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Robust Diagonal Loading Algorithm for Worst-Case Performance Optimization

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Abstract: Traditional adaptive beamforming methods are known to undergo serious performance degradation in the presence of mismatches between the assumed array response and the true array response. In this paper, we propose a robust algorithm for worst-case performance optimization which belongs to the class of diagonal loading approach, and the diagonal loading factor is computed automatically from the array observation vectors without the need of specifying any user parameters. The proposed algorithm is based on the oblique projection of the signal steering vector to mitigate the effect of noise and interference. Moreover, in order to reduce the computational cost, the weight vector is updated iteratively via the gradient descent method. The proposed algorithm provides better robustness against the signal steering vector mismatches, yields improved array output performance and has a faster convergence rate. Some simulation results are presented to compare the performance of the proposed algorithm with sample matrix inversion (SMI) algorithm. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Robust adaptive beamforming, Diagonal loading, Signal steering vector mismatches, Oblique projection method, Worst-case performance optimization.

1. Introduction

In recent decades, adaptive beamforming finds numerous applications in areas such as radar, sonar, and wireless communication systems [1-3]. However, the performance of conventional adaptive beamforming techniques is known to degrade in the presence of array signal model errors which arise due to imprecisely known wavefield propagation conditions, imperfectly calibrated arrays, array perturbations, and direction pointing errors. The same happens when the number of snapshots is relatively small. In fact, there is a close relationship between the cases of steering vector errors and small-sample errors in some situations. Recently, robust adaptive beamforming has emerged as an efficient tool that provides the solution to this mismatch problem.

There are several efficient approaches are known to provide an improved robustness against some types of mismatches. One of the classic techniques is the linearly constrained minimum variance (LCMV) beamformer [4], which provides robustness against uncertainty in the signal look direction. To account for the signal steering vector mismatches, additional linear constraints (point and derivative constraints) can be imposed to improve the robustness of adaptive beamforming [5-6]. But, the beamformers lose degrees of freedom for interference suppression. Diagonal loading [7-8] has been a popular approach to improve the robustness against mismatch errors, random perturbations, and small sample support. The main drawback of the diagonal loading techniques is the difficulty to derive a closed-form expression for the diagonal loading term which relates the amount of diagonal loading with the upper bound of the mismatch uncertainty or the required level of robustness. The uncertainty constraint is imposed directly on the steering vector. The robust Capon beamforming proposed in [9-10] precisely computes the diagonal loading level based on ellipsoidal uncertainty set of array steering vector. An Eigen decomposition batch algorithm is used to compute the diagonal loading level which would also hit the wall of computational complexity.

In this paper, we propose an improved diagonal loading algorithm for worst-case performance optimization. We show clearly that a variable diagonal loading value in our algorithm can be automatically calculated based on the array observation vectors, which is incorporated at each iterative step. We show clearly how to efficiently compute the weight vector by using Lagrange multiplier method and gradient descent method. The proposed algorithm provides a significantly improved robustness against the signal steering vector mismatches and small training sample size, and has a low complexity cost compared with SMI algorithm. Simulation results validate substantial performance improvement of our proposed robust algorithm relative to SMI algorithm.

2. Problem Formulation

2.1. Mathematical Model

We assume that there are M sensors and D unknown sources impinging from directions $\{\theta_0, \theta_1, \ldots, \theta_{D-1}\}$. The sensors receive the linear combination of the source signals in the presence of additive white Gaussian noise (AWGN). Therefore, the received signal vector is given by

$$\begin{aligned}\boldsymbol{x}(k) &= s_0(k)\boldsymbol{a}(\theta_0) + \boldsymbol{i}(k) + \boldsymbol{n}(k) \\ &= \boldsymbol{A}\boldsymbol{S}(k) + \boldsymbol{n}(k)\end{aligned}, \tag{1}$$

where $\boldsymbol{x}(k) = [x_1(k), x_2(k), \ldots, x_M(k)]^{\mathrm{T}}$ is the observed signal, $\boldsymbol{a}(\theta_0)$ is the desired signal steering vector, $\boldsymbol{A} = [\boldsymbol{a}(\theta_0), \boldsymbol{a}(\theta_1), \ldots, \boldsymbol{a}(\theta_{D-1})]$ is the array manifold, $\boldsymbol{S}(k)$ is the vector of D transmitted signal, $\boldsymbol{i}(k)$ is the interference components, and $\boldsymbol{n}(k)$ is the noise components with zero mean. The aim of adaptive beamforming algorithms is to estimate the source signal $s_0(k)$ using only the observed data $\boldsymbol{x}(k)$. We write the estimated source signal as

