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Analysis of Evaluation Stages Involved in Penman Monteith Equation

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Abstract: Evapotranspiration plays pivotal role in water management schemes and irrigation scheduling design. Water resource engineers and strategic planners are therefore highly focusing on finding out more accurate methods for its measurement. Several scientists have formulated many formulae for predicting this very important parameter but all these methods have shortfalls. To resolve the ambiguity of choice United Nations and American Society of Civil Engineers accepted the Penman Monteith equation accepted for measurement for reference evapotranspiration to come to a consensus. Here, we have provided the steps involved for effective measurement of evapotranspiration. The article is divided into five sections. Section One introduces the essence and importance of Evapotranspiration. Section Two speaks about the climatic parameters that require measurement. Section Three discusses the very important Penman – Monteith equation. Section Four elaborates on the stages those are involved for efficient measurement of this equation followed by a brief conclusion in Section Five. *Copyright © 2016 IFSA Publishing, S. L.*

Keywords: Evapotranspiration, Penman Montheith Equation, Climatic Parameters, Stages of Measurement.

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1. Introduction

Evapotranspiration comprises two physical phenomenon, evaporation and transpiration. In evaporation, water bodies' loss water in form of vapor and in transpiration, plant bodies sweat to loose water from them, both adding up to give the total amount of water lost from the earth surface. Both these are simultaneous processes which are highly influenced by the solar radiation, humidity, speed of wind and temperature (Fig.1). Evapotranspiration (ET) is an energy-driven process which increases with temperature, solar radiation, and wind and decreases with increasing humidity. Evapotranspiration plays a pivotal role in hydrological balance as it is responsible for 15 % of the atmosphere's water vapor.

Principally there are three methods by which evapotranspiration is formulated viz. mass – transfer method, temperature based method and radiation based method.

Spatially calculating ET is necessary because it is a major component in quantifying a water budget scheme and the maps provide the spatial ability to display the distribution. Evapotranspiration assessment is of outstanding importance both for planning and monitoring purposes. ET helps in determining when and how much irrigation water is needed and for designing and management of irrigation system.

Five main processes which are included in the hydrologic cycle:

1) Condensation;
2) Precipitation;
3) Infiltration;
4) Runoff;
5) Evapotranspiration.

ET varies because of a multitude of factors like wind, temperature, humidity, and water availability. Other than the primary factors, there are secondary factors which also hugely influence ET measurements and they are viz. crop type, crop length/height, soil type, period of growth, soil salinity, macro and micro mineral contents of the soil, leaf area index. When the crop is small, loss of water is predominantly because of evaporation but as the crop grows and covers the soil surface, principle component of water loss is transpiration, conclusively stating that crop development stage plays vital role in evapotranspiration losses of water. All these factors also determine ET rates which is measured in units of mm/time, where the time scale may be hours, days, months, years or even decades.

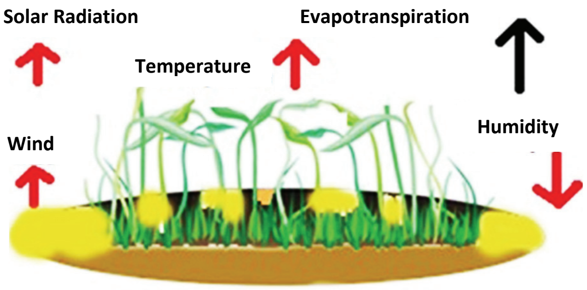

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Fig. 1. Schematic of Evapotranspiration Process.

2. Measured Parameters

Climatic data plays the most pivotal in determination of ET. The principle parameters which are the most determining factors are solar radiation, wind speed, temperature and humidity. Two meters above sea level for any extensive surface of green grass cover is taken as standard for ensuring the integrity of computation.

Other climatological and meteorological parameters like latitude, longitude, altitude, sunshine duration, soil heat flux and atmospheric pressure do significantly contribute towards ET measurement. Other than the primary factors, there are secondary factors which also hugely influence ET measurements and they are viz. crop type, crop length/height, soil type, period of growth, soil salinity, macro and micro mineral contents of the soil, leaf area index. Another factor of consideration is that the soil surface should have similar plantation for at least 100 meters circumference bringing uniformity of vegetation. As suggested by the ASCE, the definition for a reference surface is given as

"A hypothetical reference crop with an assumed crop height of 0.12 m, a fixed surface resistance of 70 s m^{-1} and an albedo of 0.23." (Allen, *et al.*, 1998).

