ISSN 1726-5479

SENSORS & TRANSDUCERS

6/09 vol. 105

Electronic Circuits, Signal Conditioning and ASICs for Sensors

International Frequency Sensor Association Publishing

IFSA

Sensors & Transducers

Volume 105, Issue 6
June 2009

www.sensorsportal.com

ISSN 1726-5479

Sensors & Transducers Journal (ISSN 1726-5479) is a peer review international journal published monthly online by International Frequency Sensor Association (IFSA).

Sensors & Transducers Journal

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Issue 6
June 2009

www.sensorsportal.com

ISSN 1726-5479

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Sensors & Transducers

ISSN 1726-5479

http://www.sensorsportal.com

New Type Small-angle Sensor Based on the TIR and SPR Theories in Heterodyne Interferomery

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Received: 15 April 2009 /Accepted: 22 June 2009 /Published: 30 June 2009

Abstract: In this paper, a new type small-angle sensor based on the total internal reflection (TIR) and surface plasmon resonance (SPR) theories in heterodyne interferomery is proposed. With the small-displacement sensor, a small rotation angle can be obtained only by measuring the variation in phase difference between s- and p-polarization states. The best theoretical sensitivity of the small-angle sensor is 2×10^4 degree/degree. And its resolution can reach 1×10^{-7} radian. The sensor has some merits, e.g., a simple optical setup, high resolution, high sensitivity, rapid measurement. *Copyright © 2009 IFSA.*

Keywords: Total internal reflection (TIR), Surface plasmon resonance (SPR), Kretchmann's configuration, Heterodyne interferometry (HI), Small-angle measurement

1. Introduction

It is well known that the small-angle measurement plays an important role in the high technology industries. In the past few decades, there have been many articles [1-8] published in order to achieve the best resolution of the small-angle measurement. Huang *et al*. proposed some methods [1-2] of the angle measurement based on the internal-reflection effect some years ago. Owing to the methods with low resolutions, Su and Chiu proposed some improved methods [3-4] of the small-angle measurement by modifying Huang's systems. Besides, Chiu *et al.* presented an instrument [5] for measuring small angles based on Huang and Ni's method [6] by using two parallelogram prisms, i.e., two elongated prisms, instead of two right-angle prisms. The instrument for measuring small angles was based on multiple total internal reflections (MTIR) in heterodyne interferometry (HI). And its resolution could reach 2.2×10^{-6} radian. And Wang *et al*. designed a reflective type small-angle sensor [7] based on multiple total-internal reflections (MTIRs) in heterodyne interferometry. Its resolution could reach 5.5×10^{-7} radian. In addition, Wang *et al*. has ever proposed some improved methods [8-9] for measuring small angles by measuring the phase difference between two right-angle prisms based on SPR technology and heterodyne interferometry. The best resolution could reach 2.4×10^{-7} [9] or even 1.2×10^{-7} radian [8].

In this paper, we try to measure small-angle rotation by combining the TIR and SPR theories in heterodyne interferomery. The small-angle sensing unit is a right-angle prism with the refractive index of 1.51509 at the wavelength $\lambda = 632.8$ nm. It deserves to be mentioned that one right-angle side of the right-angle prism isn't coated with any metal film, but the other side that is coated with two layers of metal films. In fact, the new type small-angle sensor is similar to a high sensitivity sensor used as a biosensor [10]. But the new type small-angle sensor is different from the biosensor in the Kretchmann's configuration [11]. Our sensor is with four –layer system, but that biosensor with three-layer system. With the small-displacement sensor, a small rotation angle can be obtained only by measuring the variation in phase difference between s- and p-polarization states due to the TIR and SPR effects. From experimental results, it is evident that they are in good correspondence with theoretical results. And the resolution of the method can reach 1×10^{-7} radian. Its feasibility is demonstrated.

