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Application of Improved Wavelet Thresholding Function in Image Denoising Processing

[1] Hong Qi Zhang, [2*] Hai Zhen Kang, [3] Li Hu Yi, [1] Yu Liu

[1] College of Mechanical and Electrical Engineering, Inner Mongolia Agricultural University, Hohhot, 010018, P. R. China

[2] College of Electric Power, Inner Mongolia University of Technology, Hohhot, 010051, P. R. China

[3] Inner-Mongolia Electric Power Survey and Design Institute, Hohhot, 010020, P. R. China

[*] Tel.: +86 13948413370

[*] E-mail: 429535540@qq.com

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Abstract: Wavelet analysis is a time – frequency analysis method, time-frequency localization problems are well solved, this paper analyzes the basic principles of the wavelet transform and the relationship between the signal singularity Lipschitz exponent and the local maxima of the wavelet transform coefficients mold, the principles of wavelet transform in image denoising are analyzed, the disadvantages of traditional wavelet thresholding function are studied, wavelet threshold function, the discontinuity of hard threshold and constant deviation of soft threshold are improved, image is denoised through using the improved threshold function. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Wavelet analysis, Threshold function, Denoising, Hard threshold, Soft threshold.

1. Introduction

Wavelet analysis is a branch of mathematics in 1980s when it was developed and widely applied, it is a time-frequency analysis method, the problem of time-frequency localization is addressed by wavelet analysis. Therefore, it was vigorously applied in signal analysis, image processing and other fields. Denoising method of the traditional image is the average or linear denoising one, but the signal and noise frequency band overlap, it is difficult to distinguish noise frequency band against signal and the denoising effect does not meet the expect ion. The wavelet coefficients at different scales are extracted through utilizing the time-frequency localization of wavelet transform, and multi-scale wavelet transform is implemented with regards to the signal with noise so that the wavelet coefficients related to noise are removed, signal is restructured by inverse transform of wavelet so that the goal of removing noise is attained.

At present, the method of wavelet transform denoising are as follows: modulus maxima denoising of wavelet transform, threshold denoising method based on wavelet transform, denoising method based on the correlation of adjacent scale wavelet coefficient, etc. [1].

2. The Basic Principles of the Wavelet Analysis

Wavelet transform ideas was originated from dilation and translation, the definition of wavelet transform is dilation and translation of the wavelet mother function $\psi(t)$, dilation factor is a displacement factor τ, then $\psi_{a,\tau}(t)=a^{-\frac{1}{2}}\psi(\frac{t-\tau}{a}), a>0, \tau\in R$ is wavelet basis function, which makes the inner product operation with signal $f(t)$ to be analyzed:

$$WT_x(a,\tau)=a^{-\frac{1}{2}}\int_{-\infty}^{\infty} f(t)\psi^*(\frac{t-\tau}{a})dt \quad a>0\,, \quad (1)$$

Multi-resolution analysis can provide a unified framework for constructing the wavelet and a fast algorithm of decomposition and reconstruction of wavelet function, that is, the famous Mallat algorithm [2].

The theoretical framework used to construct the multi-resolution orthogonal wavelet transform is as follows [3]:

Space $L^2(R)$ multi-resolution analysis is a spatial sequence $\{V_j\}_{j\in Z}$, which meets the following requirements:

1) $\subseteq V_{j-1}\subseteq V_j\subseteq V_{j+1}\subseteq ..., \forall j\in Z$;

2) $clos_{L^2}(\bigcup_{j\in Z} V_j)=L^2(R)$;

3) $\bigcap_{j\in Z} V_j=\{0\}$;

4) $V_{j-1}=V_j\oplus W_j \quad (j\in Z)$;

5) $f(x)\in V_j \Leftrightarrow f(2x)\in V_{j-1}$, $f(x)\in V_j \Leftrightarrow f(x+\frac{1}{2^j})\in V_j \quad (j\in Z)$