Assumptions which need to be hold for this reference surface to deliver to the best expected results are:

1) An extensive surface of green grass;
2) Of uniform height;
3) Completely shading the ground;
4) Actively growing;
5) With adequate water.

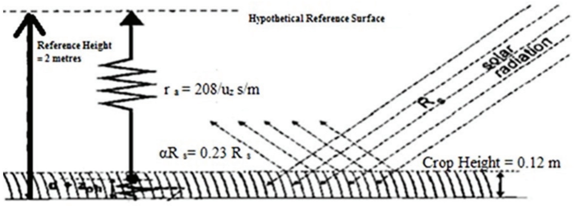

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Fig. 2. Schematic of Evapotranspiration from a reference surface.

3. Penman – Monteith Equation

Several empirical methods have been developed in the last 50 years and are in existence based on different climatic variables. In 1999, the Irrigation Association (IA) requested the EWRI – ASCE to standardize an empirical formulae for ET, based on which they came up with two equations, one for the short crop and the second one for the tall crops.

Owing to the difficulty of obtaining accurate field measurements, ET o is commonly computed from weather data. A large number of empirical or semi-empirical equations have been developed for assessing reference evapotranspiration from meteorological data. Numerous researchers have analyzed the performance of the various calculation methods for different locations. As a result of an Expert Consultation held in May 1990, the FAO Penman-Monteith (PM) method is now recommended as the standard method for the definition and computation of the reference evapotranspiration ET o. the ASCE – EWRI standardized reference ET equation for daily and hourly periods is expressed as under

$$ET_0 = \frac{0.408\times\Delta\times(R_n-G)+\gamma\times\left(\frac{C_n}{T_{mean}+273}\right)\times u_2(e_s-e_a)}{\Delta+\gamma(1+C_d u_2)} \tag{1}$$

The Penman Monteith equation, updated and recommended by FAO for Reference Evapotranspiration considering short reference crop on daily measurement basis where values of C_n and C_d are 900 and 0.34 respectively, (Allen, *et al.*, 1998) is expressed as

$$ET_0 = \frac{0.408\times\Delta\times(R_n-G)+\gamma\times(\frac{900}{T_{mean}+273})\times u_2(e_s-e_a)}{\Delta+\gamma(1+0.34u_2)} \quad (2)$$

All notations and symbols are defined in Table 1.

Table 1. Referential Values for Short and Tall crops.

Term	Value	
	short reference	**tall reference**
Reference vegetation height (m)	0.12	0.50
Height of air temperature, humidity, wind measurements (m)	2	2
Zero plane displacement height (m)	0.08	0.08
Canopy resistance, daily (s m^{-1})	70	45
Canopy resistance, daytime (s m^{-1})	50	45
Canopy resistance, night time (s m^{-1})	200	200
Latent heat of vaporization (MJ kg^{-1})	2.45	2.45

For tall reference crop, the values of C_n and C_d are 1600 and 0.38 respectively for daily measurement.

4. Evaluation Stages

This section deals with the step wise evaluation of the Penman Monteith equation where the different sub components of the equation are validated to eventually materialize the final form. We will depict each stage as step for easier comprehension. (Allen, *et al.*, 2005; Romero, *et al.*, 2009).

Step 1: Daily Mean Temperature (T_{mean})

The temperature is measured in °C. Albeit average daily temperature will suffice, but to avoid non – linearity due to saturation vapor pressure – temperature relationship, it is a better practice to account both the maximum T_{max} and minimum T_{min} temperatures of daily basis for calculating daily mean temperature T_{mean}.

$$T_{mean} = \frac{T_{max}+T_{min}}{2} \quad (3)$$

Step 2: Solar Heat Flux Density (G)

The soil heat flux, G, is the energy that is utilized in heating the soil. G is positive when the soil is warming and negative when the soil is cooling. Soil heat flux density (G) is the conduction of energy per unit area in response to a temperature gradient.

$$G = C_s\frac{T_i-T_{i-1}}{\Delta t}\Delta z \quad \ldots\ldots\ldots MJm^{-2}d^{-1}, \quad (4)$$

where C_s = KA; where K is the heat conductivity of material (W m^{-3} K^{-1}) and A is the surface area in m^2; Δz is the difference of depth between layers in meters; Δt is the difference of temperature.