2. Principle

2.1. The Phase Difference at the TIR Effect

As shown in Fig. 1, a ray of light in air is incident at θ on one side surface of a right-angle prism with refractive index n. The light ray is refracted into the prism and it propagates toward the hypotenuse surface of the prism. At that surface, there is a boundary between the prism and air. If the angle of incidence at the boundary is θ_1, then we have

$$\theta_1 = 45^\circ + \sin^{-1}\left(\frac{\sin\theta}{n}\right) . \tag{1}$$

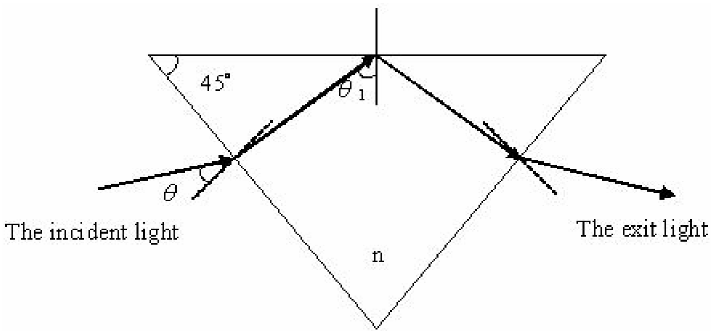

Fig. 1. A ray of light in air is incident at θ on one side surface of a right-angle prism with refractive index n.

Here the signs of θ_1 and θ are defined as positive if they are measured clockwise from a surface normal. If θ_1 is larger than the critical angle θ_C, the light is totally reflected at the boundary. According to Fresnel's equations [12], the phase difference ϕ_1 between s- and p-polarizations is given as

$$\phi_1 = \phi_s - \phi_p = 2\tan^{-1}\left\{ \frac{\sqrt{\sin^2\left[45^\circ + \sin^{-1}\left(\frac{\sin\theta}{n}\right)\right] - 1/n^2}}{\tan\left[45^\circ + \sin^{-1}\left(\frac{\sin\theta}{n}\right)\right]\sin\left[45^\circ + \sin^{-1}\left(\frac{\sin\theta}{n}\right)\right]} \right\}, \tag{2}$$

where ϕ_s and ϕ_p are the phases of s- and p-polarization components.

2.2. The Surface Plasmon Resonance (SPR) Technology

For Kretchmann's configuration of a four-layer system (BK7 glass prism-Ti-Au-air) as shown in Fig. 2, the surface plasmons are excited when α equal the resonant angle α_{sp}. From Maxwell's equations, the reflection coefficients of p-polarization and s-polarization can be expressed as [13].

$$r_{1234}^t = \frac{r_{12}^t + r_{234}^t e^{i2k_{z2}d_2}}{1 + r_{12}^t r_{234}^t 4e^{i2k_{z2}d_2}} , \tag{3}$$

$$r_{234}^t = \frac{r_{23}^t + r_{34}^t e^{i2k_{z3}d_3}}{1 + r_{23}^t r_{34}^t e^{i2k_{z3}d_3}} , \tag{4}$$

where $r_{ij}^t = \frac{E_i^t - E_j^t}{E_i^t + E_j^t}$, d_2 and $d3$ are the thicknesses of Ti and Au, respectively, and t = p , s ,

$$E_I^t = \begin{cases} n_I^2 / k_{zI} & t = p \\ k_{zI} & t = s \end{cases} , \quad I = i, j; i, j = 1,2,3. \tag{5}$$

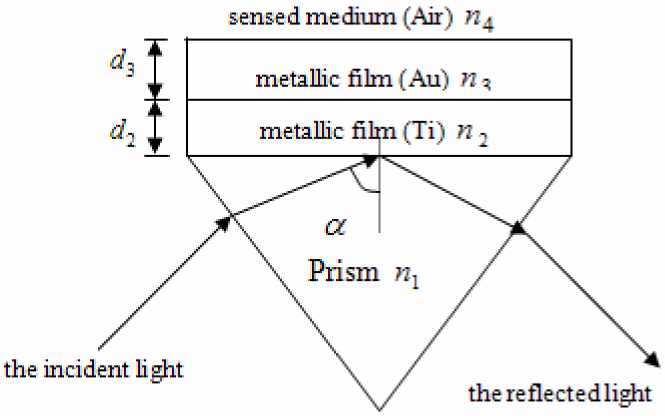

Fig. 2. The four-layer system (BK7 glass prism-Ti-Au-air).