$$y(k) = \boldsymbol{w}^{\mathrm{H}}\boldsymbol{x}(k) \quad , \tag{2}$$

where $\boldsymbol{w} = [w_1, w_2, \ldots, w_M]^{\mathrm{T}}$ is the complex weight vector, $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ stand for the transpose and Hermitian transpose, respectively. The array output SINR has the following form

$$\mathrm{SINR} = \frac{\sigma_s^2 \left|\boldsymbol{w}^{\mathrm{H}}\boldsymbol{a}(\theta_0)\right|^2}{\boldsymbol{w}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{i+n}}\boldsymbol{w}}, \tag{3}$$

where σ_s^2 is the signal power, and $\boldsymbol{R}_{\mathrm{i+n}}$ is the $M \times M$ interference-plus-noise covariance matrix

$$\boldsymbol{R}_{\mathrm{i+n}} = \mathrm{E}\{(\boldsymbol{i}(k) + \boldsymbol{n}(k))(\boldsymbol{i}(k) + \boldsymbol{n}(k))^{\mathrm{H}}\} \quad , \tag{4}$$

where $\mathrm{E}[\cdot]$ denotes statistical expectation.

2.2. SMI Algorithm

The problem of finding the maximum of (3) is equivalent to the following optimization problem

$$\min_{\boldsymbol{w}} \ \boldsymbol{w}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{i+n}}\boldsymbol{w} \ \text{ subject to } \boldsymbol{w}^{\mathrm{H}}\boldsymbol{a}(\theta_0) = 1 \tag{5}$$

From (5), the following solution can be found for the optimal weight vector

$$\mathrm{SINR}_{\mathrm{opt}} = \sigma_s^2\boldsymbol{a}^{\mathrm{H}}(\theta_0)\boldsymbol{R}_{\mathrm{i+n}}^{-1}\boldsymbol{a}(\theta_0) \tag{6}$$

Equation (6) gives an upper bound on the output SINR.

In practical applications, the exact interference-plus-noise covariance matrix $\boldsymbol{R}_{\mathrm{i+n}}$ is unavailable. Therefore, the sample covariance matrix

$$\hat{\boldsymbol{R}} = \frac{1}{N}\sum_{i=1}^{N}\boldsymbol{X}(i)\boldsymbol{X}^{\mathrm{H}}(i) \tag{7}$$

is used instead of $\boldsymbol{R}_{\mathrm{i+n}}$, where N is the number of snapshots available. This yields the generalized version of the well-known sample matrix inversion (SMI) algorithm [7]

$$\boldsymbol{w}_{\mathrm{SMI}} = \alpha\hat{\boldsymbol{R}}^{-1}\boldsymbol{a}(\theta_0), \tag{8}$$

where the scalar $\alpha = (\boldsymbol{a}^{\mathrm{H}}(\theta_0)\hat{\boldsymbol{R}}^{-1}\boldsymbol{a}(\theta_0))^{-1}$ is the normalization constant.

The fact that the sample array covariance matrix $\hat{\boldsymbol{R}}$ is used instead of the true $\boldsymbol{R}_{\mathrm{i+n}}$ is known to dramatically affect the performance of (8) as compared to the optimal beamformer in the case when the desired signal component is present in the training samples. Such a performance degradation caused by signal cancellation is commonly termed as signal self-nulling. It becomes especially strong in practical scenarios, when the knowledge of $\boldsymbol{a}(\theta_0)$ is imperfect.

3. Robust Diagonal Loading Algorithm via Worst-Case SINR Maximization

In order to overcome the above problem, we propose an improved diagonal loading algorithm, which provides excellent robustness against signal steering vector mismatches and small training data size, and improves the output performance.

