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Non-invasive and Locally Resolved Measurement of Sound Velocity by Ultrasound

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Abstract: A new method to measure sound velocity and distance simultaneously and locally resolved by an ultrasound annular array is developed in media with constant sound velocity and then applied in media with continuously changing properties. Instead of using reflectors at known positions the echoes of moving scattering particles are analyzed to determine the focus position. Although the method reaches a very high accuracy for constant sound velocity, there are systematic deviation between measurements and sound field simulations using Fermat's principle. The sound propagation has to be described with a modified wave equation. A new approach determining GREEN's functions for a half space with continuously changing properties is presented. It combines the high frequency approximation with an integral transform method. *Copyright © 2015 IFSA Publishing, S. L.*

Keywords: Ultrasound annular arrays, Measurement of sound velocity, Locally resolved, High frequency approximation, Integral transform method.

1. Introduction

A locally resolved monitoring of sound velocity allows estimating locally physical quantities like concentration or temperature or material properties like density or elasticity. This facilitates investigating and optimizing many industrial processes, like mixing or chemical reactions, as well as medical therapy like hyperthermia for cancer treatment.

In this contribution, a measurement technique is applied, which allows measuring sound velocity locally resolved using an ultrasound annular array with concentric rings. In contrast to conventional tomographic techniques, it works without any reflectors or additional receivers at known positions. Instead of evaluating various propagation paths, the focusing of the array is varied and the focus position and the sound velocity to the focus point are determined simultaneously by analyzing the echoes of moving scattering particles. This is possible because the focus position depends on the sound velocity and the parameters of the used transducer. Therefore, the time of flight to the focus is used with calibration curves for the simultaneous determination of sound velocity and focus position. The time of flight to the focus point is determined from the averaged amplitude of the echo signals: The emitted wave is reflected at each particle while the amplitude of the reflected signal is proportional to the amplitude of the incident wave (Fig. 1a). Therefore the echoes from particles within the focus area are strongest. Although a single echo nearly appears like noise averaging over a sufficient number of signals generates a clear maximum (see Fig. 1b). As particles are in movement it is possible to consider a uniform distribution of particles in average time. So the averaged echo signal amplitude is a measure for the pressure along the acoustic axis of the transducer.

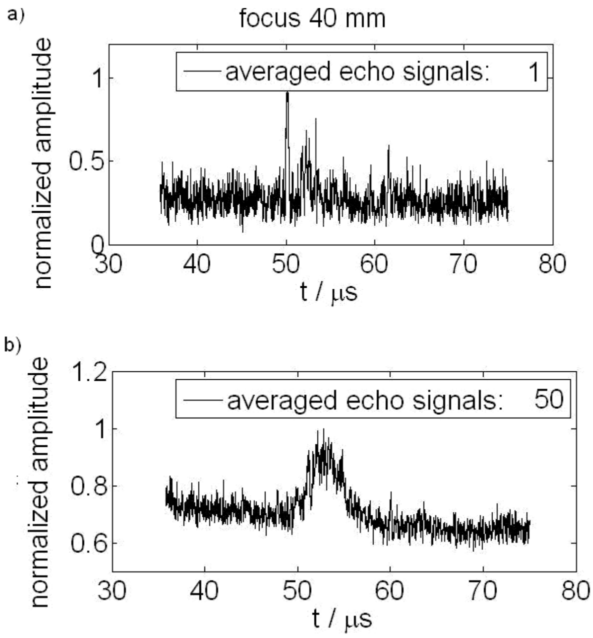

Fig. 1. Normalized averaged echo signal amplitude for different numbers of averages in water of 30 °C.

The basic set-up for the measurements is quiet simple. Only a focusing single probe and a device for pulsing and data acquisition are required. The probe can be located in the examined medium without any adjustment. The medium has to contain scattering particles much smaller than wavelength and in a low concentration, so that the properties of the propagation medium are not influenced. The method had been introduced by Lenz [1] for media with constant sound velocity using a focusing single probe.

This contribution enhances the method to annular arrays, which allow moving the focus along the acoustic axis and so measuring in different depths. At first this is qualified for media with constant sound velocity and than employed for media with non-constant sound velocity. A new method for modelling sound propagation in media with continuously changing material parameters is introduced for the evaluation. The modified wave equation caused by non-constant material parameters is solved for a point source with an integral transform method and a high frequency approximation [2].

