www.sensorsportal.com

Sensors & Transducers

http://www.sensorsportal.com

Sensitivity of Miniaturized Photo-elastic Transducer for Small Force Sensing

[1] Naceur-Eddine KHELIFA, [2] Marc HIMBERT

[1] Laboratoire Commun de Métrologie LNE-Cnam, 1 rue Gaston Boissier, F75724 Paris, France
[2] Laboratoire Commun de Métrologie LNE-Cnam, 61 rue du Landy, La plaine, 93210, France
[1] Tel.: +33 (0) 40 43 37 78, fax: +33 (0) 1 40 43 37 37
[1] E-mail: naceur.khelifa@cnam.fr

Received: 14 November 2014 /Accepted: 15 December 2014 /Published: 31 January 2015

Abstract: The sensitivity of a force sensor based on photo-elastic effect in a monolithic Nd-YAG laser depends strongly on the geometrical shape and dimensions of the laser medium. The theoretical predictions of sensitivity are in good agreement with first results obtained with a plano-concave cylindrical crystal of (4×4) mm and some values reported by other groups. However, for small size of the laser sensor, the developed model predicts sensitivity, about 30 % higher than the values given by available experiments. In this paper, we present experimental results obtained with a force sensor using a miniaturized monolithic cylindrical Nd-YAG laser of dimensions (2×3) mm with suitable optical coatings on its plane end faces. The new result of measurement concerning the sensitivity has allowed us to refine the theoretical model to treat photo-elastic force sensors with small dimensions. *Copyright © 2015 IFSA Publishing, S. L.*

Keywords: Stress birefringence, Solid-state laser, Photo-elastic effect, Small force sensor, Beat frequency, Sensitivity.

1. Introduction

The metrological characterization of scientific and industrial instrumentation employing small forces remains an intermediate target to ensure that measurements of forces lower than 0.5 N are traceable to the S. I. system of units [1-2]. Recently, a number of laboratories have shown a particular resurgence and interest to connect results of small forces measurements obtained with various devices [3-4]. Indeed, development of micro-robots for microsurgery for the retina or colonoscopy, measurement of stiffness of atomic force microscope cantilevers and handling fragile objects, all require one to know the forces exerted on the manipulated objects [5]. In these areas of activity, the implementation of forces in the range of 0.5 N to 100 μN by instruments has led to the necessity to develop ways to measure and control the forces which are generated by a given device. In fact, measurements based on different physical principles can be compared, at least in the range of their overlap. Systems of different configurations, based on the exploitation of the photo-elastic effect in a crystal, have been developed and used to measure small forces. The results obtained are encouraging despite the difficulties raised for the generation and application of small forces with good reproducibility [6-7]. It remains to resolve the difficulties in achieving monode Nd-YAG lasers and to reduce the effect of misalignment from sources of vibrations. For this purpose, it is necessary to use monolithic configuration with the resonator mirrors coated directly upon the end face of the crystal. To achieve single longitudinal mode operation of the laser emission, we need resonator length shorter than 1 mm.

Here, a focus on photo-elastic force sensors with high sensitivity is made. The reasons for this approach are related, to the fact that in a previous work we observed significant discrepancy between available experimental values of sensitivity and those predicted by the theoretical model based on some assumptions that are discussed in Section II, but also to the growing interest in many areas in the implementation of various devices for generation and measurement of small forces. To improve the theory, we started to make a photo-elastic force sensor whose dimensions lie in a range for which there is no experimental data, i.e. length×diameter is in between 3 and 12 mm^2. The ability to use a monolithic solid-state laser is that of a cylindrical Nd-YAG (neodymium-yttrium aluminum garnet) rod of 2 mm in length and 3 mm in diameter having plane end-faces. The resonance cavity of the solid state laser, represented by the end face of the crystal, was plan-plan in comparison to the first one which was plano-concave. The input face is coated for maximum reflectivity at 1064 nm and high transmission at 808 nm, while the output face is coated for about 1 % transmission at 1064 nm.

In the next Section, a brief reminder on the theoretical photo-elastic effect in a laser will be done. Section III will describe the new laser, Nd-YAG playing the role of sensitive element of the force sensor, as well as the experimental set-up. The results will be presented and discussed in connection with theoretical predictions in Section IV.