Cost function of robust adaptive beamforming algorithm minimizes the mean output power subject to a linear constraint. Thereby, the optimization problem can be formulated as

$$\min_{\boldsymbol{w}} \boldsymbol{w}^{\mathrm{H}} \hat{\boldsymbol{R}} \boldsymbol{w} \quad \text{subject to } \boldsymbol{w}^{\mathrm{H}} \boldsymbol{a}(\theta_0) = 1 \tag{9}$$

To improve the overall robustness against the signal steering vector mismatches and other types of mismatches, we consider a further extension of the constrained minimization problem (9). We assume that the norm of the error matrix $\boldsymbol{\Delta}$ can be bounded by some known constant r, $\|\boldsymbol{\Delta}\| \le r$. Then, the actual covariance matrix is $\hat{\boldsymbol{R}}_{\mathrm{d}} = \hat{\boldsymbol{R}} + \boldsymbol{\Delta}$. Based on the worst-case performance optimization, the new formulation of (9) can be written as

$$\min_{\boldsymbol{w}} \max_{\|\boldsymbol{\Delta}\| \le r} \boldsymbol{w}^{\mathrm{H}} (\hat{\boldsymbol{R}} + \boldsymbol{\Delta}) \boldsymbol{w} \quad \text{subject to } \boldsymbol{w}^{\mathrm{H}} \boldsymbol{a}(\theta_0) = 1 , \tag{10}$$

where the matrix $\boldsymbol{\Delta}$ takes into account all mismatches that may be caused, for example, by small training sample size, data nonstationarity, and quantization errors.

To solve (10), we can first solve the simpler problem [11]

$$\min_{\boldsymbol{\Delta}} - \boldsymbol{w}^{\mathrm{H}} (\hat{\boldsymbol{R}} + \boldsymbol{\Delta}) \boldsymbol{w} \quad \text{subject to } \|\boldsymbol{\Delta}\| \le r \tag{11}$$

We see that (11) sets an infinite number of inequality constraint, and stress that (11) guarantees that the distortionless response will be maintained for the worst-case mismatch, i.e., for a mismatch.

To solve (11), we can use Lagrange multiplier method to obtain

$$\boldsymbol{\Delta} = r \frac{\boldsymbol{w}\boldsymbol{w}^{\mathrm{H}}}{\|\boldsymbol{w}\|} \tag{12}$$

In order to mitigate the effect of noise and interference, we can impose another constraint on oblique projection of the signal steering vector $\boldsymbol{a}(\theta_0)$ [12]. Consequently, the beamforming problem (10) is reformulated as follows

$$\min_{\boldsymbol{w}} \boldsymbol{w}^{\mathrm{H}} (\hat{\boldsymbol{R}} + r\boldsymbol{I}) \boldsymbol{w} \quad \text{subject to } \boldsymbol{w}^{\mathrm{H}} \hat{\boldsymbol{a}}_{\mathrm{p}} = 1 , \tag{13}$$

where $\hat{\boldsymbol{a}}_{\mathrm{p}}$ is the oblique projection steering vector

$$\hat{\boldsymbol{a}}_{\mathrm{p}} = E_{\boldsymbol{a}(\theta_i)A_i} \boldsymbol{a}(\theta_0) , \tag{14}$$

where $E_{\boldsymbol{a}(\theta_i)A_i} = \boldsymbol{a}(\theta_i)(\boldsymbol{a}^{\mathrm{H}}(\theta_i)\boldsymbol{R}_{\mathrm{A}}^{+}\boldsymbol{a}(\theta_i))\boldsymbol{a}(\theta_i)\boldsymbol{R}_{\mathrm{A}}^{+}$ and $A_i = [\boldsymbol{a}(\theta_1),...,\boldsymbol{a}(\theta_{D-1})]$, here $\boldsymbol{R}_{\mathrm{A}}^{+} = [\boldsymbol{ASA}^{\mathrm{H}}]^{+}$ is the pseudo-inverse matrix.