Step 3: Wind Speed at 2 m Height (u_2)

The adjustment factor for wind speed measured at different height to wind speed at an altitude of 2 meters is given below because wind speed differs with altitude which may create ambiguities in the final result.

$$u_2 = u_z\frac{4.87}{\ln(67.8z-5.42)} = u_2(miles/hr)\times 0.477, \quad (5)$$

where u_2 is the wind speed at 2 meters height (m s^{-1}); u_z is the wind speed at z meters height (m s^{-1}).

Step 4: Slope of Saturation Vapour Pressure Curve (Δ)

$$\Delta = \frac{4098\left[0.6108exp\left(\frac{17.27T}{T_{mean}+237.3}\right)\right]}{(T_{mean}+237.3)^2}\ KPa°C^{-1}, \quad (6)$$

where T_{mean} is the mean daily temperature °C; exp = 2.7183 (base of natural logarithm).

Step 5: Atmospheric Pressure (P)

$$P = 101.3\left[\frac{293-0.0065z}{293}\right]^{5.26} \ldots..KPa \quad (7)$$

Step 6: Psychrometric Constant (γ)

Psychrometric constant γ relates partial water pressure in air to air temperature in order to estimate vapour pressure using paired dry and wet thermometer bulb temperature readings.

$$\gamma = \frac{C_p * P}{\varepsilon\lambda} = 0.665\ x\ 10^{-3}P\ \ldots\ldots KPa°C^{-1} \quad (8)$$

where C_p is the specific heat of dry air at constant pressure, 1.013 × 10^{-3} MJ kg^{-1} °C^{-1};
λ is the latent heat of vaporization, 2.45 MJ kg^{-1};
P is the atmospheric pressure kPa;

$$\lambda = 2.501-(2.361\times10^{-3})T_a \ \ldots.MJkg^{-1} \quad (9)$$

where ε is the ratio of molecular weight of water vapor to dry air = 0.622.

Step 7: Delta Term (DT) (Auxiliary Calculation for Radiation Term)

$$DT = \frac{\Delta}{\Delta+\gamma(1+0.34u_2)} \quad (10)$$

Step 8: Psi – Term (PT) (Auxiliary Calculation for Wind Term)

$$PT = \frac{\Delta}{\Delta+\gamma(1+0.34u_2)} \quad (11)$$

Step 9: Temperature Term (TT) (Auxiliary Calculation for Wind Term)

$$TT = \left[\frac{900}{T_{mean} + 273}\right] \times u_2 \tag{12}$$

Step 10: Inverse Relative Distance between Earth and Sun (d r) and Solar Declination (δ)

$$d_r = 1 + 0.033\cos\left(\frac{2\pi}{365}J\right) radian, \tag{13}$$

$$J = no. of\ days\ in\ a\ year,$$

$$\delta = 0.409\sin\left(\frac{2\pi}{365}J - 1.39\right)\ radian \tag{14}$$

Step 11: Conversion of Latitudes (φ) in Radians

$$\varphi[radians] = \frac{\pi}{180}\varphi[decimal\ degrees] \tag{15}$$

Step 12: Sunset Hour Angle (ωs)

$$\omega_s = \arccos[-\tan(\varphi)\tan(\delta)]\ radian \tag{16}$$

Step 13: Possible Day Light Hour (N)

$$Possible\ Day\ light\ hour\ N = \frac{24}{\pi}\omega_s \tag{17}$$

Step 14: Extra-terrestrial Radiation (Ra)