2. Induced Birefringence by Mechanical Stress

The birefringence is induced in an Nd-YAG laser rod under mechanical stress generated by external force applied on the crystal, as illustrated in Fig. 1.

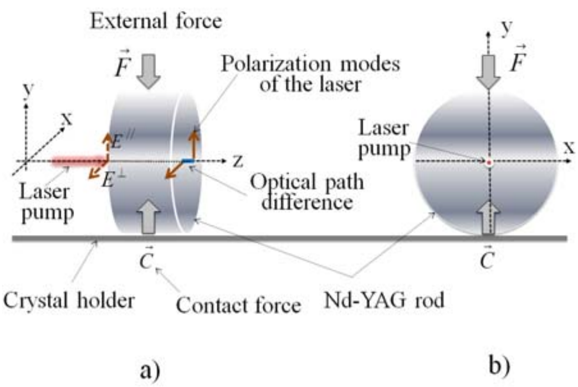

Fig. 1. Orientation of the photo-elastic rod relative to the direction of the applied force. a)- front view; b)- side view.

In order to link the force intensity to the induced birefringence (based on photo-elastic effect), we assume the stress distribution over the length of the laser rod is uniform. The stress components σ_y and σ_x along the principal directions of the rod, induced by the applied force F, are represented by [8]:

$$\begin{cases} \sigma_x = \dfrac{\alpha}{\pi \ell d} \times F \\ \sigma_y = -\dfrac{\beta}{\pi \ell d} \times F \end{cases}, \tag{1}$$

where ℓ and d are the length and diameter of the cylindrical crystal respectively, while the parameters α and β depend on the nature of the contact between the laser crystal and its support and on the orientation of the pumping beam at 808 nm relative to the principal axis of the Nd-YAG crystal.

The relative stress, along the orthogonal directions x and y, induced in the center of the rod is then given by relation:

$$\sigma_x - \sigma_y = \frac{(\alpha + \beta)}{\pi \ell d} \times F \tag{2}$$

The induced frequency shift between the frequencies of the orthogonal polarizations $E^{//}$ and $E^{\perp}$ of the oscillating laser mode is expressed as a function of external force by:

$$\Delta\nu = \nu_q^{\perp} - \nu_q^{//} = \frac{(\alpha + \beta)\, C_{PE}^{\lambda_q}}{\pi n} \times \frac{\nu_q}{\ell d} \times F\,, \tag{3}$$

where $C_{PE}^{(\lambda_q)}$ is the photo-elastic constant of the Nd-YAG crystal. For laser light of wavelength $\lambda_q = 1064$ nm (frequency $\nu_q \cong 281.76$ THz), the theoretical value of the relative stress-optic coefficient is given by [9]:

$C_{PE}^{(\lambda_q)}[111] = 1{,}25 \times 10^{-12}\ \mathrm{m}^2 \times \mathrm{N}^{-1}$. We have noted that this value is very close to the measured one [10].

The sensitivity of the force sensor is then approximated by the relation:

$$S(\ell, d) \cong \frac{(\alpha + \beta)\, C_{PE}^{\lambda_q}}{\pi n} \times \frac{\nu_q}{\ell d} \tag{4}$$

In an ideal configuration, one edge of the cylindrical rod is in contact with a flat surface and the rod is illuminated by the laser beam along its axis of revolution. In this case α and β are related via [11]:

$$(\alpha + \beta) \cong 8/\pi \tag{5}$$

These conditions are not easy to implement in the case of a monolithic laser of small diameter.

To interpret the observed differences between measurements and theoretical predictions already used before [12-13], we sought to correctly model value of the form factor $(\alpha+\beta)$.

For value of $(\alpha+\beta)$ given by (5), the sensitivity is reduced to:

$$S(\ell, d) \cong 492{,}8 \times \frac{1}{\ell\, d} \tag{6}$$

In (6), the sensitivity is in $\mathrm{MHz\,N^{-1}}$ if ℓ and d are in mm.

Precisely, the results of this approach have been confronted to experiments. As we will show in Section IV, the model used is not suitable especially for laser sensors using a small crystal. To improve this model, experimental values of sensitivity for different sizes $(\ell \times d)$ of the monolithic laser transducer are of great interest and particularly for crystal size $(\ell \times d)$ where no data are available.

