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Robust H∞ Control Design of NCS

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Abstract: Some unknown elements always exist in the analysis process for the control systems, such as unmodeled dynamics, parametric uncertainties, change of the operating environment, model reduction and linearization approximations, etc., or external disturbance. So it is significative to study the disturbance process or the uncertain systems. The emergence and the development of the robust control theory were just in such environment, and it is becoming an important research field of the control theory and its practice applications. To address instability and jump degeneration of linear network control system with time-delay and packet dropout, and considering uncertainties of network control systems model and external disturbance, the paper presents network control systems delay-dependent stochastic stability and Linear matrix inequalities (LMI) conditions by introducing Lyapunov-Krasovskii functional and relaxation matrix variable. The paper proposes the network control system delay-dependent stochastic stability and disturbance attenuation LMI conditions and robust H∞ control system perturbation attenuation performance analysis. By using simple mathematical derivation, solving two Riccati equations, we can be sub-optimal control solution. Finally, the simulation example can be found robust stability, simple and effective. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Leave linear networked control system, H∞ control, Riccati equation, Disturbance attenuation, Riccati equation method.

1. Introduction

The time-delay of the NCS is a hot topic at present. Considering the feature of the network, the delay may be constant, random, bounded or unbounded. Literature [1-3] reduces the random delay of the controlled system's forward path and feedback channel through the algorithm of the predictive control. Literature [4, 5] uses Lyapunov-Krasovskii function and LMI to get the stability condition of the time-delay. Literature [6] divides the delay into definite and continuous change parts, aiming at the uncertain system's robust stability feature with uncertain input latency, a sufficient stability condition of the system is offered through Lyapunov function. Literature [7-12] considers the Markov random delay of the system, gives a more general continuous stochastic model of the NCS. Based on this, the stabilization of the system's robust H∞ is researched through Lyapunov function, and a sufficient condition of the stochastic stability is offered [13].

In reference to the above references, the NCS with random delay and packet loss is researched in this paper. Through the algorithm of the random Lyapunov-Krasovskii functional and relaxation matrix variable, the LMI condition of the networked delay based on stochastic stability and disturbance attenuation is given [14]. Meanwhile, considering the uncertainties and external disturbance, the improved

2-Riccati equation is adopted to design the H∞ controller.

2. Issue Description

Considering a state equation of a NCS shown as:

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_1 x + D_{11} w + D_{12} u, \\ y &= C_2 + D_{21} w + D_{22} u \end{aligned} \tag{1}$$

where $u(t) \in R^r$ is the input variable, y is the observed quantity, w is the external input variable, and z is the controlled output variable. In order to make Eq. 1 simpler, the coefficient matrix of the controlled plant is marked as T.

$$T = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix}, \tag{2}$$

The following assumptions are made:

The sampling period is set as T. And the sensor adopts the time-driven approach, the controller and actuator adopt the event-driven approach [15].

The delay from the sensor to the controller is bounded random delay, and the status information $x_k - t_{rk}(r_k, k \geq 0)$ is a limited Markov chain, S={1,2,...,s}, $\Pi = [\pi_{ij}]$ is transition probability matrix, and

$$P[r_{k+1} = j, r_k = i] = \pi_{ij}$$
$$\forall i, j \in S$$

where

$$\pi_{ij} \geq 0,\ i \neq j;\ \sum_{j=1, j\neq i}^{s} \pi_{ij} = 1 - \pi_{ii}.$$

3. Main Results

3.1. Definition

Definition 1 [5] is for all $w_k \in l^2, w \neq 0$, wherein

$$l^2 = \left\{ a_k, k \geq 0 \mid \sum_{k=0}^{\infty} \left\| a_k \right\|^2 < \infty \right\},$$

the jump system is random and stable, and the controlled output $\{z_k\}$ shall satisfy

$$\|z\|_2 < \gamma \|w\|_2$$

where

$$\|z\|_2 = [\sum_{k=0}^{\infty} z_k^T z_k]^{1/2}, \gamma > 0$$

is the disturbance attenuation performance level, then the jump system is stochastic stable with γ attenuation feature.

Definition 2 For

$$w_k \in l^2, w \neq 0,$$

where

$$l^2 = \left\{ a_k, k \geq 0 \left| \sum_{k=0}^{\infty} \|a_k\|^2 < \infty \right. \right\},$$

the jump system is stochastic stable, and the controlled output $\{z_k\}$ satisfy

$$\|z\|_2 < \gamma \|w\|_2,$$

where

$$\|z\|_2 = \left[\sum_{k=0}^{\infty} z_k^T z_k \right]^{1/2}, \gamma > 0$$

is the disturbance attenuation level, then the jump system is random and stable and featured with γ attenuation feature[16].

