Sensors & Transducers, Vol. 172, Issue 6, June 2014, pp. 229-234

Sensors & Transducers

Direct Adaptive Fuzzy Variable Structure Control Design without Sliding Mode for Nonlinear Systems with Unmatched Disturbances

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Received: 11 April 20014 /Accepted: 30 May 2014 /Published: 30 June 2014

Abstract: Fuzzy adaptive sliding mode control method is a good way to resolve the control problem when the system mathematic model is unknown. However, reaching condition must be satisfied in this method. In this paper, a kind of fuzzy adaptive variable structure control method is presented in which sliding mode doesn't exist in closed-loop system. Dynamic fuzzy logical system (DFLS) is introduced to form the variable structure controller (VSC) which parameters are self-tuned on-line. Then the stability of the closed-loop system is proved by using Lyapunov stability theories. The switching logic of the VSC is not determined by the sliding mode defined directly, and the reaching condition of the sliding mode is not required anymore. Robustness, chattering free and adaptive character can be obtained. Finally, simulation results on inverted pendulum are proposed to confirm the performance under perturbations. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Sliding mode, Fuzzy, Adaptive, Variable structure control.

1. Introduction

Fuzzy adaptive sliding mode control in recent years is rapid developed and gradually perfected as one of the good techniques for uncertain systems. Its study tends to application research and control of many kinds of industrial plants [1-16]. According to different utilizations of the fuzzy logic system in the sliding mode control it can be divided into direct and indirect adaptive method. For example, Li Jichao presented the application of the indirect adaptive control method to the control system electric arc furnace [1], Liu Shanmei used direct adaptive control method in the control system of PMSM [2] and Dong Xiaomin apply direct adaptive method to the vehicle suspension system with time delay [4]. For additional example, in T. H. Ho's research direct adaptive method is applied to the hydraulic transmission and control system [13], Shahraz utilized indirect adaptive control method in nonlinear chemical process control [16]. The above applications have achieved good control effect in industry. For the details described literature about fuzzy adaptive sliding mode control one may refer to [10].

Variable structure control (VSC) mainly includes two groups: One is VSC with sliding mode in which the control switches according to a pre-designed sliding mode manifold in order to drive the system state onto the manifold. It becomes the most popular kind of VSC due to its robustness to the matched disturbances. However, it needs satisfaction of reaching condition. The other VSC is that without sliding mode, which does not require the reaching condition of sliding mode.

Fuzzy adaptive control method itself contains adaptiveness to external disturbance and uncertainty. Therefore, if the adaptive laws and the control law guarantee the stability of the tracking error given the disturbance and uncertainty, the closed-loop system has enough robustness.

Consequently, this paper uses dynamic fuzzy logic system (DFLS) for nonlinear function approximation, and then variable structure control without sliding mode is designed to guarantee that the tracking error is asymptotically stable by using Lyapunov stability theory. There is not sliding mode in the VSC and so not reaching conditions. The switching of the VSC is via tracking errors and robustness can still be achieved due to the adaptability of DFLS to uncertainties and external disturbance.

2. Problem Formulation

Consider the nonlinear system with the form of

$$\dot{x} = Ax + b[f(x) + b(x)u + d(x,t)], \quad (1)$$

where $x = [x_1, x_2, \cdots, x_n]^T$ is the state vector, $u(t)$ is the control input, $d(x,t)$ represents the unmatched disturbance and unmodelled dynamics, the constant matrices A, b are defined as

$$A = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \ b = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix},$$

and the nonlinear functions $f(x)$, $b(x)$ are unknown but satisfy the following assumption.

Assumption 1: $f(x)$, $b(x)$ are all bounded. Without loss of generality, $\bar{b}(x) > b(x) > 0$ holds with $\bar{b}(x)$ is known upper bound function.

The tracked model is

$$\dot{x}_d = Ax_d + b[-\sum_{i=0}^{n-1} a_i x_d^{i+1} + r(t)], \quad (2)$$

where $x_d = [x_d^1, x_d^2, \cdots, x_d^n]^T$ is the tracked state, $a_0, a_1, \cdots a_{n-1}$ are Hurwitz coefficients, $r(t)$ is the tracked reference signal. Denote error vector

$$e = [e_1, e_2, \cdots, e_n]^T, \quad e_i = x_i - x_d^i \ (i = 1, 2, \cdots, n), \quad (3)$$

then the problem is to design fuzzy adaptive variable structure controller $u(t)$ so that the error vector e asymptotically stable.