3. Experiment

For technical reasons, we used a cylindrical crystal having a length of 2 mm and a diameter of 3 mm with parallel end faces (Fig. 2). The pumping face is coated to have HR@1064 nm and HT@808 nm and the second one is coated with HT@808 nm and HR (99.5 %) @1064 nm. The laser is bonded to its holder formed by a rectangular channel having a width of 3.5 mm and a depth of 3 mm, machined in an aluminum part. The temperature of the rod and the holder assembly is stabilized to better than ± 0.02 °C by using a proportional-integral-derivative (P. I. D) controller.

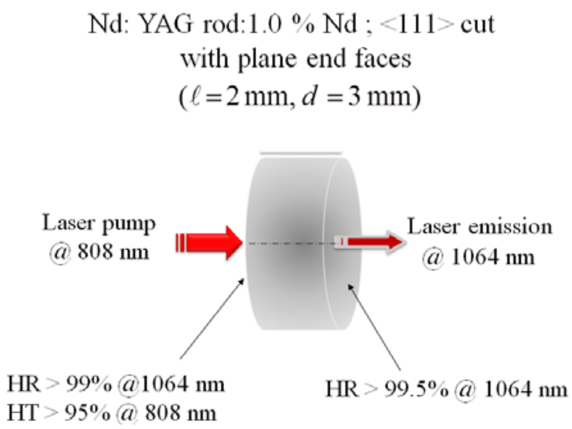

Fig. 2. Monolithic configuration of the Nd-YAG laser used as sensing element of the force sensor.

In this experiment a new procedure is employed for loading the laser rod by using a mass standard. In fact, a rectangular (3×2) mm flexible blade made from a sheet of HAVAR material (a non-magnetic alloy of Co/Cr/Ni/Fe/W/..., having a high strength) with a thickness of 10 µm is bonded to the upper portion of the laser and plays the role of a pan as in an electromagnetic compensation balance.

The photography of Fig. 3 gives an overview of the experimental set-up. One can find more details on the experimental design in [12].

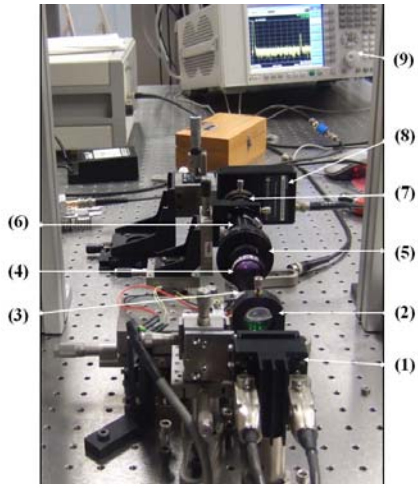

Fig. 3. View of the photo-elastic force sensor device.

The various components as indicated in this Fig. 3 are: (1) - laser diode system @ 808 nm; (2) - lens; (3) - Nd-YAG rod; (4) - bandpass filter for residual beam @ 808 nm; (5) - $\pi/4$ polarizer; (6) - adjustable pinhole; (7) - lens; (8) - nanosecond photo-detector; (9) - spectrum analyzer.

4. Results

The response and the sensitivity of this photo-elastic sensor were analyzed by measuring the deviation of the beat note frequency when applying, on the top of the crystal laser, deadweight linked to a mass standard, m_e.

4.1. Response of the Sensor

Mass standard of 0.1 g (≈ 1 mN) to 20 g (≈ 200 mN) were used to study the response of the photo-elastic sensor. As shown in Fig. 4a, the response is almost linear over the range studied and is checked on a wider range. In fact, same results were reported before for other size of photo-elastic Nd-YAG crystal of the force sensor [6-7, 13].

Using the least squares method, we deduce, from results reported in Fig. 4a, the mean value of the sensitivity and its uncertainty to:

$$S_{\exp} \cong 0.5947\ (0.0034)\ \ \mathrm{MHz \times g^{-1}}$$

In terms of force this value is equivalent to:

$$S_{\exp} \cong 60.63\,(0.35)\ \text{MHz}\times\text{N}^{-1}$$

To examine the response of the sensor to the low forces, we have considered only the beat frequency measurements observed when the crystal of the force sensor is loaded by mass standards smaller than 2 g (about 20 mN). From Fig. 4b; which represents a part of Fig. 4a, we deduce the sensitivity of the sensor by the same method used before.