3.2. NCS Robust H∞ Disturbance Attenuation Analysis and Controller Design

3.2.1. H∞ Disturbance Attenuation Analysis

Theorem 1[6]: Concerning the random delay

$$\tau_m \leq \tau_i \leq \tau_M, i \in S,$$

if the matrix

$$P_i > 0, Q_1 > 0, Q_2 > 0, T_i = \begin{bmatrix} T_{1i} & T_{2i} & T_{3i} \end{bmatrix}$$

exists, H_i, N_i shall meet the following LMI:

$$\psi_i = \begin{bmatrix} \Theta_i + \gamma^{-1} C_i^T C_i & * \\ \gamma^{-1} D_2^T C_i + B_2^T T_i & \gamma^{-1} D_2^T D_{2i} - \gamma I \end{bmatrix} < 0, \tag{3}$$

In the sum form

$$C_i = \begin{bmatrix} C_1 & D_{1i} & 0 \end{bmatrix},$$

concerning the linear jump system and given disturbance attenuation level γ, the system is featured with γ disturbance attenuation feature, then for all

$$w_k \in l^2, w \neq 0 : \|z\|_2 < \gamma \|w\|_2 .$$

Proof : select the following Lyapunov functional:

$$V(x_k,r_k)=V_1(x_k,r_k)+V_2(x_k,r_k)+V_3(x_k,r_k)+V_4(x_k,r_k).$$

where

$$V_1(x_k,r_k)=x_k^T P_{rk} x_k$$
$$V_2(x_k,r_k)=\sum_{l=k-\tau_i}^{k-1} x_l^T Q_1 x_l$$
$$V_3(x_k,r_k)=\sum_{\theta=-\tau_i}^{-1}\sum_{l=k+\theta}^{k-1}\delta_l^T Q_2 x_l$$
$$V_4(x_k,r_k)=(1-\pi_m)\sum_{\theta=-\tau_M+1}^{-\tau_m+1}\sum_{l=k+\theta-1}^{k-1}\left\{x_l^T Q_1 x_l+(l+1-k-\theta)\delta_l^T Q_2 \delta_L\right\}$$

then

$$2\varepsilon_k^T T_i^T\left\{A_{si}x_k-\delta_k-B_{1i}\sum_{l=k-\tau_i}^{k-1}\delta_l+B_2 w_k\right\}=0$$

where

$$\varepsilon_k=\begin{bmatrix}x_k^T & x_{k-\tau_i}^T & \delta_k^T\end{bmatrix}^T$$
$$x_{k-\tau_i}=x_k-\sum_{l=k-\tau_i}^{k-1}\delta_l$$
$$\delta_l=x_{k+1}-x_l, A_{si}=A_1+B_{1i}-I$$

where T_i is a matrix with certain dimensions.

From Eq.4, we can obtain

$$E[V(x_{k+1},\tau_{k+1})\,|\,F_k]-v(x_k,\tau_k)\le\varepsilon_k^T\Theta_i\varepsilon_k+2\varepsilon_k^T T_i^T B_2 w_k.$$

Assuming the zero initial condition, that is,

$$x_k = 0, k = -\tau_M, -\tau_M + 1, \ldots, -1,$$

and the definition

$$J_N=\gamma^{-1}\|z\|_2^2-\gamma\|w\|_2^2=E\left[\sum_{j=0}^{N-1}(\gamma^{-1}z_j^T z_j-\gamma w_j^T w_j)\right].$$

For

$$V_0(\phi,\tau_0)=0,$$

there are

$$E[V_N(x_N,r_N)]=E\left[\sum_{j=0}^{N-1}[V_{k+1}(x_{k+1},r_{k+1})-V_k(x_k,r_k)]\right]>0$$

therefore we can get

$$J_N=E\left[\sum_{j=0}^{N-1}[V_{k+1}(x_{k+1},r_{k+1})-V_k(x_k,r_k)]+\gamma^{-1}z_k^T z_k-\gamma w_k^T w_k\right]$$
$$-E[V_N(x_N,\gamma_N)]$$
$$\le E\left[\sum_{j=0}^{N-1}[V_{k+1}(x_{k+1},r_{k+1})-V_k(x_k,r_k)]+\gamma^{-1}z_k^T z_k-\gamma w_k^T w_k\right]$$
$$=\sum_{k=0}^{N-1}\left[\eta_k^T\Psi_i\eta_k\right]$$