V_j and W_j are the scale space and wavelet space respectively, W_j is the orthogonal complement of V_j in V_{j-1} .., and $W_i\perp W_j (i\neq j)$, V_j can be expressed as: $V_j=\ldots\oplus W_{j+2}\oplus W_{j+1}$. $L^2(R)$ can be expressed as: $L^2(R)=\underset{j\in Z}{\oplus} W_j=\ldots\oplus W_{-1}\oplus W_0\oplus W_1\oplus\ldots$, for any function $f(x)\in L^2(R)$, which has only but one decomposition:

$f(x)=\ldots+g_{-1}(x)+g_0(x)+g_1(x)+\ldots$, in the subspace V_0, assuming a low-pass smoothing function $\phi(t)$, integer displacement collection of which is $\{M^{-j/2}\phi(M^{-j}t-k)|k\in Z\}$ and the collection constitutes a specification orthogonal basis, $\phi(t)$ is dubbed as scaling function, that is:

$$\phi_{j,k}(t)=2^{j/2}\phi(2^{-j}t-k), \quad j,k\in Z\,, \quad (2)$$

with any $f\in L(R)$, and $P_j f(t)\in V_j$, $Q_j f\in W_j$, P_j is named as projection operator of the scale space, then Q_j the projection operator of wavelet space. When j equals zero, $P_0 f(t)$ stands for the projection of $f(t)$ in V_0, then

$$P_0 f(t)=\sum_k c_k^{(0)}\phi_{0k}(t)\,, \quad (3)$$

where $c_k^{(0)}$ is the weighted linear combination, whose expression is $c_k^{(0)}=\langle\phi_{0k}(t), P_0 f(t)\rangle$, $c_k^{(0)}$ is called the discrete approximation of $f(t)$ under the condition of the resolution $j=0$, likewise, in the V_1 subspace, $c_k^{(1)}$ is called the discrete approximation of $f(t)$ under the prerequisite of the resolution $j=1$, $P_0 f(t)$ is called the smooth approximation of $f(t)$ in V_0, which is a general picture of $f(t)$ under the condition of the resolution $j=0$. In the W space, assuming $\psi(t)$ a low-pass function in the W_0 subspace, a collection of integer bits $\langle M^{-j/2}\psi(M^{-j}t-k); k\in Z\rangle$ constitutes the orthonormal basis in W_0, then

$$\psi_{1k}(t)=\frac{1}{\sqrt{2}}\psi(\frac{t}{2}-k); \quad k\in Z\,, \quad (4)$$

Form a set of orthonormal basis in W_1, $Q_1 f(t)$ is projection of the $f(t)$ in the W_1, then

$$Q_1 f(t)=\sum_k d_k^{(1)}\psi_{1k}(t)\,, \quad (5)$$

where linear weight is:

$$d_k^{(1)}=\langle Q_1 f(t), \psi_{1k}(t)\rangle\,, \quad (6)$$

Because

$$V_0=V_1\oplus W_1\,, \quad P_0 f(t)=P_1 f(t)+Q_1 f(t)\,, \quad (7)$$

where $Q_1 f(t)$ is the difference approximation of smooth V_0, V_1 with the adjacent two level, which reflects the nuances between the two levels of detail. $Q_1 f(t)$ is the detail function when $j=1$, $d_k^{(1)}$ is the discrete details when $j=1$, $\psi(t)$ represents a wavelet function with the feature of bandpass. Multi-resolution analysis can be interpreted as multiple bandpass filter group, smooth component $c_k^{(1)}$ and detail component $d_k^{(1)}$ are as follows:

$$c_k^{(1)}=\sum_n h(n-2k)c_n^{(0)}\,, \quad (8)$$

$$d_k^{(1)}=\sum_n g(n-2k)c_n^{(0)}\,, \quad (9)$$

Similarly, $c_k^{(2)}, d_k^{(2)}$ and so on can be concluded. Thanks to a two-dimensional signal of the image signal, 2D wavelet transform of the image is, in the final analysis, a transform of discrete two-dimensional wavelet of the image. Two-dimensional wavelet transform algorithm and two-dimensional image reconstruction are respectively shown in Fig. 1 and Fig. 2, a one-dimensional wavelet of the two-dimensional image data are transformed in the horizontal direction and the vertical direction independently [4].