$S \cong 0.5686\ (0.0119\,\text{MHz}\times\text{g}^{-1}$. Or, in terms of sensitivity to a force: $S = 57.97\,(1.21)\,\text{MHz}\times\text{N}^{-1}$.

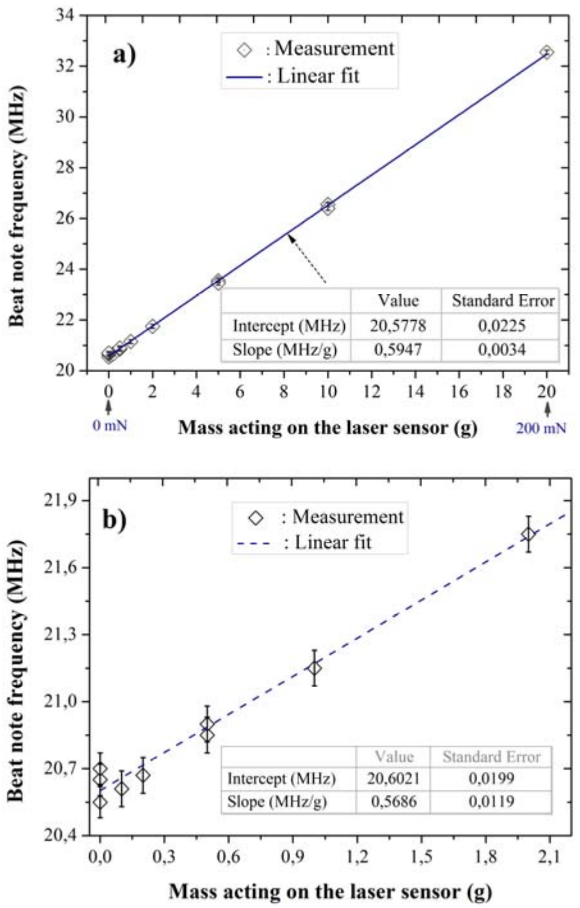

Fig. 4. Response of the sensor under the action of deadweight. b) Is a zoom, at low loads, of Fig. 4a.

Comparing these two results, we can conclude that the sensor sensitivity to weak forces does not change significantly (lower by about 3 % compared with the value deduced from a wider range of loading). However, we get an uncertainty on the derived sensitivity three times larger, because of the level of uncertainty on the frequency of the observed beat note between the two orthogonal polarizations.

To study the reproducibility of measurements of frequencies we made several series of measurements using a calibrated mass of 5 g, by measuring the frequency of the beat signal when the laser crystal was free and when it was subjected to the action of the net weight of the mass standard, m_e placed and staying on the top of the crystal in a perfect balance in the local gravitational field. The condition to take correctly a frequency measurement is that mass standard placed on the crystal must remain truly balanced on the rod so that the load is distributed equally in each cycle of mass insertion and removal.

This phase of measurement is the most critical because it affects the reproducibility of the stress exerted by the dead weight on the crystal. Despite the fact that the procedure of insertion and removal of the mass standard is performed manually, the measurements turn out to be reproducible. The results of the beat frequency measurement are reported in Table 1.

Table 1. Mean values of observed beat frequency when the crystal is unloaded then loaded with deadweight of 5 g.

State of the laser crystal	Mean value (MHz)	Repeatability (MHz)	Reproducibility (MHz)
Unloaded	20.55	0.04	0.08
loaded	23.51	0.05	0.1

The mean value and the associated uncertainty of repeatability are derived from the average of an actual range of ten values of measured frequencies for each state of the laser crystal.

The uncertainty of reproducibility is evaluated from the measurement of several repeated cycles of applying and removing the mass standard of 5 g. During successive cycles of loading and withdrawal of the force transducer, the mean frequency of the beat signal changes from 20.55 to 23.51 MHz. These values are obtained by averaging the observed frequencies measured alternately when the mass standard is inserted then removed. For a given situation of the sensitive element of the transducer, the frequency of the beat note is measured with a repeatabilty better than 50 kHz. However, the reproducibility of the frequencies observed during a series of about fifteen successive loadings and removals is of the order of 100 kHz. This large scatter is mainly generated by the reproducibilty of internal stress distribution, induced in the center of the laser sensor, after each cycle of measurement when the sensing element is loaded by mass standard. A priori, one could reduce this limitation by improving the system of loading and unloading the sensitive element of the transducer.