3. Dynamic Fuzzy Logic Systems

As we know, fuzzy logic system has strong approximation ability. Its output is attached to a dynamic filters will form dynamic fuzzy logic system (Dynamic Fuzzy Logic Systems, DFLS). Lixin Wang studies a kind of DFLS in which the closed-loop system stability and the error convergence are validated [17]. J. X. Lee uses DAFLS to online approximation and control, thus adaptive controller is constructed based on it that has low-pass filtering function [18]. The similar analysis is also studied by Shitong Wang [19].

Using singleton fuzzifier, product inference rules and average defuzzifier method, a dynamic fuzzy logic system (DFLS) can be described as follows [18],

$$\dot{\hat{g}}(x) = -\xi[\hat{g}(x) - \Theta^T p], \quad (4)$$

where $g(x)$ is the nonlinear scalar function to be approximated, $\Theta = [\theta_1, \theta_2 \cdots \theta_M]^T$ is the adjustable parameter of the DFLS which can be turned on-line by the adaptive laws, $\hat{g}(x)$ is the approximation output. $\xi > 0$ is the real scalar to be designed, $p(x) = [p_1(x), p_2(x), \cdots p_m(x)]^T$ is the fuzzy basis function vector, M is the amount of fuzzy rules, H is the state vector dimension.

Let parameters a_h^m, b_h^m depicted the membership functions of x_h, then the fuzzy basis function is of the form

$$p_m(x) = \frac{\prod_{h=1}^{H} \mu_{mh}(x_h)}{\sum_{m=1}^{M}\prod_{h=1}^{H} \mu_{mh}(x_h)}, \quad \mu_{mh}(x_h) = \exp[-(\frac{x_h - a_h^m}{b_h^m})]. \quad (5)$$

The following lemma [17] is introduced for its later application.

Lemma 1: For smooth nonlinear vector field $g(x): \ R^n \to R$, there is a parameter

$$\Theta^* = \arg\min_{\theta\in\Omega_\theta}\left[\sup_{x\in R^n}\left|g(x) - g^*(x)\right|\right],$$

to guarantee $\forall\varepsilon \in R, \ \varepsilon > 0$, $\left\|g(x) - g^*(x)\right\| < \varepsilon$, where $g^*(x)$ is the approximation output of fuzzy logic system of DFLS (4), namely

$$g^*(x) = \Theta^{*T} p. \quad (6)$$

The fuzzy inference rules of the fuzzy logic system are designed as follows.

if x_1 is a_1^1 and x_2 is a_2^1 and …… x_H is a_H^1 then $\hat{g}(x)$ is θ_1, if x_1 is a_1^2 and x_2 is a_2^2 and …… x_H is a_H^2 then $\hat{g}(x)$ is θ_2,

……

if x_1 is a_1^M and x_2 is a_2^M and …… x_H is a_H^M then $\hat{g}(x)$ is θ_M.

In the following SMC controller design, the DFLS described by (4) and the above rules of the fuzzy logic system are adopted directly for the approximation of unknown dynamics.

4. Controller Design

For the system (1), it is convenient to attribute the unmatched disturbance and uncertainty factors into one nonlinear function approximation for identification with the form of

$$g(x) = \frac{f(x) + [b(x) - \bar{b}(x)]u + d(x,t)}{\bar{b}(x)}. \qquad (7)$$

In this way a DFLS is constructed to approximate $g(x)$ which input vector is $x = [x_1, x_2, \cdots x_n]^T$.