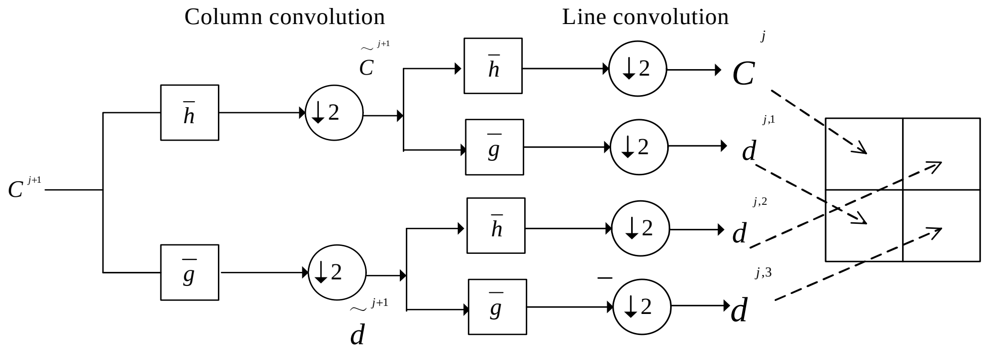


Fig. 1. Two-dimensional wavelet decomposition.

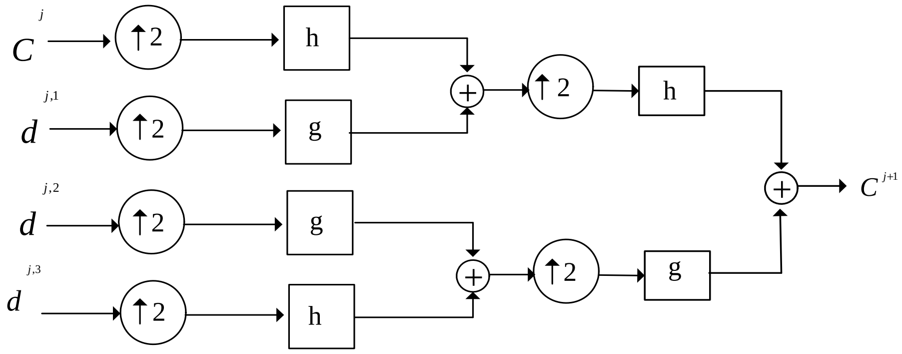


Fig. 2. Two-dimensional wavelet reconstruction.

3. Denoising Processing Through using Wavelet Transform Image

3.1. The Principle of Denoising for Wavelet Transform

Singular points (point mutations) in the signal typically contain important features of the signal, S. Mallat established a relationship between the characterization of signal singularity Lipschitz exponent and local maxima mode of wavelet transform coefficients in 1992 [5]. Signal $f(x) \in L^2(R)$ has the following characteristics in the vicinity of

$$\left| f(X_0+h) - P_n(X_0+h) \right| \le A|h|^{\alpha}, \quad n < \alpha < n+1 \tag{10}$$

where h is the sufficiently small amount, A is the constant, $P_n(x)$ is the polynomial of n degree over $f(X_0)$ point, $f(x)$ is the Lipschitz index α in x_0. As long as the Lipschitz index α value is greater, the function is smoother and singularity is smaller, vice versa. If the function is less than 1 at one point the Lipschitz index of 1, function at this point is singular. The absolute value of the function of wavelet transform coefficients in different scales measures the Lipschitz exponent α. Assume $0 \le \alpha \le 1$, the sufficient and necessary conditions that the function $f(x)$ has uniform Lipschitz index α in $[a,b]$ is a constant ($K > 0$), $x \in [a,b]$. Wavelet transform should meet:

$$\left| W_{2^j} f(x) \right| \le K\left(2^j\right)^{\alpha}, \tag{11}$$

Using log on both sides of the equation:

$$\log_2 \left| W_{2^j} f(x) \right| \le \log_2 K + j\alpha, \tag{12}$$

Variation between the value of wavelet transform and Lipschitz exponent is

If $\alpha > 0$, then the coefficients in modulus maxima of the wavelet transform increase with the rise of scale;

If $\alpha < 0$, then the coefficients in modulus maxima of the wavelet transform decrease with the rise of scale;

If $\alpha = 0$, then the coefficients in modulus maxima of the wavelet transform fail to vary with scale changes.

The white noise is random distribution of singular everywhere with a negative Lipschitz exponent [6]

$$\alpha = -\frac{2}{2} - \varepsilon, \quad \forall\ \varepsilon > 0, \tag{13}$$

assume $n(x)$ is a white noise, the variance of which is σ^2, then:

$$|Wn(s,x)|^2 = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} n(u)n(r)\psi_s(x-u)\psi_s(x-r)dudr, \tag{14}$$

where $E[n(u)n(r)] = \sigma^2\delta(u-r)$

$$E(|Wn(s,x)|^2) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} E[n(u)n(r)]\psi_s(x-u)\psi_s(x-r)dudr = \frac{\sigma^2\|\psi\|_2^2}{S}, \tag{15}$$

the formula (15) shows that: $E(|Wn(s,x)|^2)$ is inversely proportionate to Scale S. With the increase in scale S and plummet of the value of modulus maxima of the wavelet transform, then the signal on these maxima Lipschitz exponent is negative. They are noise, the average density of modulus maxima points of the wavelet transform is:

$$d_s = \frac{1}{s\pi}\left(\frac{\|\psi^{(2)}\|_2}{2\|\psi^{(1)}\|_2} + \frac{\|\psi^{(1)}\|_2}{\|\psi\|_2}\right), \tag{16}$$

where in $\psi^{(1)}, \psi^{(2)}$ are the first derivative of $\psi(x)$ and the second derivative of $\psi(x)$ respectively. The equation (16) shows that: average density d_s is inversely proportionate to scale S, wherein scale $s = 2^j$, $j = 1, 2, 3, \ldots$, average density of modulus maxima point will decrease with increasing scale, at least half of the modulus maxima point can not pass the large-scale one. If it is modulus maxima in the scale of 2^j and is not maxima points in the larger scale 2^{j+1} modulus, these points are noise [7].

Through the above-mentioned analysis, signal and noise in the wavelet transform with the transform of scale will display different characteristic. Maxima value of the signal modulus increases with increase of scale, but modulus maxima of the noise decreases with increase of scale. After several wavelet transform, the modulus maxima of the signal can maintain a rather high value, modulus maxima of noise become small. Setting a threshold, we retain the maximum value that is greater than the threshold value, and eliminate the maximum value that is smaller than the threshold value, we calculate estimated wavelet coefficients by using residual maxima. We use the wavelet coefficient to reconstruct the signal so as to achieve the purpose of eliminating the noise [8]. The basic idea of the wavelet transform denoising can be expressed by the Fig. 3.

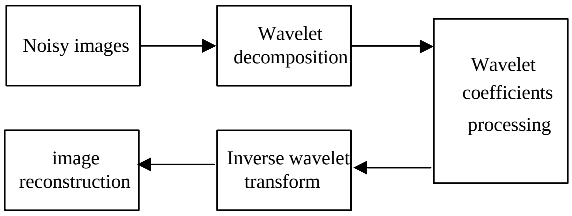


Fig. 3. Wavelet denoising diagram.