Experimental sensitivity of the transducer, determined here by using only one mass standard, is given by:

$$S^{*}_{\exp} = \frac{\Delta\nu^{(load)} - \Delta\nu^{(free)}}{m_e} \cong 0.592\ \text{MHz}\times\text{g}^{-1}$$

Or, $S^*_{\exp} \cong 60.35\ \text{MHz}\times\text{N}^{-1}$; this value is very close to $S_{\exp} = 60.63\ \text{MHz}\times\text{N}^{-1}$, deduced from linear fit of results reported in Fig. 4a.

The associated uncertainty of repeatability is evaluated to:

$$u_{repeat}\,(S_{\exp}) \cong 1.30\ \text{MHz}\times\text{N}^{-1} \tag{7}$$

If one considers the most pessimistic situation related to the reproducibility of frequencies measurement in a series of a mass standard insertions and removals, one obtains:

$$u_{reprod}\,(S_{\exp}) \cong 2.61\ \text{MHz}\times\text{N}^{-1} \tag{8}$$

This last uncertainty is the main limitation in terms of measurement for this kind of force sensor.

4.2. Sensitivity and Size of the Sensing Element of the Force Sensor

The new values of sensitivity and his uncertainty are reported in Fig. 5 and compared to the experimental measurements available for different force transducer reported before by others authors [11, 14]. In this figure, only results obtained with a photo-elastic force sensor using monolithic Nd-YAG laser are considered for comparison. It can be seen, from this figure, that the predicted sensitivities by (6) corresponding to the ideal situation, discussed in Section II, are clearly superior to experimental values when the dimensions ($\ell \times d$) of the laser sensor are small. As one can see from Fig. 5, this is the case of the two available experimental values of sensitivity, corresponding to $(\ell \times d) = 3$ and $6\,\text{mm}^2$.

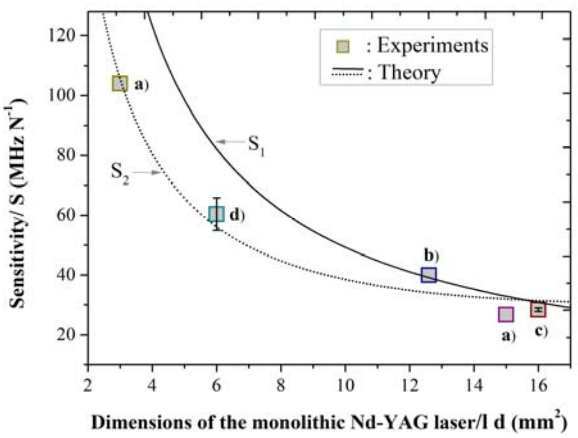

Fig. 5. Sensitivity of photo-elastic force sensor versus dimensions of the monolithic sensing element Nd-YAG laser. a) - [11]; b) - [14]; c) - [13]; d) - this work.

In Fig. 5, the solid line is given by $S_1 = 492{,}8/(\ell\, d)$ and corresponds to a simple theoretical model (Section II) while the dot one, $S_2 = 310/(\ell\, d) + 3\ell\, d/4$, is an empirical relation which provides sensitivity values close enough to experimental results. Thus, the new results show that, when the sizes of the sensor are small, the sensitivity of the photo-elastic force sensor depends, in a relatively complex way, on the nature of the contact and on the localization of the applied force associated with the calibrated weight. Therefore, for the small size of the sensitive element, it is difficult to obtain high sensitivity with good reproducibility required for the measurement of small forces at the micronewton level with acceptable accuracy.

However, this type of sensor should prove useful for force measurement over a range of five orders of magnitude, namely from less than 100 µN to more than 10 N. This wide range is very useful when trying to make comparisons with results obtained with other devices used in small force measurement.

Table 2. Values of the physical quantities used for the determination of the deadweight F_{dw}.