We consider a linear combination of $\hat{\boldsymbol{R}}$ and $\boldsymbol{R}_{\mathrm{i+n}}$, which has the form [13]

$$\hat{\boldsymbol{R}}_{\mathrm{d}} = \alpha \boldsymbol{R}_{\mathrm{i+n}} + \beta \hat{\boldsymbol{R}} , \tag{15}$$

where $\alpha > 0$ and $\beta > 0$. In our paper, we assume the initial value of $\boldsymbol{R}_{\mathrm{i+n}}$ is identity matrix $\boldsymbol{I}$ [14]. We rewrite (15) as:

$$\hat{\boldsymbol{R}}_{\mathrm{d}} = \alpha \boldsymbol{I} + \beta \hat{\boldsymbol{R}} \tag{16}$$

Then, we can obtain the enhanced covariance matrix as follow

$$\tilde{\boldsymbol{R}} = \hat{\boldsymbol{R}} + \frac{\alpha}{\beta} \boldsymbol{I} \tag{17}$$

Let $\tilde{\boldsymbol{R}}$ instead of $\boldsymbol{R}_{\mathrm{i+n}}$ in (5), the problem can be converted to

$$\min_{\boldsymbol{w}} \boldsymbol{w}^{\mathrm{H}} \tilde{\boldsymbol{R}} \boldsymbol{w} \quad \text{subject to } \boldsymbol{w}^{\mathrm{H}} \hat{\boldsymbol{a}}_{\mathrm{p}} = 1 \tag{18}$$

Using the Lagrange multiplier method to solve the optimization problem, we obtain the Lagrange function $J(\boldsymbol{w}, \lambda)$

$$J(\boldsymbol{w}, \lambda) = \frac{1}{2} \boldsymbol{w}^{\mathrm{H}} \tilde{\boldsymbol{R}} \boldsymbol{w} + \lambda (\boldsymbol{w}^{\mathrm{H}} \hat{\boldsymbol{a}}_{\mathrm{p}} - 1) \tag{19}$$

The gradient vector of (17) is given by

$$\boldsymbol{\psi} = \tilde{\boldsymbol{R}} \boldsymbol{w} + \lambda \hat{\boldsymbol{a}}_{\mathrm{p}} \tag{20}$$

Let the gradient vector (20) is equal to zero, we can get the optimal weight vector

$$\boldsymbol{w}_{\mathrm{opt}} = -\lambda \left(\hat{\boldsymbol{R}} + \frac{\alpha}{\beta} \boldsymbol{I} \right)^{-1} \hat{\boldsymbol{a}}_{\mathrm{p}} \tag{21}$$

Contrast (13) and (21), we derive that diagonal loading factor *r* can be replaced by *α/β*. So the first step is to obtain *α* and *β* by minimizing the mean squared error of $\tilde{\boldsymbol{R}}$:

$$\mathrm{MSE} = \mathrm{E}\left\{ \left\| \tilde{\boldsymbol{R}} - \boldsymbol{R} \right\|^2 \right\} , \tag{22}$$

where $\boldsymbol{R} = \mathrm{E}\{\boldsymbol{x}(k)\boldsymbol{x}^{\mathrm{H}}(k)\}$ denotes the theoretical covariance matrix of the array output vector. The MSE minimization problem is derived as follows [10]

$$\mathrm{E}\left\{\left\|\tilde{\boldsymbol{R}}-\boldsymbol{R}\right\|^2\right\}=\mathrm{E}\left\{\left\|\alpha\boldsymbol{I}-(1-\beta)\boldsymbol{R}+\beta\left(\hat{\boldsymbol{R}}-\boldsymbol{R}\right)\right\|^2\right\} \\ =\left\|\alpha\boldsymbol{I}-(1-\beta)\boldsymbol{R}\right\|^2+\beta^2\mathrm{E}\left\{\left\|\hat{\boldsymbol{R}}-\boldsymbol{R}\right\|^2\right\} \quad (23)$$

The unconstrained minimization of (22) can be written as

$$\min_{\alpha,\beta}\ \mathrm{E}\left\{\left\|\tilde{\boldsymbol{R}}-\boldsymbol{R}\right\|^2\right\}=\min_{\alpha,\beta}\ \alpha^2M-2\alpha(1-\beta)tr(\boldsymbol{R})+\chi(\beta), \quad (24)$$

where $\chi(\beta)=(1-\beta)^2\left\|\boldsymbol{R}\right\|^2+\beta^2\mathrm{E}\left\{\left\|\hat{\boldsymbol{R}}-\boldsymbol{R}\right\|^2\right\}$.