This paper is divided in five sections. Section 2 explains how to measure sound velocity and focus position simultaneously. It motivates the modelling of sound propagation in media with non-constant material properties which is discussed in Section 3. Section 4 shows some measurement results for media with constant sound velocity and a linear temperature gradient in water. Section 5 gives a summary and perspectives.

2. Method to Measure sound Velocity and Distance Simultaneously

2.1. Sound Field Calculation by GREEN'S Functions

GREEN's functions are a beneficial tool for modelling sound propagation in homogeneous media, which describe the impulse response of a point source in a space with a specific geometry. Exact solutions had been found for half spaces, plates and layered media by means of integral transform [3], [4]. Also approximated harmonic GREEN's functions are calculated giving the transfer function in the Fourier transformed domain [5]. GREEN's functions for a half space are also used in this contribution

$$G_{\omega}(R,\theta) = \frac{1}{R} S(\theta) e^{jkR}, \qquad (1)$$

with the wave number k, die distance R between source and observation point and the angle θ between the vector between source and observation point and the normal vector of the surface where the source is located. The directivity pattern $S(\theta)$ concerns the boundary conditions at the interface. An example for an interface of water and PZT (lead- zirconate-titanate) is shown in Fig. 2.

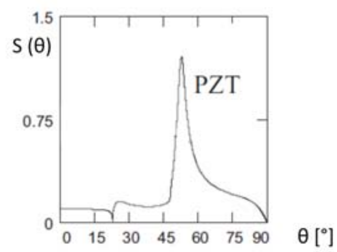

Fig. 2. Directivity pattern of a point source on a water-PZT-interface.

2.2. Annular Arrays for Focusing

Annular arrays need far less elements for a well focusing as the widely spread linear array. The array used in this contribution consists of 6 elements (see Fig. 3 for structure and dimensions) and reaches an extension of the sound beam in the range of the wavelength.

Focusing works by a superposition of measured signals where each is delayed with a calculated time: The sound paths from each element to the designed focus point are determined (Fig. 4). With a known sound velocity of the propagation medium these paths allow to calculate the differences in time of flight, which are used as delay times for each element so that all waves arrive at the focus point at the same time.

Element	r_i [mm]	r_a [mm]
1	0	3.57
2	5.05	6.18
3	7.14	7.98
4	8.74	9.44
5	10.1	10.7
6	11.28	11.83

Fig. 3. Structure and dimensions of the curved annular array, centre frequency: f_m = 9 MHz, Curvature: R = 50 mm; r_i and r_a are the inside and outside radii of the rings.

As there is positive interference of course the pressure at this point becomes maximal.

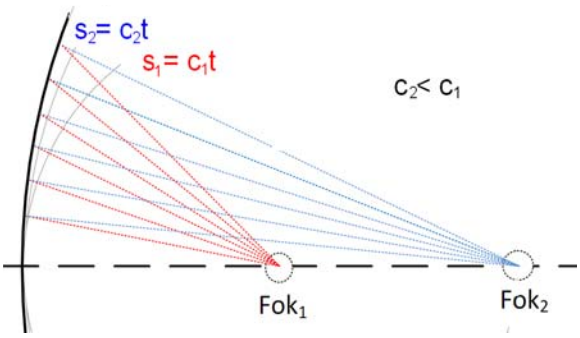

Fig. 4. Focusing sound paths to focus points for different sound velocities.

Of course if e.g. the temperature in the medium changes this causes a change of sound velocity. The focus moves because the positive interference arises at another point (Fig. 4, red and blue lines).

As calculated sound fields are in the Fourier transformed domain also the delay times have to be transformed into phase shifts. A superposition of the phase shifted sound fields generates the focus at the designed point. Fig. 5 shows the sound fields of the used array, driven with the same delay times for different sound velocities. They were chosen so that the focus would arise at a distance of 40 mm in a medium with a sound velocity of c = 1500 m/s.