Afterwards, design the VSC controller as

$$u(t) = -\bar{b}^{-1}(x)\sum_{i=0}^{n-1} a_i y_d^i + \bar{b}^{-1}(x) r(t) - \hat{g}(x) - [\varepsilon_h + k\bar{b}^{-1}(x)\|e\|]\operatorname{sgn} e_n, \qquad (8)$$

where the parameter k satisfies

$$k > \max_{i=1,\cdots n-1} \lambda_i + \frac{n+1}{2}, \qquad (9)$$

and the adaptive law is constructed as

$$\begin{aligned} \dot{\Theta} &= \alpha[G^{-1}]^T p\bar{b}(x)\sum_{i=1}^{n} e_i, \\ \dot{\hat{g}}(x) &= -\beta[\hat{g}(x) - \Theta^T P] + \alpha(1 + p^T G^{-1} p)\bar{b}(x)\sum_{i=1}^{n} e_i \end{aligned} \qquad (10)$$

with $G \in R^{M\times M}$, $G = G^T$, $G > 0$, $\alpha > 0$, $\beta > 0$ all free positive matrix or scalars. In the equations (8) and (9), $\lambda_i > 0$ $(i = 1, \cdots, n-1)$ is optional argument to be designed, $\varepsilon_h > |\varepsilon|$ where $\varepsilon = g(x) - g^*(x)$ is the optimal approximation error. According to Lemma 1, the error ε can be small enough, thus ε_h may be determined as arbitrary positive scalar by the situation of the system uncertainties and disturbances.

The fuzzy adaptive VSC controller comprises of DAFLS approximator to $\hat{g}(x)$, Adaptive law, VSC controller and the reference signal. DAFLS is formulated by the equation (10-2) and the adaptive law is given by the equation (10-1). The VSC controller (8) forms a complicated function. In next section the overall system stability will be analyzed.

5. Stability Analysis

The main result is concluded as the following theorem.

Theorem 1: For the nonlinear system (1) and the given tracked reference signal (2), the tracking error of the closed-loop system formulated by the fuzzy adaptive VSC (8-10) is asymptotically stable if the parameters λ_j satisfy

1) $\lambda_j > 0, \quad j = 1, 2, \cdots n-1$,

2) $D = \begin{bmatrix} \lambda_1 & 0 & 0 & \cdots & \pm 1 \\ 0 & \lambda_2 & 0 & \cdots & \pm 1 \\ 0 & 0 & \lambda_3 & \cdots & \pm 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \pm 1 & \pm 1 & \pm 1 & \cdots & 1 \end{bmatrix} > 0$.

Proof: It is easy to see that $D = D^T$, $D > 0$. Choose a Lyapunov function of the tracking error with the form of

$$V(t) = \frac{\alpha}{2} e^T D e + \frac{1}{2}(\hat{g} - \Theta^T p)^2 + \frac{1}{2}\tilde{\Theta}^T G \tilde{\Theta}, \qquad (11)$$

where $\tilde{\Theta} = \Theta^* - \Theta$ is the error between Θ and its corresponding optimal parameter Θ^*. If $D > 0$, obviously $\|V(t)\| \to \infty$ when $\|e\| \to \infty$.

Differentiating the Lyapunov function (11) with respect to time t one can get

$$\begin{aligned} \dot{V}(t) &= \alpha \cdot \dot{e}^T D e + \beta \cdot (\dot{\hat{g}} - \dot{\Theta}^T p)(\hat{g} - \Theta^T p) + \dot{\Theta}^T G \tilde{\Theta} \\ &= \alpha[e_2, e_3 \cdots e_n, \dot{e}_n]^T [\lambda_1 e_1 \pm e_n, \lambda_2 e_2 \pm e_n \cdots \\ &\qquad \lambda_{n-1} e_{n-1} \pm e_n, \ \sum_{i=1}^{n-1} c_i e_i \pm e_n] + \\ &\quad \beta(\dot{\hat{g}} - \dot{\Theta}^T p)(\hat{g} - \Theta^T p) + \dot{\Theta}^T G \tilde{\Theta}. \end{aligned}$$