Computing the gradient of (24) and then equating it to zero, this yields the optimal solution for α (β is fixed)

$$\alpha_0=\frac{(1-\beta)tr(\boldsymbol{R})}{M} \quad (25)$$

Next, inserting (25) into (24), we can gain another unconstrained minimization problem

$$\min_{\beta}\ \frac{(1-\beta)^2\left\{\left\|\boldsymbol{R}\right\|^2M-tr^2(\boldsymbol{R})\right\}}{M}+\beta^2\mathrm{E}\left\{\left\|\hat{\boldsymbol{R}}-\boldsymbol{R}\right\|^2\right\} \quad (26)$$

The optimal solution for β is derived by computing the gradient of (26) and equating it to zero

$$\beta_0=\frac{\eta}{\eta+\rho}, \quad (27)$$

where $\eta=\frac{\left\|\boldsymbol{R}\right\|^2M-tr^2(\boldsymbol{R})}{M}$ and $\rho=\mathrm{E}\left\{\left\|\hat{\boldsymbol{R}}-\boldsymbol{R}\right\|^2\right\}$.

To estimate α_0 and β_0 from the available training sample data, we need estimation value of ρ and η, respectively. In practice, the exact covariance matrix $\boldsymbol{R}$ is unavailable. Therefore, $\boldsymbol{R}$ is replaced by $\hat{\boldsymbol{R}}$ to estimate η

$$\hat{\eta}=\frac{\left\|\hat{\boldsymbol{R}}\right\|^2M-tr^2(\hat{\boldsymbol{R}})}{M} \quad (28)$$

Let $\hat{\boldsymbol{r}}_m$ and $\boldsymbol{r}_m$ denote the mth columns of $\hat{\boldsymbol{R}}$ and $\boldsymbol{R}$ respectively. Consequently, we have

$$\mathrm{E}\left\{\left\|\hat{\boldsymbol{R}}-\boldsymbol{R}\right\|^2\right\}=\sum_{m=1}^{M}\mathrm{E}\left\{\left\|\hat{\boldsymbol{r}}_m-\boldsymbol{r}_m\right\|^2\right\} \quad (29)$$

According to [13], we can estimate ρ as

$$\hat{\rho}=\frac{1}{N}\sum_{m=1}^{M}\left[\frac{1}{N}\sum_{k=1}^{N}\left\|\boldsymbol{x}(k)\right\|^2\cdot\left|x_m(k)\right|^2-\left\|\hat{\boldsymbol{r}}_m\right\|^2\right], \\ =\frac{1}{N^2}\sum_{k=1}^{N}\left\|\boldsymbol{x}(k)\right\|^4-\frac{1}{N}\left\|\hat{\boldsymbol{R}}\right\|^2 \quad (30)$$

where $\boldsymbol{x}_m(k)$ denotes the mth element of $\boldsymbol{x}(k)$. Applying (28) and (30), we can obtain the estimation value of α_0 and β_0, respectively

$$\hat{\beta}_0=\frac{\hat{\eta}}{\hat{\eta}+\hat{\rho}} \quad (31)$$

$$\hat{\alpha}_0=\frac{\left(1-\hat{\beta}_0\right)tr\left(\hat{\boldsymbol{R}}\right)}{M} \quad (32)$$

Eventually, the estimation value of $\tilde{\boldsymbol{R}}$ is obtained

$$\tilde{\boldsymbol{R}}_{\mathrm{opt}}=\hat{\boldsymbol{R}}+\frac{\hat{\alpha}_0}{\hat{\beta}_0}\boldsymbol{I} \quad (33)$$

Substituting (33) into (21), we can derive as

$$\hat{\boldsymbol{w}}_{\mathrm{opt}}=-\lambda\left(\hat{\boldsymbol{R}}+\frac{\hat{\alpha}_0}{\hat{\beta}_0}\boldsymbol{I}\right)^{-1}\hat{\boldsymbol{a}}_{\mathrm{p}} \quad (34)$$

Inserting (34) into the linear constraint of (18), we can drive the parameter λ

$$\lambda=-\frac{1}{\hat{\boldsymbol{a}}_{\mathrm{p}}^{\mathrm{H}}\left(\hat{\boldsymbol{R}}+\frac{\hat{\alpha}_0}{\hat{\beta}_0}\boldsymbol{I}\right)^{-1}\boldsymbol{a}_{\mathrm{p}}} \quad (35)$$