The basic process of the wavelet transform denoising:

a) The decomposition process is to select a wavelet basis and determine the level of wavelet decomposition, decompose the signal, and calculate the modulus maxima of wavelet transform coefficients in each scale.

b) We make processing threshold to large value of mold in the maximum scale. So long as its absolute value of the amplitude is greater than the threshold, we keep it. Otherwise, it should be removed.

c) The wavelet coefficients are computed according to the modulus maxima of each scale.

d) Signal is reconstruct by using wavelet coefficients.

3.2. Selection of Wavelet Denoising Threshold Function

Wavelet thresholding method is the first multi-scale wavelet decomposition of the signal, values of the signal wavelet coefficients are relatively large, and the wavelet coefficient values of the noise are relatively small. So we can set the appropriate threshold λ. We deal with the wavelet coefficients that are greater than and less than the threshold respectively. We use the wavelet coefficients to reconstruct the signal through inverse transform in order to achieve the purpose of eliminating the noise.

During the process of denoising in the wavelet threshold, processing strategies of wavelet coefficients and methods of estimation are reflected by the threshold function. The wavelet thresholding functions include hard threshold function, soft threshold function and improved threshold value function [9].

1) Hard threshold function is expressed as:

$$\widehat{W_{j,k}} = \begin{cases} W_{j,k} & |W_{j,k}| \geq \lambda \\ 0 & |W_{j,k}| < \lambda \end{cases}, \tag{17}$$

2) Soft threshold function is expressed as:

$$\widehat{W_{j,k}} = \begin{cases} sgn(W_{j,k})(\left|W_{j,k}\right| - \lambda) & \left|W_{j,k}\right| \geq \lambda \\ 0 & \left|W_{j,k}\right| < \lambda \end{cases}, \quad (18)$$

3) Semi-soft threshold function put forward by Gao is expressed as:

$$\widehat{W_{j,k}} = \begin{cases} sgn(W_{j,k})\dfrac{\lambda_2(\left|W_{j,k}\right| - \lambda_1)}{\lambda_2 - \lambda_1} & \lambda_1 < \left|W_{j,k}\right| < \lambda_2 \\ 0 & \left|W_{j,k}\right| < \lambda_1 \\ W_{j,k} & \left|W_{j,k}\right| > \lambda_2 \end{cases}, \quad (19)$$

4) Threshold function proposed by X.-P. Zhang.

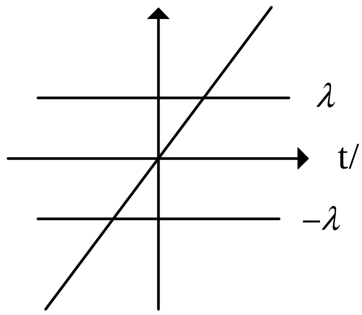
A proposed is the expansion of the soft threshold function. It has a higher order through improvement, threshold function can be expressed as follows:

$$\widehat{W_{j,k}} = \begin{cases} W_{j,k} + (\lambda - \dfrac{\lambda}{2k+1}) & W_{j,k} < -\lambda \\ \dfrac{1}{(2k+1)\lambda^{2k}} W_{j,k}{}^{2k+1} & \left|W_{j,k}\right| < \lambda \\ W_{j,k} - (\lambda - \dfrac{\lambda}{2k+1}) & W_{j,k} > \lambda \end{cases}, \quad (20)$$

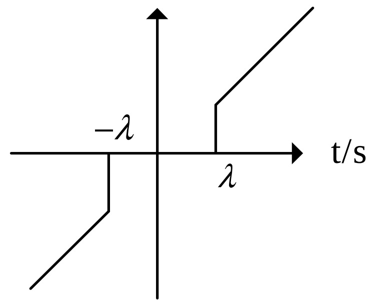
5) A type of improved compromise method between hard and soft threshold. Threshold function of which is expressed as:

$$\widehat{W_{j,k}} = \begin{cases} sgn(W_{j,k})(\left|W_{j,k}\right| - \mu\lambda) & \left|W_{j,k}\right| \geq \lambda \\ 0 & \left|W_{j,k}\right| < \lambda \end{cases}, \quad (21)$$

