in the ideal case, the output signal of the micro-machined gyro is directly proportional to the input signal of the sensitive axis, and this is the basic principle for the sensitive angular velocity of micro-machined gyro [4, 5].

3. Errors Caused by Axial Acceleration

3.1. Analysis on the Mechanism of Errors

According to the basic principle for using Coriolis acceleration to measure the angular velocity of micro-machined gyro, a kinetic analysis was made on the following assumptions. When a capacitive micro-machined gyro was working in a stable state, the single mass was being driven to vibrate along the X-direction, if $x(t)$ was used to indicate the displacement of steady vibration of the single mass, the equation of its motion would be:

$$x(t) = B_x \sin(\omega_x t) \tag{2}$$

For ω_x, a formula could be drawn:

$$\omega_x^{\ 2} = \frac{k_x}{m_x} \tag{3}$$

For B_x, formulas could be drawn:

$$B_x = \frac{F_0}{m_x \omega_x^{\ 2} \sqrt{(1-\lambda_x^{\ 2})^2 + (2\xi_x \lambda_x)^2}} \tag{4}$$

$$2\xi_x \omega_x = \frac{c_x}{m_x} \tag{5}$$

$$\lambda_x = \frac{\omega_d}{\omega_x} \tag{6}$$

In the formulas of (2), (3), (4), (5) and (6), c_x was the damping coefficient, k_x was the elastic constant, m_x was the inertial mass, F_x was the borne driving force, and ω_d was the angular frequency of the borne driving force. When the signals of the acceleration were produced along the X direction in the system, under the force of acceleration, the driving single mass superposed the same-phase vibration caused by the signals of the acceleration based on the driving vibration. Its equation of motion could be expressed as:

$$x(t) = B_x \sin(\omega_x t) + B_{xa} \sin(\omega_{xa} t - \varphi_{xa}) \tag{7}$$

When the angular velocity Ω was input into the system, according to the measurement principle for Coriolis acceleration, the Coriolis acceleration coupled on the detecting mass could be indicated as follows:

$$a_k = 2\Omega[B_x \omega_x \cos(\omega_x t) + B_{xa}\omega_{xa} \cos(\omega_{xa} t - \varphi_{xa})] \tag{8}$$

It could be concluded from formula (7) and (8) that the Coriolis acceleration on the detecting mass could be divided into two items: one item was the necessary part for resolving the angular velocity; and the other item was the interferential term caused by acceleration in the system, that is, the measurement errors caused by the axial acceleration [3,6].

3.2. Sweep Verification on the Caused Measurement Errors

For the single mass capacitive vibratory micro-machined gyro, as described in formula (2), the errors of gyro resulted from the existence of the axial acceleration, to be specific, interferential item $B_{xa}\omega_{xa}\cos(\omega_{xa} t - \varphi_{xa})$ in formula (8). In order to verify the authenticity of such caused errors, a test of sine frequency sweep vibration was designed in this paper. More specifically, a three-axis vibratory micro-machined gyro was driven to make a unidirectional vibration along its sensitive axis on the high frequency electro dynamic vibratory test stand to observe the real output of the gyro under the condition without angular velocity sensitivity through a sine frequency sweep vibration, and gather the corresponding relationship between the output ranging from the start oscillation frequency of 5 Hz to the end frequency of 200 Hz and the vibration frequency.

In the vibratory test, the connection among the equipments involved in the test was shown in Fig. 1.

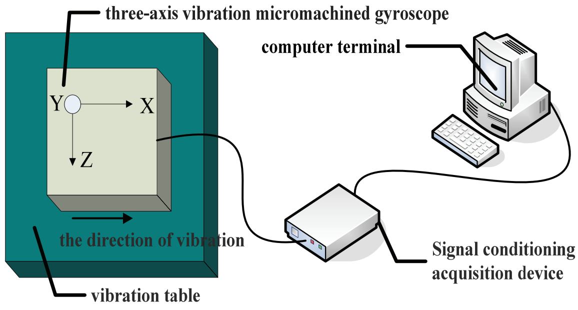

Fig. 1. Diagrammatic sketch of the connection of the equipments involved in the vibratory verification.

The three-axis vibratory micro-machined gyro was fixed on the high frequency electrodynamic vibratory test stand, and the gyro should be fixed on the position according to the principle of assuring one sensitive axis in the gyro parallel to the vibratory direction of the vibratory test stand. Moreover, the power supply and signal output of the micro-machined gyro were connected to the signal conditioning and collecting device by a cable. In addition, the signal conditioning and collecting device provided power for the micro-machined gyro and made spontaneous adjustment, transformation and storage of the output of the gyro, and finally uploaded them to the computer terminal for data analysis and waveform reproduction.