In Eq(5), $k_{zi(j)}$ is the component of the wave vector in medium $i(j)$ in the z direction and is given as

$$k_{zi(j)} = k_0 (n_{i(j)}^2 - n_1^2 \sin^2 \alpha) , \tag{6}$$

where n_1 is the refractive index of prism, n_2 is the refractive index of Ti metal, n_3 is the refractive index of Au metal, n_4 is the refractive index of air and k_0 is the wave vector in vacuum. If the amplitude reflection coefficients r_{1234}^p and r_{1234}^s are written as

$$r_{1234}^p = \left|r_{1234}^p\right| e^{i\phi_p}, r_{1234}^s = \left|r_{1234}^s\right| e^{i\phi_s} , \tag{7}$$

then the phase difference variation ϕ_2 between s and p polarization components is

$$\phi_2 = \phi_s - \phi_p . \tag{8}$$

Besides, the reflectivities of p and s polarization components are $R_p = \left|r_{1234}^p\right|^2$ and $R_s = \left|r_{1234}^s\right|^2$, respectively.

2.3. The Small-angle Sensing Unit

In the paper, a right-angle prism is designed as a small-angle sensing unit as shown in Fig. 3. It deserves to be mentioned that one right-angle side of the right-angle prism isn't coated with any metal film, but the other side that is coated with 2nm Ti-film and 45.5 nm Au-film, i.e. $d_2 = 2nm$ and $d_3 = 45.5nm$. The permittivities of Bk-7 glass prism ($\varepsilon_1 = n_1^2$), Ti film ($\varepsilon_2 = n_2^2$), Au metal ($\varepsilon_3 = n_3^2$), and air ($\varepsilon_4 = n_4^2$) are $\varepsilon_1 = (1.51509)^2$, $\varepsilon_2 = -3.84 + 12.5i$, $\varepsilon_3 = -12 + 1.26i$, and $\varepsilon_4 = (1.0003)^2$, respectively.

If the new type small-angle sensor is mounted on a rotary stage, we can choose the resonant angle α_{sp} as the incident angle α_0, i.e. $\alpha_0 = \alpha_{sp} = 43.81°$. It is because that the phase difference variation is very sensitive around the resonant angle α_{sp} [9]. If the incident angle α is equal to α_{sp}, we can obtain $\theta = \theta_0 \approx 1.803°$, where θ is the incident angle at the hypotenuse surface of the prism. At this moment, we choose $\theta = \theta_0$ as the initial incident angles of the incident beam, i.e., the initial position of the rotary stage. If the rotary stage is rotated a small angle $\Delta\theta$, the incident angle at one side of the right-angle prism (the surface of the side is uncoated metal, denoted as 1st surface) is $\beta = 45° + \sin^{-1}[\frac{\sin(\theta_0 + \Delta\theta)}{n}]$ and the incident angle at the other side (the surface of the side coated two layers of metals, denoted as 2nd surface) is $\alpha = 45° - \sin^{-1}[\frac{\sin(\theta_0 + \Delta\theta)}{n}]$. As the rotary stage is rotated a small angle $\Delta\theta$, it will induce the phase difference variation due to the TIR effect $\Delta\phi_1$ at 1st surface and the phase difference variation $\Delta\phi_2$ due to the SPR effect at 2nd surface. Therefore, the total phase difference variation $\Delta\phi$ is given by:

$$\Delta\phi = \Delta\phi_1 + \Delta\phi_2 \tag{9}$$

3. Experimental Setup and Results

Fig. 4 shows the experimental setup of the new type small-angle sensor based on the TIR and SPR theories for use in heterodyne interferomery. In the experiment, a heterodyne light source with the beat frequency of 2 kHz [14] is incident on a beam-splitter BS and is divided into the reflected and transmitted lights. The transmitted light passes through an analyzer AN_r, which transmission axis is at $45°$ with respect to the x axis, then enters a photo-detector PD_r. The light measured by PD_r is the reference signal. On the other hand, the reflected light enters the hypotenuse of a right-angle prism. At first, the light is incident on a side of the right-angle prism (the surface of the side is uncoated metal). Following the reflected light is incident on the other side that is coated two layers of metals. Finally, the light is detected by a photo-detector PD_t when it passes through the hypotenuse of the right-angle prism and an analyzer AN_t. Here, the signal measured by PD_t is the test signal. The two signals are sent to a lock-in amplifier (SR830; Stanford Research Systems, Sunnyvale, Calif.), and their phase difference ϕ can be achieved. As the rotary stage is rotated a small angle $\Delta\theta$, the phase difference ϕ' can be measured. Thus we can obtain the total phase difference variation $\Delta\phi$ as follows:

$$\Delta\phi = \phi' - \phi \tag{10}$$

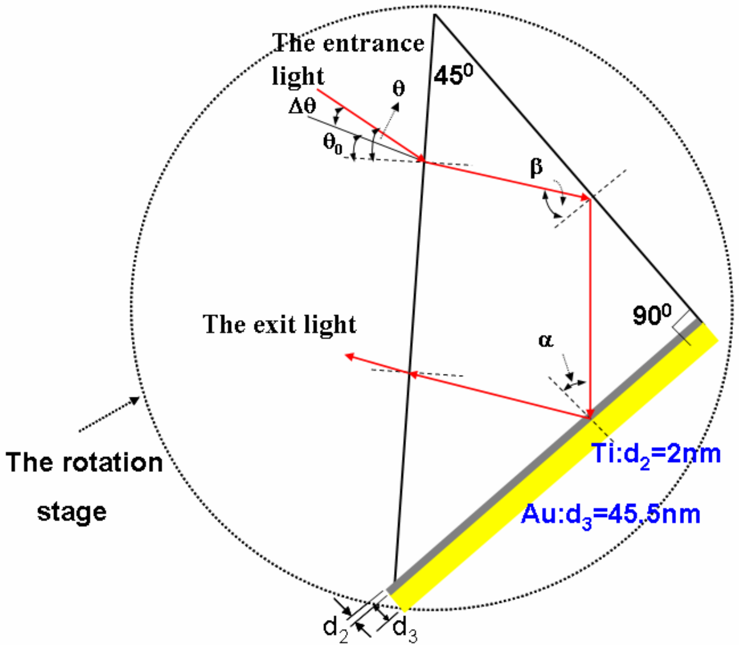

Fig. 3. The new type small-angle sensor.

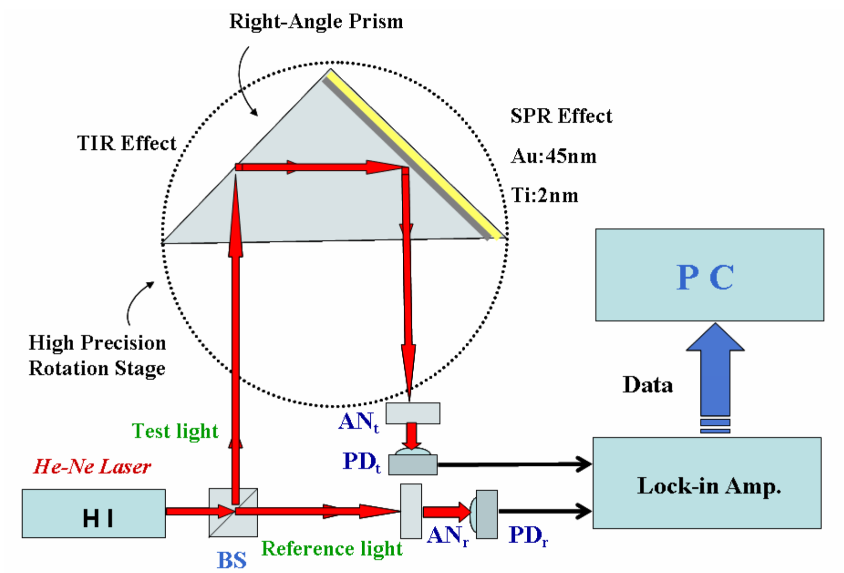

Fig. 4. The experimental setup.

Fig. 5 shows the experimental curve of the phase difference versus the rotating angle $\Delta\theta$ of the rotary stage. It is evident that only evaluating the phase difference variation can perform small-angle measurement. And the experimental results and the theoretical curve are in good agreement.

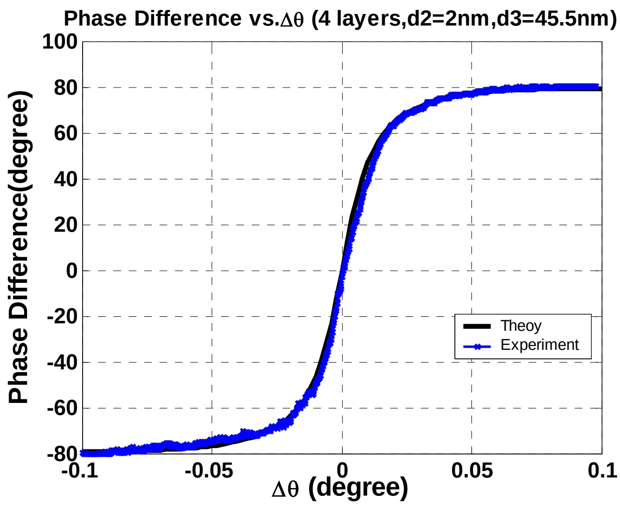

Fig. 5. The experimental and theoretical results.