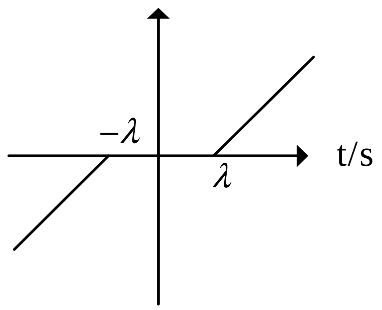
$\mu \in [0 \quad 1]$. If μ = 0, it is hard threshold function, if μ =1, then it is soft threshold function. The several threshold functions are shown in Fig. 4.



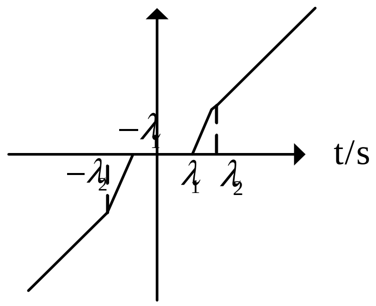
(a) Signal



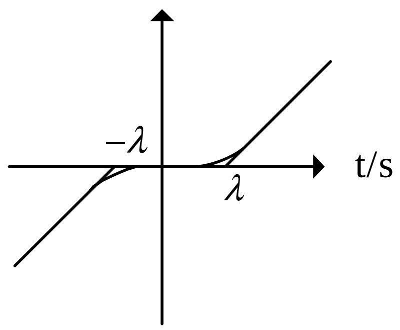
(b) Hard threshold function



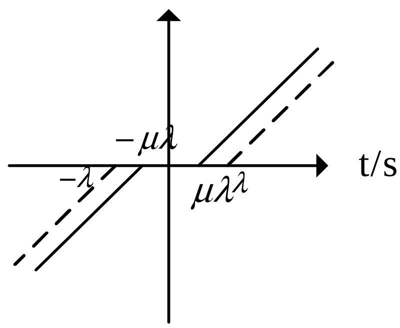
(c) Soft threshold function



(d) semi-soft threshold function



(e) Zhang improved Threshold Function



(f) Hard and soft threshold compromises

Fig. 4. Several threshold functions.

3.3. An Improved Threshold Function

The several threshold functions have achieved noticeable results and been widely used, by analyzing the hard threshold function. We know that $W_{j,k}$ is discontinuous at $-\lambda$ and λ, a reconstructed image may produce Gibbs effect. In the soft threshold function, although the overall $\widehat{W_{j,k}}$ continuity is good, but when $|W_{j,k}| \geq \lambda$, there is a certain deviation between $W_{j,k}$ and $\widehat{W_{j,k}}$. It will affect the degree of approximation between the reconstructed image and the original image. This paper has improved threshold function on the basis of the compromise method of the hard threshold value and the soft one to make the function continuous on $\pm\lambda$. We use a function to adjust the coefficient that is smaller than the threshold value so that the coefficients smaller than the threshold value gradually tend to zero. It improves discontinuity of the hard threshold and constant deviation of soft threshold in order to prevent the useful information from being filtered out as noise and keep the edge detail.

Expression of the function is as follows:

$$\widehat{W_{j,k}} = \begin{cases} sgn(W_{j,k})(|W_{j,k}| - \lambda + \gamma) & |W_{j,k}| \geq \lambda \\ \dfrac{1}{\exp(\frac{2k}{|\lambda|})\lambda^{2k}} W_{j,k}{}^{2k+1} & |W_{j,k}| < \lambda \end{cases}, \quad (22)$$

where

$$\gamma = \frac{\lambda}{\exp\left(2k / |W_{j,k}|\right)}, \quad (23)$$

New threshold function is shown in Fig. 5.