Using (35), the optimal weight vector is rewritten as

$$\hat{\boldsymbol{w}}_{\mathrm{opt}}=\frac{\left(\hat{\boldsymbol{R}}+\frac{\hat{\alpha}_0}{\hat{\beta}_0}\boldsymbol{I}\right)^{-1}\hat{\boldsymbol{a}}_{\mathrm{p}}}{\hat{\boldsymbol{a}}_{\mathrm{p}}^{\mathrm{H}}\left(\hat{\boldsymbol{R}}+\frac{\hat{\alpha}_0}{\hat{\beta}_0}\boldsymbol{I}\right)^{-1}\hat{\boldsymbol{a}}_{\mathrm{p}}} \quad (36)$$

In order to avoid computing the covariance matrix inversion in (36), we adopt the gradient descent method to update the weight vector

$$\boldsymbol{w}(k+1)=\boldsymbol{w}(k)-\mu\boldsymbol{\psi}(k), \quad (37)$$

where μ is the optimal step size. Let $\tilde{\boldsymbol{R}}_{\mathrm{opt}}$ instead of $\tilde{\boldsymbol{R}}$ in (20), we rewrite the gradient vector

$$\hat{\boldsymbol{\psi}}=\left(\hat{\boldsymbol{R}}+\frac{\hat{\alpha}_0}{\hat{\beta}_0}\right)\boldsymbol{w}(k)-\frac{1}{\hat{\boldsymbol{a}}_{\mathrm{p}}^{\mathrm{H}}\left(\hat{\boldsymbol{R}}+\frac{\hat{\alpha}_0}{\hat{\beta}_0}\boldsymbol{I}\right)^{-1}\hat{\boldsymbol{a}}_{\mathrm{p}}}\hat{\boldsymbol{a}}_{\mathrm{p}} \quad (38)$$

Inserting (38) into (37), we can rewrite the weight vector

$$\boldsymbol{w}(k+1)=\left(\boldsymbol{I}-\mu\cdot\left(\hat{\boldsymbol{R}}+\frac{\hat{\alpha}_0}{\hat{\beta}_0}\right)\right)\boldsymbol{w}(k)+\frac{\mu\hat{\boldsymbol{a}}_{\mathrm{p}}}{\hat{\boldsymbol{a}}_{\mathrm{p}}^{\mathrm{H}}\left(\hat{\boldsymbol{R}}+\frac{\hat{\alpha}_0}{\hat{\beta}_0}\boldsymbol{I}\right)^{-1}\hat{\boldsymbol{a}}_{\mathrm{p}}} \quad (39)$$

4. Simulation Results

In this section, some simulation results are presented to evaluate the performance of the proposed algorithm. We assume a uniform linear array of $M=10$ omnidirectional sensors spaced half a wave length apart. The sample covariance matrix is computed based on $N=100$ data snapshots. All results are calculated based on 100 independent simulation runs. We assume two interfering sources with random waveforms and directions of arrival (DOAs) – 50º and 50º, respectively. The proposed beamforming algorithm is compared to SMI beamformer.

4.1. Example 1: Exactly Known Signal Steering Vector

In this example, the plane wave signal is assumed to imping on the array from $\theta_0=0^\circ$. Fig. 1 displays the performance of the methods tested versus the number of snapshots for the fixed $\mathrm{SNR}=10\,\mathrm{dB}$. Fig. 2 shows the performance of the algorithms versus the SNR for the fixed training data size $N=100$. In this scenario, we note that our improved diagonal loading algorithm outperforms the SMI algorithm and makes the mean output SINR close to the optimal value.

4.2. Example 2: Signal Look Direction Mismatch

We assume that the both the presumed and actual signal spatial signatures are plane waves impinging from the DOAs $\theta_0=0^\circ$ and $\theta_0=2^\circ$, respectively. Fig. 3 displays the performance of the methods versus the number of snapshots for $\mathrm{SNR}=10\,\mathrm{dB}$. The performance of the algorithms versus the SNR for the fixed training data size $N=100$ is shown in Fig. 4.