2.3. Simultaneous Determination of Sound Velocity and Focus Position

It has been shown, that a change of sound velocity displaces the focus position resulting in a different time of flight to the focus. If the parameters of the transducer are known the time of flight can be calculated as a function of sound velocity. Fig. 6 shows functions for several sets of delay times. The labels mean that the used delay times would cause a focus in a calibration medium with c = 1500 m/s at the corresponding depth.

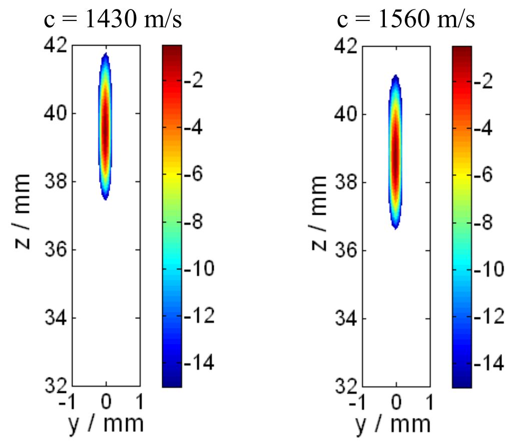

Fig. 5. Sound fields for different sound velocities.

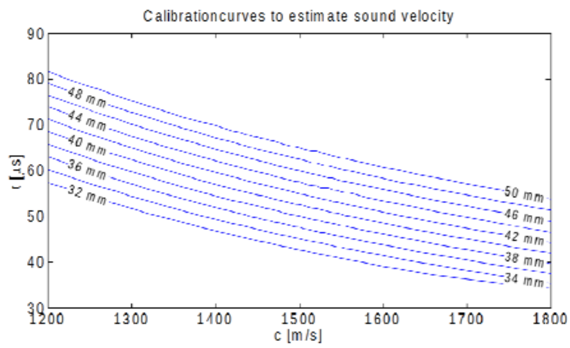

Fig. 6. Calibration curves.

The measured time of flight to the focus point can be used to determine the sound velocity by these curves. Of course then the real focus position emerges.

If the delay times are varied the focus is moved along the acoustic axis of the array. So the sound velocity can be measured at each point, which allows measuring the distribution of sound velocity.

3. Sound Propagation in Media with Non-constant Properties

As mentioned in the previous section the parameters of the transducer have to be known to calculate the sound field of the transducer. The focus position also has to be predicted for media with continuously changing properties where additional considerations are required.

3.1. Simultaneous Determination of Sound Velocity and Focus Position

The most obvious way to model the sound propagation in media with continuously changing properties is applying Fermat's principle. It says that a wave propagates along that path which results in a minimal propagation time.

It can be used to derive Snell's law but if sound velocity changes continuously the propagation path becomes curved and has to be determined by a calculus of variation [6]. The time of flight T to a point $P(x,y)$ and the path length S to this point are:

$$T = \frac{1}{d}\ln[\frac{c(x)}{c_0}\frac{1+\sqrt{1-a^2c_0{}^2}}{1+\sqrt{1-a^2c^2(x)}}], \quad (2)$$

$$S = \frac{1}{da}[\arcsin(c(x)a) - \arcsin(c_0 a)], \quad (3)$$

with

$$a = \frac{2y}{\sqrt{(y^2d - 2c_0x - x^2d)^2 + 4y^2(c_0 + dx)^2}}$$

for a linear sound velocity gradient ($c(x) = c0+d\,x,$) and a point source in the origin of coordinates. This had been used to derive fields of point sources for linear sound velocity gradients. The impulse response is assumed by

$$G_\delta(x,y,t) = \frac{1}{S(x,y)}\delta(t - T(x,y)) \quad (4)$$

with the Dirac function δ. In the Fourier transformed domain this is

$$G_\omega(x,y,\omega) = \frac{1}{S(x,y)}e^{j\omega T(x,y)} \quad (5)$$

Fig. 7 shows the phase plots of two examples where sound velocity increases (left) and decreases (right) in positive direction x.

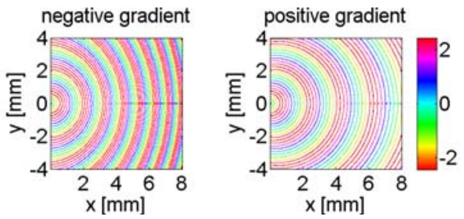

Fig. 7. Phase plot for positive and negative sound velocity gradient.