For each day and for different latitudes, extra – terrestrial Radiation can be calculate by knowing the solar constant, solar declination and time of the year

$$\mathrm{R_a} = \frac{24(60)}{\pi}\mathrm{G_{sc}d_r}[\omega_s \sin(\varphi)\sin(\delta) + \cos(\varphi)\sin(\omega_s)] \ldots\ldots \mathrm{MJm^{-2}d^{-1}} \tag{18}$$

where G_{sc} is the solar constant = 0.0820 MJ m^{-2} min^{-1}

Step 15: Daily Mean Solar Radiation (Rs)

We provide below the conversion relationship between Radiation in MJ and W per square meter per day.

$$R_S\ MJm^{-2}day^{-1} = R_s \times 0.0864\ Wm^{-2}day^{-1} \tag{19}$$

$$R_s = \left(a_s + b_s\frac{n}{N}\right)R_a \ldots \mathrm{MJ}m^{-2}d^{-1}, \tag{20}$$

where *n* is the actual duration of sunshine (hours);
N is the maximum possible duration of daylight hours (hours);
n/ N is the relative sunshine duration;
a_s is the regression constant expressing fractional R_a received by earth on an overcast day ($n = 0$);
$a_s + b_s$ is the fractional R_a reaching the earth on a clear day ($n = N$).

$$R_s = k_{rs}\sqrt{T_{max} - T_{min}}\,R_a \ldots \mathrm{MJ}m^{-2}d^{-1}, \tag{21}$$

where k_{rs} is the adjustment coefficient (0.16 to 0.19 [°$C^{0.5}$])

With no adjustment for station elevation

$$R_s = (a_s + b_s)R_a \tag{22}$$

Step 16: Clear Sky Solar Radiation (Rso)

$$R_{so} = (0.75 + 2 \times 10^{-5}z)R_a \ldots \mathrm{MJ}m^{-2}d^{-1} \tag{23}$$

Step 17: Net Solar or Net Shortwave Radiation (Rns)

$$R_{ns} = (1 - \alpha)R_s \ldots\ldots \mathrm{MJ}m^{-2}d^{-1}, \tag{24}$$

where α is the albedo or canopy reflection coefficient (0.23)

Step 18: Net Outgoing Long Wave Solar Radiation (Rnl)

$$R_{nl} = \sigma\left[\frac{(T_{max} + K)^4 + (T_{min} + K)^4}{2}\right](0.34 - 0.14\sqrt{e_a})\left(1.35\frac{R_s}{R_{so}} - 0.35\right) \ldots. \mathrm{MJ}m^{-2}d^{-1} \tag{25}$$

where σ is the Stefan Boltzman constant (4.903 ×10^{-9} MJ K^{-4} m^{-2} day^{-1}), K = 273 Kelvin

Step 19: Net Radiation (Rn)

$$R_n = R_{ns} - R_{nl} \ldots\ldots\ldots\ldots. \mathrm{MJ}m^{-2}d^{-1} \tag{26}$$

The net radiation in equivalent of evaporation (mm) *(R_{ng})*

$$R_{ng} = 0.408 \times R_n \tag{27}$$

Step 20: Mean Saturation Vapour Pressure Derived from Air Temperature T (es)

$$Sat. Vapour\ Pressure\ at\ T\ °\ C\ temp.\ e^{\circ}(\mathrm{T}) = 0.6108 \times 2.7183^{\left(\frac{17.27T}{T+237.3}\right)} \ldots\ldots.. \mathrm{KPa} \tag{28}$$

$$e^{\circ}(T_{max}) = 0.6108 \times 2.7183^{\left(\frac{17.27T_{max}}{T_{max}+237.3}\right)} \tag{29}$$

$$e^{\circ}(T_{min}) = 0.6108 \times 2.7183^{\left(\frac{17.27T_{min}}{T_{min}+237.3}\right)} \tag{30}$$

$$e_s = \frac{e^{\circ}(T_{max}) + e^{\circ}(T_{min})}{2} \ldots\ldots\ldots \mathrm{KPa} \tag{31}$$