Denote $S_a = \sum_{i=1}^{n-1} (\lambda_i e_i e_{i+1} \pm e_{i+1} e_n)$, $S_b = \sum_{i=1}^{n} |e_i|$. Then on account of that

$$\begin{aligned} &\sum_{i=1}^{n-1} \lambda_i (e_i - e_{i+1})^2 = \sum_{i=1}^{n-1} \lambda_i (e_i^2 + e_{i+1}^2) - 2\sum_{i=1}^{n-1} \lambda_i e_i e_{i+1} \ge 0 \\ &\Rightarrow \frac{1}{2} \max_{i=1,\cdots n-1} \lambda_i \left[\sum_{i=1}^{n-1} (e_i^2 + e_{i+1}^2)\right] \ge \sum_{i=1}^{n-1} \lambda_i e_i e_{i+1} \\ &\Rightarrow \max_{i=1,\cdots n-1} \lambda_i \ \|e\|^2 \ge \sum_{i=1}^{n-1} \lambda_i e_i e_{i+1}, \end{aligned}$$

and also $\frac{1}{2}\left[\|e\|^2 + ne_n^2\right] \ge \sum_{i=1}^{n-1} e_n e_{i+1}$, $\left[\sum_{i=1}^{n} |e_i|\right]^2 \ge \sum_{i=1}^{n} e_i^2$, the following inequality can be given,

$$\|S_a\| \le \left\{ \max_{i=1,\cdots n-1} \lambda_i + \frac{n+1}{2} \right\} \|e\|^2, \|S_b\| \ge \|e\|, \quad (12)$$

Substitute (10) into (12), one can get

$$\begin{aligned}\dot{V}(t) &= \alpha \cdot S_a + \alpha(\sum_{i=1}^{n-1}|e_i| + e_n)\dot{e}_n - \beta[\hat{g}(x) - \Theta^T p][\hat{g}(x) \\ &- \Theta^T p] + \alpha(1 + p^T G^{-1} p)\bar{b}(x)[\hat{g}(x) - \\ &\Theta^T p]\sum_{i=1}^{n} e_i - \alpha \cdot p^T G^{-1} p\bar{b}(x)[\hat{g}(x) - \\ &\Theta^T p]\sum_{i=1}^{n} e_i - \alpha \cdot \bar{b}(x)(\Theta^{*T} p - \Theta^T p)\sum_{i=1}^{n} e_i \\ &= \alpha \cdot S_a + \alpha \cdot S_b \operatorname{sgn}(e_n)\dot{e}_n - \\ &\alpha \cdot \bar{b}(x)[g^*(x) - \hat{g}(x)]\sum_{i=1}^{n} e_i.\end{aligned}$$

Afterwards substitute the controller (8), one get

$$\begin{aligned}\dot{V}(t) &\le \alpha \cdot S_a + \alpha \cdot S_b \operatorname{sgn}(e_n) \cdot \dot{e}_n - \\ &\alpha \cdot \bar{b}(x)[g^*(x) - \hat{g}(x)]\sum_{i=1}^{n} e_i \\ &\le \alpha \cdot S_a + \alpha \cdot S_b \cdot \bar{b}(x)[g(x) - \hat{g}(x) - \\ &(\varepsilon + k\bar{b}^{-1}(x)\|e\|)\operatorname{sgn} e_n] - \\ &\alpha \cdot \bar{b}(x)[g^*(x) - \hat{g}(x)]\sum_{i=1}^{n} e_i \\ &\le \alpha \cdot (S_a - k\|e\|^2),\end{aligned}$$

namely

$$\dot{V}(t) \le \alpha\left\{ \max_{i=1,\cdots n-1} \lambda_i + \frac{n+1}{2} \max_{i=1,\cdots n-2} c_i \right\} \|e\|^2 - \alpha k\left\{ \min_{i=1,\cdots,n} c_i \right\}\|e\|^2. \quad (13)$$

Obviously, the derivative of Lyapunov function (11) is negative.

$\dot{V}(t) \equiv 0$ if and only if $\|e\| = 0$. That is, $\forall e \in R^n$, $e \ne \bar{0}$, $\dot{V}(t) \ne 0$. Hence the overall system (1) is asymptotically stable by Lyapunov stability theories.

The controller (8-10) is a kind of adaptive variable structure controller, in which dynamic fuzzy logic system is used as on-line approximation by (7). The reaching condition of sliding mode is no longer required to be satisfied by controller design. Even in the presence of strong interference, the uncertainties and unmodeled dynamics, the proof of Theorem 1 based on the controller is still valid, so the system also has strong robustness.