Quantity	Value	Uncertainty
Local gravity ($\text{m}\times\text{s}^{-2}$)	$g_{\ell oc} = 9.8093810$	3×10^{-7}
Mass standard (g)	$m_e = 0.1$	5×10^{-6}
Mass density ($\text{kg}\times\text{m}^{-3}$)	$\rho_e = 7850$	4×10^{-3}
Air density ($\text{kg}\times\text{m}^{-3}$)	$\rho_a = 1.1950$	2×10^{-4}

4.3. Force Measurement

The force acting on the laser sensor when a mass standard m_e is balanced on the rod, corresponds to the dead weight in the local gravity g_{loc}.

It is given by:

$$F_{dw} = m_e g_{loc}\,(1 - \rho_a/\rho_e)\,, \tag{9}$$

i.e. the weight in vacuum minus the buoyancy correction. ρ_a and ρ_e are respectively the air density and the density of mass standard. Using the values measured in the laboratory, as given in Table 2, the force exerted on the laser by the action of $m_e = 0.1$ g is theoretically: $F_{dw} = 9.808\times10^{-4}$ N.

The dominant component of the overall uncertainty associated with this value of force comes from that of the standard mass, m_e.

$$u\,(F_{dw}) \cong (\partial F_{dw}/\partial\, m_e)\, u\,(m_e) \cong 10^{-8}\ \text{N}$$

When this force acts on the sensing element, the shift induced on the frequency of the beat note is about 60 kHz, i.e., at the level of the reproducibility of frequency measurement. The measured force is determined from the experimental sensitivity of the force sensor and measurement of the frequency shift $\Delta\nu^{load} - \Delta\nu^{unload}$, induced by the action of an unknown force. The intensity of this force is then calculated from:

$$F_{\exp} = (\Delta\nu^{load} - \Delta\nu^{unload}) \big/ S_{\exp} \qquad (10)$$

In the case of deadweight associated to $m_e = 0.1$ g, the values of force determined from (10) and expressions of $S_{\exp}$ and $S^*_{\exp}$ are respectively given by:

$$\begin{cases} F_{\exp} = 9.90 \times 10^{-4} \text{ N} \\ F^*_{\exp} = 9.94 \times 10^{-4} \text{ N} \end{cases}$$

These experimental values differ by about 1 % from that calculated using (9). Considering the uncertainty of repeatability in the determination of the experimental sensitivity, the uncertainties associated with these values of force are at the same order, or:

$$u_{repeat}(F) \cong 10^{-3} \text{ N}$$

We note that this uncertainty is at the level of the measured value for the applied force. On the other hand, the uncertainty of reproducibility is twice as important.

Since the sensor output is governed by the stress developed in the photo-elastic material, reading could be erroneous if the contact is located at one end of the crystal and not along one of its edges. The range of the applied force, at the micronewton level, can be improved by increasing the contact area of the sensor or by using photo-elastic material with higher constant of optical stress.

As perspective, we plan to consider a force sensor based on a dynamometer using a passive diamagnetic spring having a very low stiffness to be coupled to the photo-elastic force sensor in order to apply a quantifiable vertical force on the photo-elastic crystal. The objective is to make the optical system traceable for its lowest range of measurement (1 - 100 µN) by realizing a connection of traceability of the two sensors. The magnetic force sensor consists of a magnetic and rigid microcapillary in a passive diamagnetic levitation as reported in [15]. For this work, we hope to have a miniaturized monolithic Nd-YAG laser with a length smaller than 1 mm.

5. Conclusions

In this report, we have presented the photo-elastic force sensor used to characterize the response, in terms of sensitivity and its uncertainty, especially when the dimensions of the sensing element are small. The new determination of sensitivity was used to obtain the best fit between theoretical predictions and experimental values of sensitivity for the photo-elastic force sensors. Hence, the result is novel in the sense that it provides new information on the behavior of the sensitivity for a given sensor size, which showed the overall effect of the nature of the contact, represented by the geometrical factor, on the sensitivity of the sensor. To adjust the theoretical model so it agrees with the results of a set of measurements, we have reconsidered the contribution of the shape factor to the value of the sensitivity. Thus, fitting this parameter, we have reduced the observed differences between experimental values of sensitivities, based on photo-elastic effect in a monolithic Nd-YAG laser, and theoretical predictions.