Step 21: Actual Vapour Pressure (ea) Derived from Dew Point Temperature

$$e_a = e^0(T_{wet}) - \gamma_{psy}(T_{dry} - T_{wet}) \tag{32}$$

Step 22: Actual Vapour Pressure (e_a) Derived from Psychrometric Data

$$e_a = e^0(T_{dew}) = 0.6108\exp[\frac{17.27T_{dew}}{T_{dew} + 273.3}] \quad (33)$$

$$\gamma_{psy} = a_{psy}P \quad (34)$$

Step 23: Actual Vapour Pressure (e_a) Derived from Relative Humidity RH

$$e_a = \frac{e^{\circ}(T_{min})\left[\frac{RH_{max}}{100}\right] + e^{\circ}(T_{max})[\frac{RH_{min}}{100}]}{2} \ldots kPa \quad (35)$$

$$e_a = \frac{RH_{mean}}{100}\left[\frac{e^{\circ}(T_{max}) + e^{\circ}(T_{min})}{2}\right] \ldots kPa \quad (36)$$

Step 24: Vapour Pressure Deficit

$$Vapour\ Pressure\ Deficit = e_s - e_a \quad (37)$$

Step 25: Overall ET_o Equation

Radiation Term (ET_{rad})

$$ET_{rad} = DT \times R_{ng} \quad (38)$$

Wind Term (ET_{wind})

$$ET_{wind} = PT \times TT(e_s - e_a) \quad (39)$$

Final Reference Evapotranspiration (ET_o)

$$ET_o = ET_{wind} + ET_{rad} \quad (40)$$

Table 2. All symbols and variables used in PM equation.

Symbol	Connotation	Unit
C_n	Numerator constant for the reference crop type	
C_d	Denominator constant for the reference crop type	
C_s	Solar constant	
K	Heat Conductivity of Material	**W m^{-3} K^{-1}**
A	Surface Area	**m^2**
Δt	Difference of Temperature	**°C**
Δz	Difference of depth	**metres**
P	Atmospheric Pressure	**kPa**
z	Elevation above sea level	**Metres**
γ	Psychrometric Constant	**kPA °C^{-1}**
λ	Latent Heat of Vaporization, 2.45	**MJ kg^{-1}**
C_p	Specific heat at constant pressure, 1.013*10 $^{-3}$	**MJ kg^{-1} °C^{-1}**
ε	Ratio molecular weight of water vapor, 0.622	-
T_{mean}	Mean Air Temperature	**°C**

Table 2. Continued.

Symbol	Connotation	Unit
T_{max}	Maximum Air Temperature	**°C**
T_{min}	Minimum Air Temperature	**°C**
e°(T)	Saturation Vapour Pressure at air Temp T	**kPa**
T	Air Temperature	**T**
exp	Exponential base of natural logarithm, 2.7183	-
e_s	Saturation Vapour Pressure	**kPa**
λ	Latent Heat of Vaporization, 2.45	**kPa**
C_p	Specific heat at constant pressure, 1.013*10 $^{-3}$	**kPa**
ε	Ratio molecular weight of water vapor, 0.622	**kPa**
T_{mean}	Mean Air Temperature	**kPa °C^{-1}**
T_{max}	Maximum Air Temperature	**%**
T_{min}	Minimum Air Temperature	**%**
e°(T)	Saturation Vapour Pressure at air Temp T	**kPa**
T	Air Temperature	**MJ m^{-2} day^{-1}**
exp	Exponential base of natural logarithm, 2.7183	**MJ m^{-2} day^{-1}**
e_s	Saturation Vapour Pressure	**MJ m^{-2} day^{-1}**
e_a	Actual Vapour Pressure	**MJ m^{-2} day^{-1}**
$e^{\circ}(T_{max})$	Sat. Vapour Pressure at mean daily max air temp	**MJ m^{-2} day^{-1}**
$e^{\circ}(T_{min})$	Sat. Vapour Pressure at mean daily min air temp	**MJ m^{-2} day^{-1}**
Δ	Slope of Sat. Vapour Pressure Curve at air Temp T	**MJ m^{-2} min^{-1}**
RH_{max}	Maximum Relative Humidity	-
RH_{min}	Minimum Relative Humidity	**Radian**
$e_s - e_a$	Vapour Pressure Deficit	**Hour**
R_a	Extra-terrestrial Radiation	**Hour**
R_s	Solar or Shortwave Radiation	-
R_{so}	Clear – sky Solar Radiation	-
δ	Solar Declination	**Radian**
ω_s	Sunset hour angle	**Radian**
b_s	Regression Constant, n = 0	-
$a_s + b_s$	Regression Constant, n = N	-
α	Albedo or canopy reflection coefficient	-
σ	Stefan Boltzman constant, 4.903 * 10 $^{-9}$	**MJ K^{-4} m^{-2} day^{-1}**
k_{rs}	Adjustment Coefficient	**°C$^{-0.5}$**
u_2	Wind Speed at 2 m above ground surface	**m s^{-1}**
u_z	Wind Speed at *z* m above ground surface	**m s^{-1}**