$\widehat{W_{j,k}}$ of improved function is continuous at the $\pm\lambda$, so oscillation phenomenon can be avoided when the signal is reconstructed , when $|W_{j,k}| < \lambda$, we use a coefficient of the inverse of the exponential function to control the trend of $|W_{j,k}|$ close to zero so that it overcomes the discontinuity appeared in the hard threshold function, when $|W_{j,k}| \geq \lambda$, we add an offset term $\gamma = \frac{\lambda}{\exp\left(2k / |W_{j,k}|\right)}$ in the expression so as to compensate for the deviation between the $W_{j,k}$ and $\widehat{W_{j,k}}$.

$$\begin{aligned} &\lim_{k \to \infty} sgn(W_{j,k})(|W_{j,k}| - \lambda + \frac{\lambda}{\exp\left(2k / |W_{j,k}|\right)}) \\ &= sgn(W_{j,k})(|W_{j,k}| - \lambda) \end{aligned}, \quad (24)$$

$$\begin{aligned} &\lim_{k \to 0} sgn(W_{j,k})(|W_{j,k}| - \lambda + \frac{\lambda}{\exp\left(2k / |W_{j,k}|\right)}) \\ &= W_{j,k} \end{aligned}, \quad (25)$$

Equation (24) and Equation (25) show that when $k \to \infty$, Equation (22) is a soft threshold function, when $k \to 0$, Equation (22) is a hard threshold function. Therefore, a new threshold function is the one between soft and hard threshold function, it can be selected by adjusting the value of k.

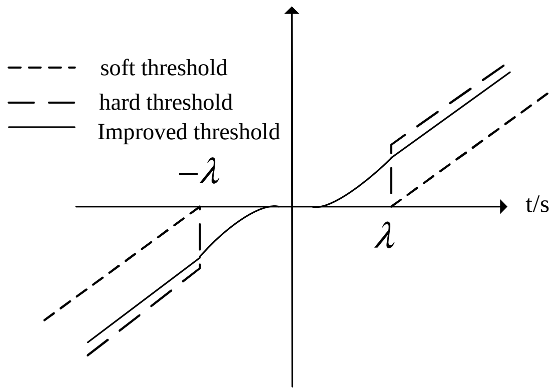


Fig. 5. Improved threshold function.

4. Test Results

In this paper, we use microstructure image of 12Cr1MoV steel as a test object. In the image acquisition process, the noise is mixed with the image so that the picture quality deteriorates. We use soft threshold and hard threshold and improved threshold function to process images. We select sym4 of the wavelet basis to category the image into four layers, experimental results of which are shown in Fig 6.

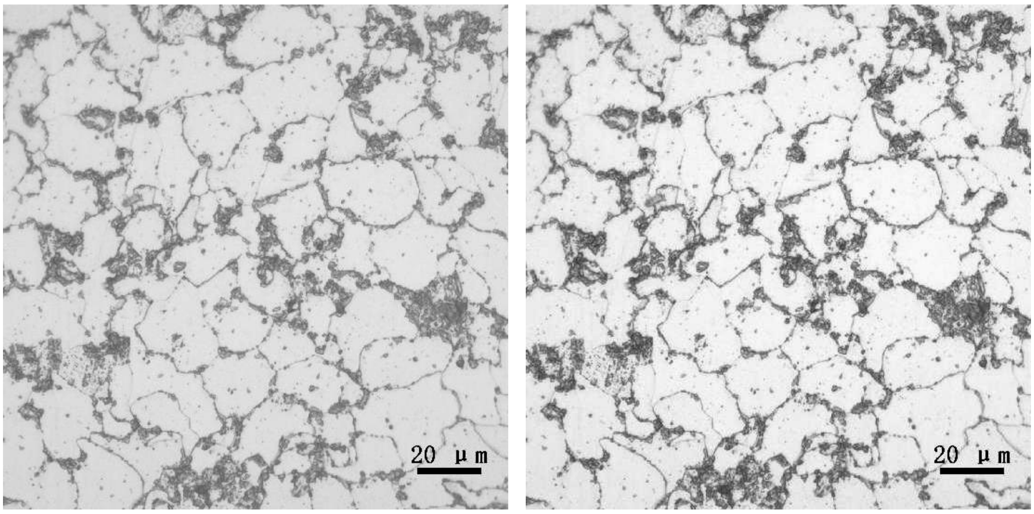
We select the signal to noise ratio (SNR) and peak signal-to-noise ratio (PSNR) as measure of the test index. Three threshold function test results are shown in Table 1.