Although Fermat's principle gives the correct time of flight of the wave there is no information about the amplitude of the wave.

3.2. Derivation of the Modified Wave Equation

The sound propagation in liquids is based on two fundamental equations, the equations of motion and elasticity:

$$-\nabla p = \rho_0 \dot{\vec{v}}, \qquad \chi\dot{p} = \nabla\vec{v}, \quad (6)$$

with pressure p, particle velocity v, and the material parameters mass density ρ_0 and elasticity χ. If the material parameters are constant, differentiating these equations with respect to location and time leads to the well-known wave equation. If these parameters are functions of location an additional term appears in the gradient of (6). This additional term also appears in the wave equation:

$$\Delta p + \rho_0\nabla\left(\frac{1}{\rho_0}\right)\cdot\nabla p - \rho_0\chi\ddot{p} = 0 \quad (7)$$

Also the definition of potential Φ is modified:

$$\vec{v} = \frac{1}{\rho_0}grad\Phi \quad (8)$$

The following examination shall be done for a one-dimensional dependence of the material properties in direction z.

3.3. One-dimensional Solution

Considering a plane wave propagating in the direction of the gradient of the material properties the wave equation for the potential is obtained

$$\frac{d^2}{dz^2}\Phi + \rho_0\frac{d}{dz}\left(\frac{1}{\rho_0}\right)\frac{d}{dz}\Phi - \rho_0\chi\frac{d^2}{dt^2}\Phi = 0 \quad (9)$$

If the mass density depends linearly on z this equation has a solution in the form of a generalized power series [7]:

$$\Phi = \sum_{n=1}^{\infty} c_n(z - z_0)^{n+K} \quad (10)$$

However, there are already numerical problems in evaluating the coefficients and the solution with respect to the convergence for this one-dimensional case. So it seems not to be feasible finding an exact solution for a two- or three-dimensional problem.

3.4. High Frequency Approximation and Integral Transform

High frequency approximation had been developed in geophysics and is applied in techniques like ray tracing [8]. In this contribution, harmonic GREEN's functions shall be derived with this approach in combination with an integral transform method. This allows calculating a transfer function for a point source for a specific geometry. Just the axially symmetric problem is solved because it is much easier manageable than the general problem. The wave equation in cylindrical coordinates *(r, θ, z)* is used:

$$0=\left[\frac{1}{r}\left(\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right)+\frac{\partial^2}{\partial z^2}\right]\Phi+\ldots$$
$$\rho_0\frac{\partial}{\partial z}\left(\frac{1}{\rho_0}\right)\frac{\partial}{\partial z}\Phi-\rho_0\chi\frac{\partial^2}{\partial t^2}\Phi \quad (11)$$

The single derivative to r in the second term vanishes due to its scalar multiplication with the gradient of the mass density, having only a component in z direction. Applying a Hankel transform as described in [3] with respect to r

$$\Phi^{H0}(\xi,z,t)=\int_0^\infty \Phi(r,z,t)J_0(j\omega\xi r)rdr \quad (12)$$

leads to a one-dimensional wave equation in the transformed domain, which is denoted by the index H0:

$$0=\frac{d^2}{dz^2}\Phi^{H0}+\rho_0\frac{d}{dz}\left(\frac{1}{\rho_0}\right)\frac{d}{dz}\Phi^{H0}-\ldots$$
$$\xi^2\omega^2\Phi^{H0}-\rho_0\chi\frac{\partial^2}{\partial t^2}\Phi^{H0} \quad (13)$$

The high frequency approximation assumes that equation (8) can be solved by the following ansatz

$$\Phi=A(x,z)e^{j\omega(t-T(x,z))} \quad (14)$$

Now, Φ is replaced with this ansatz and the terms are arranged according to its powers of ω.

$$0=A\left[\omega^2\left[\rho_0\chi-\left[\frac{dT}{dz}\right]^2-\xi^2\right]\right]-\ldots$$
$$j\omega\left[\frac{d^2T}{dz^2}+2\frac{1}{A}\frac{dA}{dz}\frac{dT}{dz}+\rho_0\frac{d}{dz}\left(\frac{1}{\rho_0}\right)\frac{dT}{dz}\right]\ldots \quad (15)$$
$$\ldots+\rho_0\frac{d}{dz}\left(\frac{1}{\rho_0}\right)\frac{dA}{dz}+\frac{d^2A}{dz^2}$$