In the process of vibration, the corresponding relationship between the vibratory frequency and the samples of the time axis for signal collection was shown in Table 1.

Table 1. Vibration test schedule.

Frequency /Hz	The moment /minutes: seconds	The length / seconds
5	0:0	0
20	0:56	56
40	1:24	84
60	1:41	101
80	1:52	112
100	2:01	121
120	2:08	128
140	2:15	135
160	2:20	140
180	2:25	145
200	2:29	149

Furthermore, in the test, three sine frequency sweep vibration were done respectively along the X, Y and Z axis of the micro-machined gyro with a vibratory control spectrum of 1.75 g, that is, the axial acceleration was 17.15 ms^{-2} (the acceleration of gravity of the place where the test was carried could be calculated as 9.8 ms^{-2}). Finally, three output curves of the gyro were got, as respectively shown in the Figs. 2, 3 and 4.

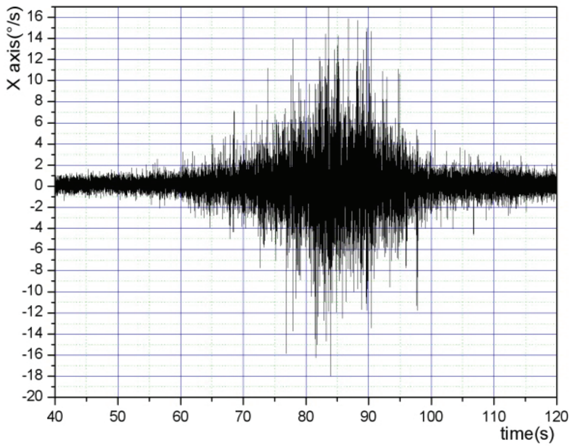

Fig. 2. Output curve of the X-axis of the gyro.

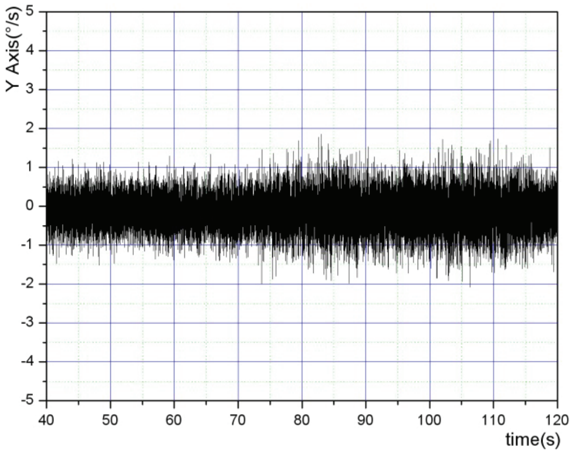

Fig. 3. Output curve of the Y-axis of the gyro.

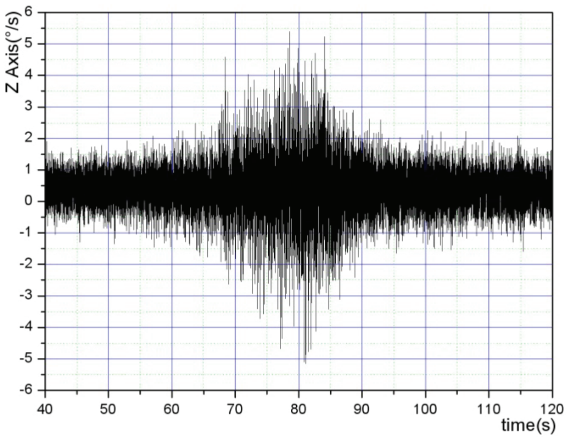

Fig. 4. Output curve of the Z-axis of the gyro.

From the three curves, we could find:

1) With the frequency ranging from 20 Hz to 60 Hz, the output of the gyro enlarged gradually and then decayed gradually;

2) The degree of sensitivity to the vibratory acceleration on the three axes were different.

Therefore, it can be concluded that there indeed exists errors caused by the axial acceleration in the single mass capacitive micro-machined vibratory gyro. Moreover, this kind of errors is sensitive to the frequency, and the sensitivity to the frequency of the samples in the test focus within the range from 20Hz to 60Hz. Meanwhile, because of the nuances in the MEMS manufacturing processes, the differences of the degree of sensitivity of the testing samples to the axial acceleration on the different sensitive axis also exist.