7. Conclusions

This article throws light on stepwise evaluation for the measurement of evapotranspiration (ET). The valedictory results are greatly influenced by the day to day weather conditions and also it is highly effected by the geographical location where the measurement is being carried out. In order to obtain even better results, the constants need to be calibrated according to the planetary position. A parallel study ongoing to calibrate these constants for geographical location of southern West Bengal, India. With a reliable measurement of ET, we will be able to design and implement controllers scheduling our day to day irrigation plan and water management schemes and other water resources applications.

References

[1]. Allen R. G., L. S. Pereira, D. Raes, M. Smith, Crop evapotranspiration: guidelines for computing crop water requirements, *FAO - Food and Agriculture Organization of the United Nations*, Rome, 1998, pp. 1-15.

[2]. Allen R. G., Walter I. A., Elliot R. L., Howell T. A., Itenfisu D., Jensen M. E., Snyder R., The ASCE standardized reference evapotranspiration equation, *ASCE*, 2005.

[3]. Dukes M. D., M. L. Shedd, S. L. Davis, Smart Irrigation Controllers: Programming Guidelines for Evapotranspiration-Based Irrigation Controllers AE445, Gainesville, *University of Florida Institute of Food and Agricultural Sciences*, 2009. http://edis.ifas.ufl.edu/pdffiles/ae445 Accessed Dec. 8, 2009.

[4]. Smith M., Allen R. G., Monteith J. L., Pereira L. S., Perrier A., Pruitt W. O., Report on the expert consultation on procedures for revision of FAO guidelines for prediction of crop water requirements, *Land and Water Development Division, United Nations Food and Agriculture Service*, Rome, Italy, 1992.

[5]. Smajstrla A. G., B. J. Boman, D. Z. Haman, F. T. Izuno, D. J. Pitts, F. S. Zaxueta, Basic Irrigation Scheduling in Florida, *University of Florida Institute of Food and Agricultural Aciences*, AE 111. Gainesville, 1997, pp. 1-10. http://edis.ifas.ufl.edu/pdffiles/ae111 Accessed Dec. 8, 2009.

[6]. Lincoln Zotarelli, Michael D. Dukes, Consuelo C. Romero, Kati W. Migliaccio, Kelly T. Morgan, AE 459, Step by Step Calculation of the Penman-Monteith Evapotranspiration (FAO-56 Method), *University of Florida, IFAS Extension*, pp. 1-10.

[7]. Faruk Bin Poyen, Apurba Kumar Ghosh, Palash Kundu, Review on Different Evapotranspiration Empirical Equations, *International Journal of Advanced Engineering, Management and Science (IJAEMS)*, Vol. 2, Issue 3, March 2016, pp. 17-24.

[8]. Brian J. Boman, E. W. Stover, Outline for Managing Irrigation of Florida Citrus with High Salinity Water, *University of Florida, IFAS Extension,* AE 217, 2002, pp. 1-4.

[9]. Simonne E. H., *et al*., Principle and Practices of Irrigation Management for Vegetables, *University of Florida, IFAS Extension*, 2010, pp. 17-27.

[10]. Blaney H. F., Criddle W. D., Determining water requirements in irrigated areas from climatological and irrigation data, *United States Department of Agriculture, Soil Conservation Service*, Washington, DC, 1950.

[11]. Lincoln Zotarelli, Michael D. Dukes, Consuelo C. Romero, Kati W. Migliaccio, Kelly T. Morgan, Step by Step Calculation of the Penman-Monteith Evapotranspiration (FAO-56 Method), AE 459, *University of Florida, IFAS Extension*, 2010.