Assuming large ω the first two lines are equated to zero independently and the frequency-independent third line is neglected. So T can be determined directly from the first line and with this solution A is determined from the second line

$$T(\xi,z)=\pm\int_{z_0}^{z}\sqrt{\rho_0(z)\chi(z)+\xi^2}\,dz, \quad (16)$$

$$A(\xi,z)=A_0\sqrt{\frac{\left(\rho_0(z_0)\chi(z_0)-\xi^2\right)\rho_0(z)}{\left(\rho_0(z)\chi(z)-\xi^2\right)\rho_0(z_0)}}, \quad (17)$$

with the solely free parameter A_0. Note that this leads to the solution for a homogeneous medium if ρ_0 and χ are constants. All methods of generalized ray theory explained in [3] like the derivation of source functions for point source acting on an interface considering the boundary conditions can be applied to this solution. Finally, the inverse transformation has to be done:

$$\Phi(r,z)=-\omega^2\int_0^\infty A(\xi,z)e^{j\omega(t-T(\xi,z))}J_0(j\omega\xi r)\xi d\xi \quad (18)$$

Current work is on an evaluation of this integral with a steepest descent approximation as it is described in [9]. The approximation facilitates the integral into a solvable form. This causes a neglect of surface waves being not of interest for the presented application. However, the method is complicated because of the integral expression of T in the exponent. Though, the integral can be evaluated by a finite series expansion resulting in additional terms containing higher powers of ξ.

4. Measurement Results

4.1. Measurements for Constant Sound Velocity

Primarily, sound velocity was measured in different media with constant temperature, as described in Section II, to compare measurements and simulations. It is striking that the measured curve for *Θ = 6 °C* fits very well to the simulated one, whereas the measured curve for *Θ = 60 °C* deviates (Fig. 8). Therefore, additional measurements with water-ethanol solutions were done. Different sound velocities can be adjusted with different concentrations of ethanol, without a change in temperature [10].

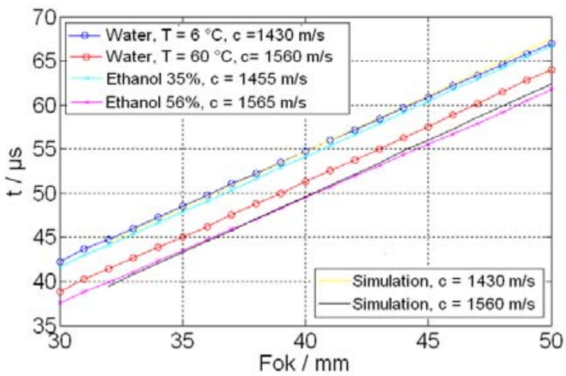

Fig. 8. Comparison of measured and calculated times of flight as a function of the used set of delay times for different media.

It can be recognized that the measured curve of a water-ethanol solution with a sound velocity of 1565 m/s in Fig. 8 agrees to the simulated curve. This implicates that the working temperature of the probe has to be taken into account calculating the calibration curves.

To analyze the accuracy measurements with different numbers of averages have been done. Table 1 shows that the relative error of sound velocity is less than 1 %.

Table 1. Measurement capability.

Number of averaged signals	Standard deviation ± 3σ of time of flight	Absolute error of sound velocity
50	500 ns	13 m/s
200	250 ns	6 m/s
1000	150 ns	4 m/s
5000	100 ns	2.5 m/s

4.2. Measurements for a Sound Velocity Gradient

An evident way to achieve a sound velocity gradient is to generate a temperature gradient with water, because the sound velocity as a function of temperature is well known for water [11] and it can be generated in a stable state. Fig. 9 shows the general set-up. Water in a basin located at the bottom is kept at a temperature of 6 °C. A second smaller basin is placed above. It contains a metal plate at its bottom for good thermal conduction and a heat source at its top. This generates a vertical layered arrangement of warm water above cooler water, whereby a fluid flow is avoided. The temperature is measured by an array of temperature sensors to determine the sound velocity profile in the experimental set-up.