6. Simulation

The inverted pendulum is a typical nonlinear, coupled system with disturbances, which mathematical model is of the form

$$\dot{x}_1 = x_2, \ \dot{x}_2 = f(x) + b(x)u, \quad (14)$$

where the system state vector $x = [\theta \quad \dot{\theta}]^T$ with θ its pendulumangle excursion and u control voltage of the cart.

It is estimated the upper bound function of nonlinear control gain $b(x)$ as $\bar{b}(x) = 280$, the controller is designed as (8), the other parameters $\alpha = \beta = 10$, $\varepsilon = 0.01$, $k = 10$, $G = I$ are selected. The tracked model is set by

$$\dot{x}_d = \begin{bmatrix} 0 & 1 \\ -200 & -30 \end{bmatrix} x_d + \begin{bmatrix} 0 \\ 200 \end{bmatrix} r(t),$$

where $x_d = [x_{d1} \quad x_{d2}]^T$ is the state of the tracked model, $r(t)$ is the reference signal. Let $r(t) = 0$, the control volt curve, angel excursion curve and its derivative curve of the inverted pendulum are shown in Fig. 1.

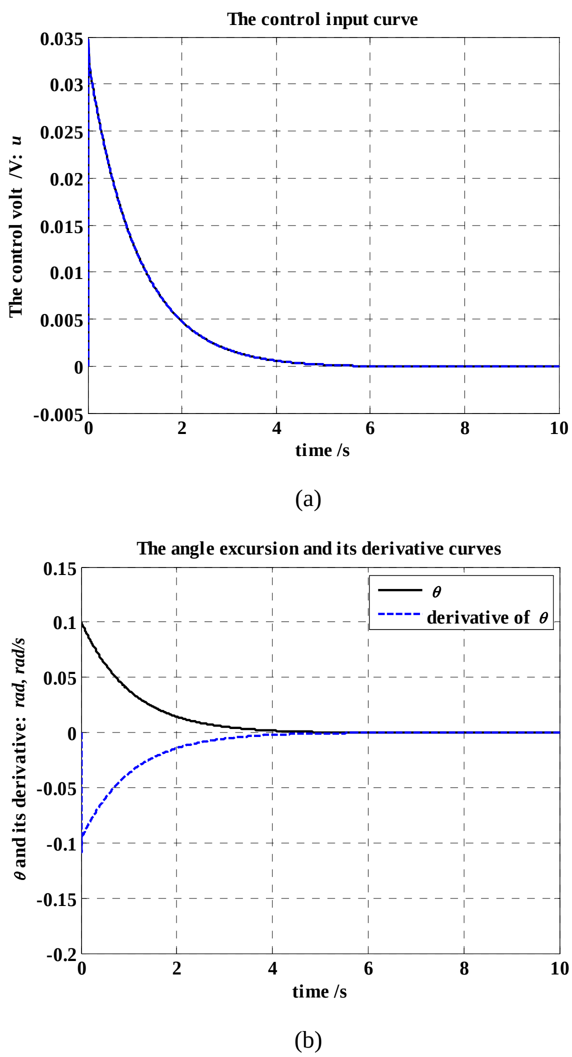
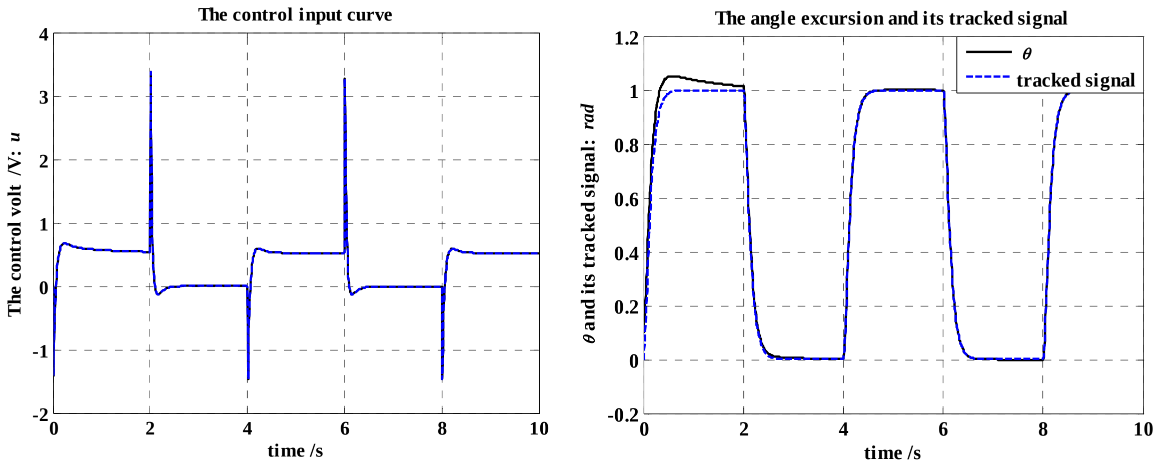