4. Discussion

At the moment, let us discuss the sensitivity of the new type small-angle sensor, the sensitivity *S* of the system is defined as

$$S = \frac{d\phi}{d\theta}, \tag{11}$$

where $d\phi$ is the variation of the phase difference and $d\theta$ is a small-angle change made by rotating the rotary stage. As shown in Fig. 6, we can also obtain the curves of sensitivity *S* versus θ. The best theoretical sensitivity of the small-angle sensor is 2×10^4 degree/degree over the measurement range $-0.03^\circ \le \Delta\theta \le 0.03^\circ$.

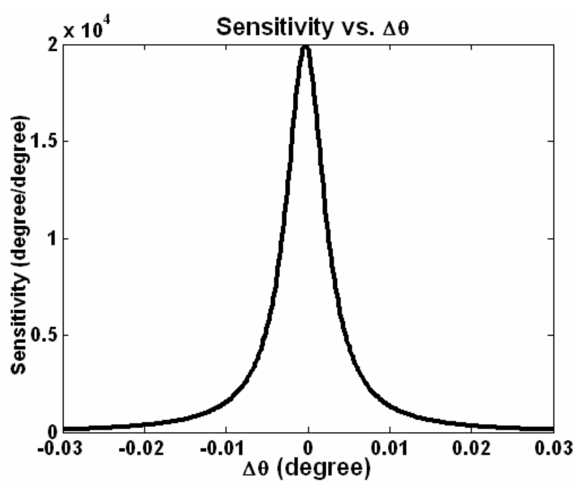

Fig. 6. The sensitivity of the sensor.

Besides, the angular resolution R can be defined as

$$R = \frac{d\theta}{d\phi}\Delta\Phi\,, \tag{12}$$

where $d\phi$ is the variation of the phase difference, $\Delta\Phi = 0.01°$ is the resolution of the lock-in amplifier is equal to $0.01°$ and $d\theta$ is a small-angle change made by the rotation of the rotary stage. As shown in Fig. 7, the resolution of the sensor will reach 1×10^{-7} radian over the measurement range $-0.03^{\circ} \le \Delta\theta \le 0.03^{\circ}$.

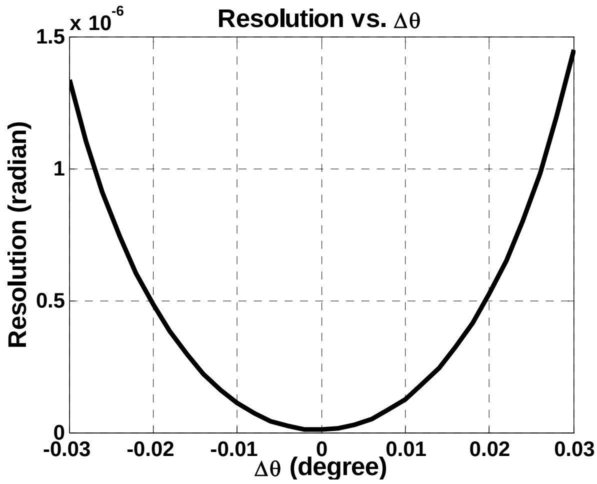

Fig. 7. The resolution of the sensor.

5. Conclusions

A new-type small-angle sensor based on the TIR and SPR theories in heterodyne interferometry is proposed. In the paper, the small-angle measurement can be performed only by evaluating the phase difference variation ($\Delta\phi$**)** between s and p polarizations due to the TIR and SPR effects. The best resolution of the method can reach 1×10^{-7} radian. The new-type small-angle sensor has some merits, e.g., a simple optical setup, easy operation, high measurement accuracy, high resolution, rapid measurement, and high stability etc. And its feasibility is demonstrated.

Acknowledgements

This study was supported in part by the National Science Council in Taiwan under contract No. NSC-94-2622-E-231-025-CC3 & NSC 97-2221-E-231 -001 -MY2.

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- Theory, principles, effects, design, standardization and modeling;
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- Sensor instrumentation;
- Virtual instruments;
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