In this example, we note the SMI algorithm is very sensitive even to slight mismatches, and the proposed algorithm can improve the performance of SMI algorithm. Our improved diagonal loading algorithm consistently enjoys better performance, and provides better robustness against signal steering vector mismatches and the small training sample size.

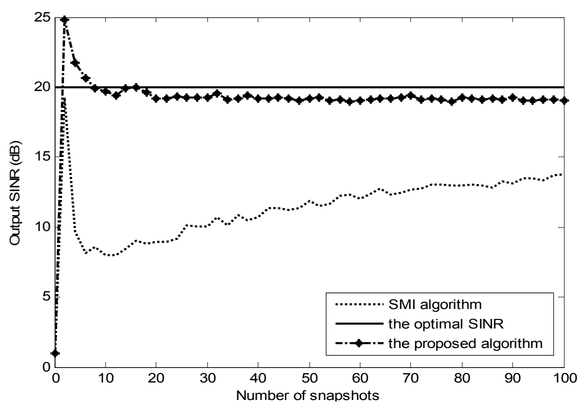

Fig. 1. Output SINR versus N.

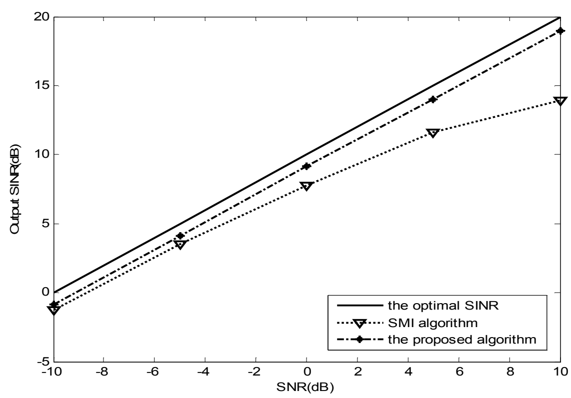

Fig. 2. Output SINR versus SNR.

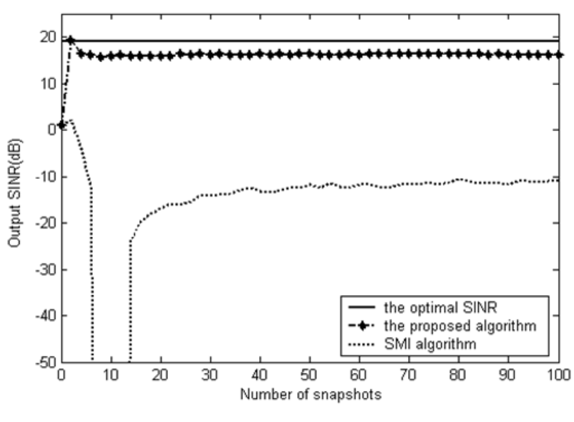

Fig. 3. Output SINR versus N.

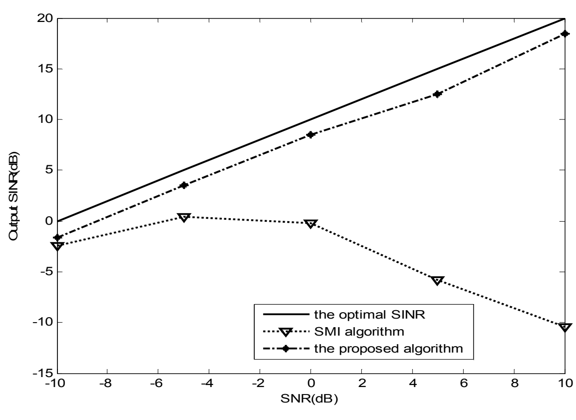

Fig. 4. Output SINR versus SNR.

6. Conclusions

In this paper, we present a robust adaptive beamforming algorithm via the oblique projection method and worst-case performance optimization. The proposed algorithm belongs to the diagonal loading method, but the diagonal loading factor is derived instead of relaying on experimental experience. In order to reduce the computational cost, the weight vector is iteratively updated by the gradient descent method. Our proposed algorithm provides well robustness against signal steering vector mismatches and small training data size, and enjoys a significantly improved performance as compared with SMI algorithm. The numerical experiments have been carried out to illustrate the superior performance of the proposed method for signal steering vector mismatches.

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