Fig. 1. The control input curve and the angle excursion curves.

Consequently, $r(t)$ is set as square wave with 4 second period. The corresponding control volt curve, angle excursion curve and the tracked state x_{d1} are shown in Fig. 2. One can conclude that the tracking error converges quickly and the tracking effective is good.

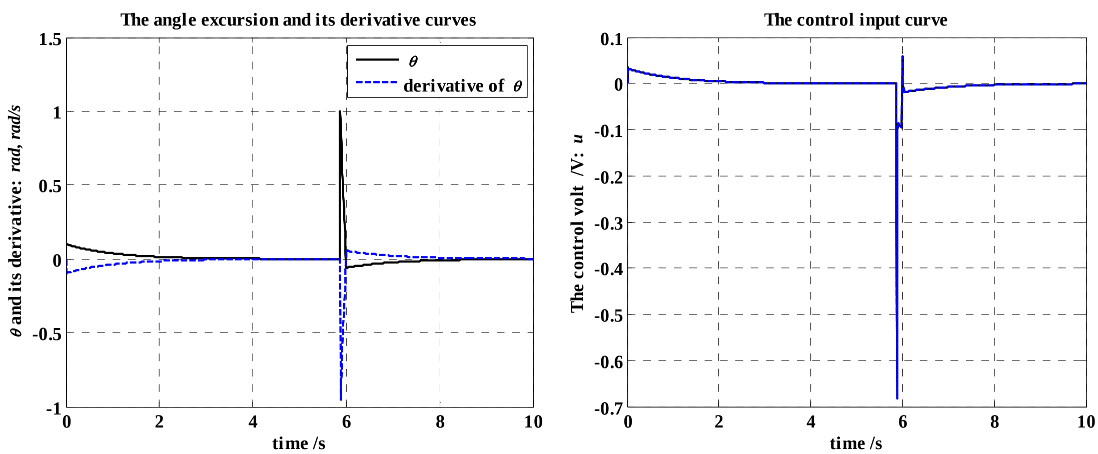
Finally, a pulse disturbance with unit magnitude value acts on the pendulum. Under the proposed controller in this paper, the control volt, the pendulum angle excursion curve and its angle speed signal are shown in Fig. 3. From Fig. 1-3 it is shown that the proposed controller can guarantee the stability of the tracking error was verified by the above simulation results.

Additionally, from these results the robustness and adaption ability to disturbance were also tested. As a result the proposed fuzzy adaptive VSC control without sliding mode is feasible.



(a) (b)

Fig. 2. The control volt and tracking signals when the reference signal is square wave.



(a) (b)

Fig. 3. The control volt and angle excursion curves when disturbed.

7. Conclusions

Through numerical simulation, it is shown that DFLS has not only strong robustness, but also a certain filtering function to unmodeled dynamics and disturbances, which thus greatly weakens the chattering of sliding mode control. The approximation capability of DFLS makes the system robust performance better and disturbance compensation ability strengthened. These guarantee the tracking performances. Therefore, in the absence of sliding mode the reaching condition does not need to be required while in the overall closed-loop system fuzzy adaptive VSC is implemented.

Acknowledgements

This work is supported by the National Natural Science Foundation of China (61304024), the Fundamental Research Funds for the Central Universities (3142013055), the Natural Science

Foundation of Hebei Province (F2013508110) and the Science and Technology plan projects of Hebei Provincial Education Department (Z2012089).

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