Finally, we can conclude that this type of force sensor can provide high sensitivity, when using small size for sensitive element but we observe a slight deterioration in the reproducibility of the measurements. To avoid aging of the sensing element, the force range should be adjusted in order to prevent deformations that could cause distortion of the crystal as a result of lateral forces or torques. We plan to study these aspects in order to characterize the two sensors we have used for potential applications in industry. However, we believe that it can be quite suitable for the detection and measurement of force in the range of 1 N - 0.1 mN. Also, it can be used and integrated to other systems in the view of traceable small force measurements to the International System of Units.

Acknowledgements

The authors acknowledge the financial support of the Direction of Scientific and Technological Research (DRST) of the national metrology and testing laboratory (LNE). Thanks to Daniel Truong for machining work and in particular the new crystal holder.

References

[1]. M.-S. Kim, J. R. Pratt, SI traceability: Current status and future trends for forces below 10 microNewtons, *Measurement,* 43, 2010, pp. 169-182.

[2]. C. Diethold, F. Hilbrunner, Force measurement of low forces in combination with high dead loads by the use of electromagnetic force compensation, *Meas. Sci. Technol*., 23, 2012, pp. 1-7, http://iopscience.iop.org/0957-0233/23/7/074017.

[3]. G. Xiao, X. Long, B. Zhang, G. Li, Precise force measurement method by Y-shaped cavity dual-frequency laser, *Chinese Optics Letters,* 9, 10, 101201, 2011, pp. 1-4.

[4]. C. F. Tuan, F.-L. Pan, Y.-C. Lin, S.-S. Pan, C.-L. Wu, Sub-milligram weight subdivision and application in force calibration of nanoindenter, in *Proceedings of the XIX IMEKO World Congress, Fundamental and Applied Metrology*, Lisbon, 6-11 September 2009, pp. 393-396.

[5]. M. Appleyard, C. Mosse, T. Mills, G. Ducan, D. Castillo, C. Swain, The measurement of forces exerted during colonoscopy, *Gastrointestinal Endoscopy,* 52, 2000, pp. 237-240.

[6]. W. Holzapfel, M. Finnemann, High-resolution force sensing by a diode-pumped Nd-YAG laser, *Optics Lett*., 18, 13, 1993, pp. 2062-2064.

[7]. N. Khelifa, Small-Force Measurement by Photo-Elastic Transducer, *Optics and Photonics Journal,* 4, 1, 2014, pp. 14-20.

[8]. M. M. Frocht, Photoelasticity, *John Wiley and Sons,* New York, 1, 1948, pp. 144-149.

[9]. W. Holzapfel, L. Hou, S. Neuschaefer-Rube, Error effects in microlaser sensors, in *Proceedings of the XVI IMEKO World Congress*, Austria, Vol. 3, 2000, pp. 85-90.

[10]. J. Ding, Q. Feng, L. Zhang, S. Zhang, Laser frequency splitting method for high-resolution determination of relative stress-optic coefficient and internal stresses in Nd:YAG crystals, *Applied Optics*, Vol. 47, No. 30, 2008, pp. 5631-5636.

[11]. W. Holzapfel, S. Neuschaefer-Rube, M. Kobusch, High-resolution, very broadband force measurement by solid-state laser transducers, *Measurement,* 28, 4, 2000, pp. 277-291.

[12]. N.-E. Khelifa, M. Himbert, Induced birefringence in a solid-state laser: Towards a photo-elastic Nd-YAG transducer for small force measurement, in *Proceedings of the 3rd International Conference on Optics Photonics and Their Applications (ICOPA'13)*, Algiers, 9-11 December, No. 60, 2013, pp. 1-15.

[13]. N.-E. Khelifa, M. Himbert, Sensitivity of Photo-Elastic Nd-YAG laser for Small Force Sensing, in *Proceedings of the 5th International Conference on Sensor Device Technologies and Applications (SENSORDEVICES' 14)*, Lisbon, Portugal, 16-20 November 2014.

[14]. J. Ding, L. Zhang, Z. Zhang, S. Zhang, Frequency splitting phenomenon of dual transverse modes in Nd: YAG laser, *Opt. & Laser Technol*., 42, 2010, pp. 341-346.

[15]. A. Cherry, J. Abadie, E. Piat, Analysis of a passive microforce sensor based on magnetic springs and upthrust buoyancy, *Sensors and Actuators A: Physical*, 169, 1, 2011, pp. 27-36.