Table 1. Using different algorithms denoising SNR and PSNR value.

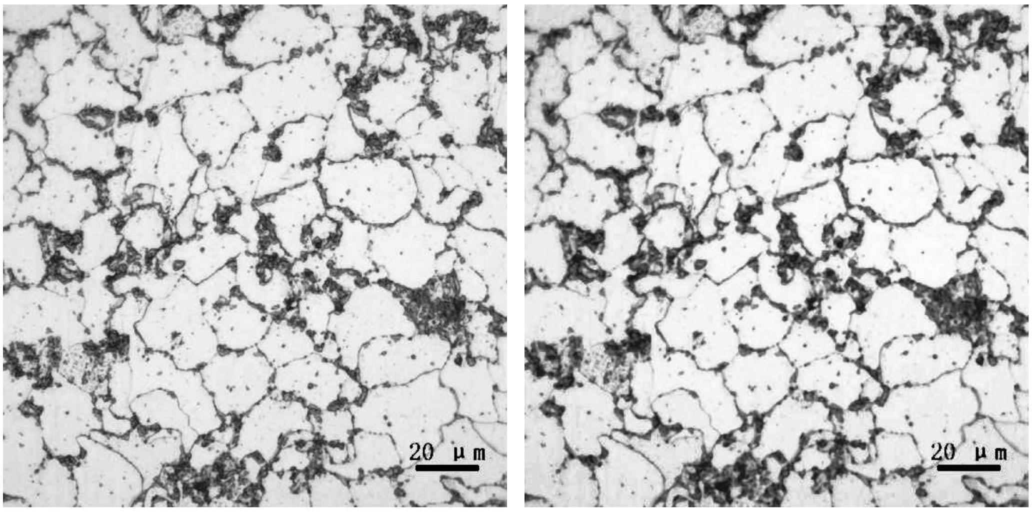
Detection index	SNR	PSNR
Hard threshold	14.4258	35.0318
Soft threshold	14.5306	37.6319
Improved methods	14.8513	38.7511

5. Conclusions

Testing laboratory study showed that a soft threshold denoising image signal-to-noise ratio (SNR) and peak signal-to-noise ratio (PSNR) is higher than that of denoising image of the hard threshold soft threshold. During the hard threshold processing, when the high frequency coefficients is quantified, the wavelet coefficients will create numerical mutation. Mutation of this value is a high-frequency noise. It will be introduced to the image when the image reconstructs. Because the soft thresholding method is continuous, the image effect is smoother. The edge details of image blur, but general trend still reflects the original one of the image. De-noising effect of soft threshold is better than the hard one. Hard threshold method well preserved the edge details of the image, but the continuity of the wavelet coefficients is poor, the reconstructed signal may create the mutation or shock phenomenon, the noise has not been completely suppressed. Improved threshold function in removing noise and retaining the details of the image information is appropriate, PSNR of the denoising image is evidently improved. The improved method obtains satisfactory denoising effects.



(a) Original image (b) Hard threshold



(c) Soft threshold (d) Improved methods

Fig. 6. Wavelet transform denoising images.

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