4. Design on the Preconditioning Circuit of the Single Mass Capacitive Vibratory Micro-machined Gyro

4.1. Introduction on the Design

From the Figs. 2, 3 and 4, the errors caused by the axial acceleration in the single mass capacitive vibrating micro-machined gyroscope cannot be ignored, especially in the experiments for the design and research of the aircraft, because the experiments sometimes must be conducted under an environment accompanied by strong impact and vibration. Under such an experiment environment, although the micro-machined gyro shares the ability to resist the overload and impact, if the errors caused by the axial acceleration cannot be suppressed or weakened, great negative impact on the results of the measurement on the angular parameters will be brought in, even more seriously, the correct resolving of the altitude of aircraft cannot be conducted by using the collected data.

In the reality, there are two methods to resolve the errors caused by the axial acceleration. One method is to change the single mass into a double mass by taking advantage of the symmetrical characteristics in the design on the testing capacity structure of the gyro to suppress the errors. However, this method fundamentally changes the structure of the micro-machined gyro described in this paper, and its manufacturing process is more complex comparatively. In view of the characteristics that the errors caused by the axial acceleration are sensitive to the frequency, the other method is to make a deep preconditioning of circuit board level on the signal of the output of the single mass capacitive vibratory micro-machined gyro by combining with the frequency scope concerned by the parameters of the gyro in the actual application, and build an anti-aliasing filtering module to suppress the errors caused by the axial acceleration with the elliptic high-order low-pass filter.

4.2. Design on the Elliptic High-Order Filtering Module

The error caused by the axial acceleration is sensitive to the frequency, and the range of the sensitive frequency focus within 20 Hz to 60 Hz in the verification link of the test. In the actual application, because the angular velocity is a kind of slowly varying sensing signal, when conducting a measurement on the angular velocity, the range of frequency concerned mainly ranges from direct current to 20 Hz. Therefore, a low-pass filtering module with higher order and better quality factor is adopted to make a preconditioning on the output signal of such kind of micro-machined gyro, so as to suppress the errors caused by the axial acceleration to the minimum before the output signal of the gyro is collected and stored, and reduce the output signal beyond 20 Hz in a wide-range. At the same time, building an anti-aliasing filter module can effectively restrain the aliasing phenomenon in the link of AD sampling, and farthest restore the real changing condition of the dynamic parameters of angular velocity. In this paper, a ten-order elliptic filter LTC1569 is adopted in the pre-condition circuit to build the filtering module, which shares the characteristics of better quality factor, comparatively steep frequency response curve and smaller intra band roughness, and its typical design is shown in Fig. 5. In this design, a 2 V referential voltage is needed for the pin 3 of LTC1569, the 1 μF shunt capacity is respectively distributed from the pin 3, 4, and 7 to the electrical reference, and the pin 5 can be taken as a programmable input programmable control of the chip set real cut-off frequency of the theoretical value of a divider, four or sixteen divider [7].

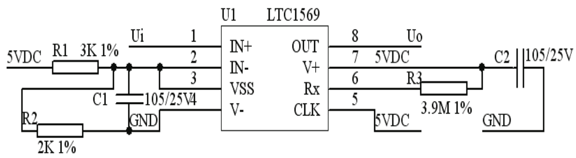

Fig. 5. Design on ten-order low-pass filtering circuit.

According to the Fig. 3.1, for the design on the cut-off frequency, there is a formula:

$$f_{cutoff} = \frac{128\ K\,(10\ K\ /\ R_3)}{1,4\,OR\ 16} Hz \tag{9}$$

In this paper, the cut-off frequency of the design on the circuit is about 20 Hz. According to the illustration of the frequency division in the chip materials about the cut-off frequency of the 5th base pin in LTC1569, in formula (9), the cut-off frequency is calculated by sixteen frequency division, R3 is a 3.9 MΩ resistor, and the following formula is got:

$$f_{cutoff} = \frac{128K(10K/3.9M)}{16} Hz = 20.5Hz \tag{10}$$

A test of frequency response is conducted on the preconditioning circuit as shown in Fig. 5 by using the signal generator and oscilloscope, and a frequency response curve of single channel is acquired discretely, aiming at the characteristic of the input signal frequency range, finally the results of the sweep on the frequency is obtained, as shown in Table 2.

Table 2. Frequency response test results list.

Test Frequency	Input Signal v_{p-p}	Output Signal v_{p-p}	Attenuation rate	Attenuation (dB)
(Hz)	(V)	(V)		
5	2.1667	2.197	1.013984	0.12
10	2.2084	2.159	0.996446	-0.03
20	2.2267	2.112	0.974754	-0.222
31.3	2.2001	1.519	0.701066	-3.08
60	2.2301	0.05	0.023077	-32.7

Then, a Bode diagram, as shown in the Fig. 6, can be concluded from Table 2. From this Figure, we can find that the actual cut-off frequency is about 30 Hz, fading down -32.7 dB at 60 Hz, whose sound interval is twice of that of the cut-off frequency.