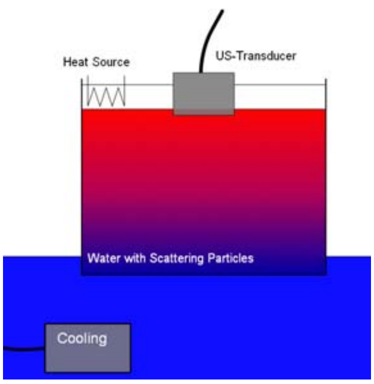

Fig. 9. Experimental set-up for a sound velocity gradient.

The sound field for this gradient and the time of flight to the focus were calculated applying Fermat's Principle. Fig. 10 shows the comparison of calculated and measured times of flight to the focus as a function of the used set of delay times, corresponding with a focus (Fok) in the calibration medium water of 20 °C (sound velocity gradient in water from 40 °C at the transducer to 6 °C at a distance of 50 mm).

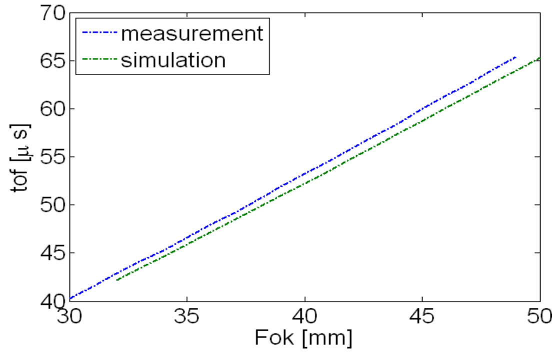

Fig. 10. Comparison of measured and calculated times of flight as a function of the used set of delay times for a temperature gradient.

Although the notable difference is just in the range of one microsecond this would cause an error of more than 100 m/s in determining the sound velocity.

For additional examinations, the sound velocity was measured conventionally via measuring the time of flight to a reflector at a known position. Moving the reflector along the acoustic axis allows a stepwise reconstruction of the sound velocity profile. Additionally the sound velocity was determined from temperature sensors again. Fig. 11 shows a comparison of the two conventional methods to determine the mean sound velocity between the probe and a reflector at distance z. First, the time of flight for various reflector distances is measured (blue line). Second, the temperatures are measured at various locations and converted to a sound velocity according to [11] and averaged over the propagation path (green line). The systematic deviation can be seen here, too.

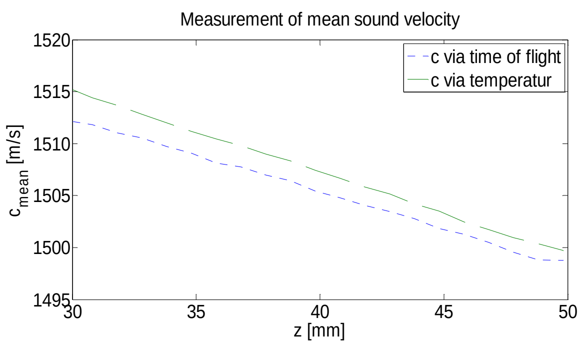

Fig. 11. Comparison of two conventional methods to determine the mean sound velocity between the probe and a reflector at distance *z*.

Both deviations (see Fig. 10 and Fig. 11) result from the deficient assumption that wave propagation can be described with the wave equation for homogeneous media.

5. Summary and Perspective

This contribution discusses a non-invasive method to measure the sound velocity locally resolved along the acoustic axis of the used annular array. The three-dimensional distribution of sound velocity can be obtained by scanning.

The method has been qualified for media with constant sound velocity and then extended to media with continuously changing properties. It has been shown that the continuous change of material properties has to be taken into account for the modelling of sound propagation. Fermat's principle gives the correct time of flight, but to obtain information about the amplitude, the modified wave equation has to be solved. The potential of a point source has been calculated in the Hankel transformed domain. The inverse transform is actually realized and will allow calculating GREEN's functions for media with continuously changing properties.

Due to the assumed change of material properties in only one dimension a change of these properties in other dimensions would cause a lateral deviation of the focus position. This effect has to be examined in further works.

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