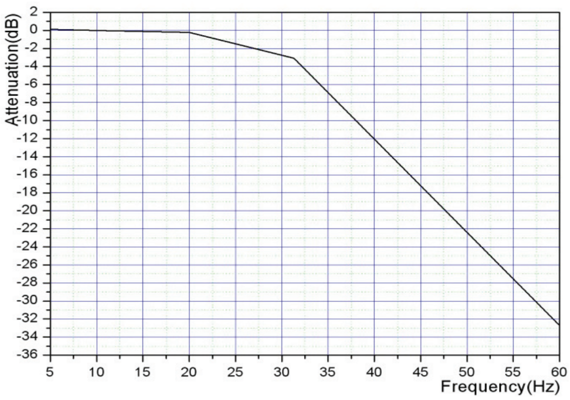

Fig. 6. Bode diagram of the frequency response.

5. Preconditioning Circuit in Engineering Application

5.1. Vibratory Comparison of Sine Frequency Sweep

As described in 3.1 section of the paper, errors caused by the axial acceleration could not be ignored; and as described in 4.1 of the paper, the anti-aliasing filtering module built with elliptic high-order low-pass filter could suppress the errors caused by the axial acceleration. Thus, a single directional sine frequency sweep was done on a three-axis vibratory micro-machined gyro vibrating along its sensitive axis through the high-frequency electro dynamic vibratory test stand, and a transverse comparison was conducted on the output of the gyro with the filtering module and those without the filter module, Fig. 7

was the results of the comparison. It could be seen that the output of the sensitive axis of the gyro added with the high-order low-pass filtering module was greatly attenuated, and the waveform was normal and available in the frequency range from DC to 20 Hz.

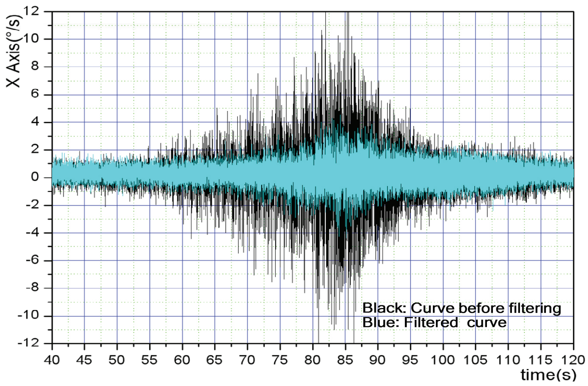

Fig. 7. Diagrammatic sketch of the comparison between the vibratory curves in sine frequency sweep.

5.2. Engineering Application

In a large-sized ground test of aircraft, with a certain aim, tests on the parameters of angular velocity were conducted respectively on two single mass capacitive vibrating micro-machined gyros with or without the output preconditioning circuit in the aircraft. The changing curves of the comparative data collected from the certain sensitive axis after the tests were shown in the Fig. 8.

6. Conclusions

Taking a single mass capacitive vibratory micro-machined gyro as the research object, a deep analysis was done in this paper on the errors of angular rate measurement caused by the axial acceleration in this type of gyro. Moreover, a preconditioning circuit with high-order low-pass filtering module was built by combining with the actual application, which suppressed the errors effectively and provided some important testing data and references for the engineering application of the single mass capacitive vibratory micro-machined gyro.

Now, the single mass capacitive vibratory micro-machined gyro with preconditioning circuit has been applied as a mature product into many large-sized ground tests for aircraft. It has acquired the information about the parameters of the angular velocity successfully and correctly, and been applied into the corresponding link in resolving the attitude of the aircraft.

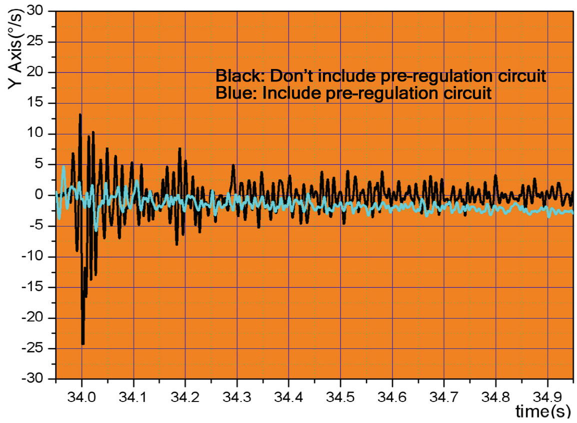

Fig. 8. Diagrammatic sketch of the comparison between the output curves of angular velocity of a sensitive axis in a flight test.

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