where

$$\eta_k=\begin{bmatrix}\xi_k^T & w_k^T\end{bmatrix}^T.$$

Then,

$$\sum_{k=0}^{N-1}\left[\eta_k^T\Psi_i\eta_k\right]<0,$$

and for any nonzero

$$w_k\in l^2(0,\infty), z\in l^2(0,\infty),$$

there is

$$\|z\|_2<\gamma\|w\|_2.$$

According to the definition 2, the theorem 1 can be proved.

3.2.2. Design of Status Feedback Controller

H∞ design method can be roughly divided into two categories. The first category is the analytical solution based on function approximation theory; the second category is seeking the second-best solution through the analytical method, then relying on the repeated iteration to close to the optimal solution. The paper adopts the improved 2-Riccati equation belonging to the second category, and here mainly aims at the formal standard H∞ control issue. The method only needs to solve two algebraic Riccati equations, and it is a design method for the controller with a more application [17-21].

Before designing the H∞ controller, the controlled object in the issue description shall be made the following assumptions: (A, B_2) can be calm, and (A, C_2) can be detected;

1) $D_{12} = \begin{bmatrix} 0 \\ I_p \end{bmatrix}$, and $D_{21} = \begin{bmatrix} 0 & I_q \end{bmatrix}$,

$$D_{11} = \begin{bmatrix} D_{1111} & D_{1112} \\ D_{1121} & D_{1122} \end{bmatrix}, D_{1111} \in R^{(m-p)\times(r-q)}, D_{1122} \in R^{p\times q}$$

2)
$$rank\begin{bmatrix} -jwI + A & B_2 \\ C_1 & D_{12} \end{bmatrix} = n + p$$
$$rank = \begin{bmatrix} -jwI + A & B_1 \\ C_2 & D_{21} \end{bmatrix} = n + q, \forall w \in R$$

3) $D_{22} = 0$

If the stable solution X_∞ and Y_∞ for the following algebraic Riccati equation exist, namely

$$X_\infty = Ric\begin{bmatrix} A - BR^{-1}D_{1*}^T C_1 & -BR^{-1}B^T \\ -C_1^T C_1 + C_1^T D_{1*} R^{-1} D_{1*}^T C_1 & -A^T + C_1^T D_{1*} R^{-1} B^T \end{bmatrix}$$

$$Y_\infty = Ric\begin{bmatrix} A^T - C^T \tilde{R}^{-1} D_{*1} B_1^T & -C^T \tilde{R}^{-1} C \\ -B_1 B_1^T + B_1 D_{*1}^T \tilde{R}^{-1} D_{*1} B_1^T & -A + B_1 D_{*1}^T \tilde{R}^{-1} C \end{bmatrix}$$

The matrix F and H are defined as

$$F = \begin{bmatrix} F_{11} \\ F_{12} \\ F_2 \end{bmatrix} = -R^{-1}\left[D_{1*}^T C_1 + B^T X_\infty\right]$$

$$H = \begin{bmatrix} H_{11} & H_{12} & H_2 \end{bmatrix} = -\left[B_1 D_{*1}^T + Y_\infty C^T\right]\tilde{R}^{-1}$$

If the above controlled plant in theorem 2[7] meets the above assumptions, then a closed-loop stable controller exists and the sufficient and necessary condition meeting for $\|\phi(s)\|_\infty < \gamma$ is satisfying the following four conditions[22-25]:

1) $\max(\bar{\sigma}\left[D_{1111}, D_{1112}\right], \bar{\sigma}\left[D_{1111}^T, D_{1121}^T\right] < \gamma$;

2) $X_\infty \geq 0$;

3) $Y_\infty \geq 0$;

4) $\rho(X_\infty, Y_\infty) < \gamma^2$.

where $\bar{\sigma}(\cdot)$ represents the maximum singular value of the matrix, and $\rho(\cdot)$ represents the maximum eigenvalue of the matrix.

When the above conditions are satisfied, the H_∞ controller is

$$K = F_l(K_a, S)$$

where $S \in BH_\infty^{p\times q}$ and K_a is expressed as

$$K_a = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} = \begin{bmatrix} \hat{A} & \hat{B}_1 & \hat{B}_2 \\ \hat{C}_1 & \hat{D}_{11} & \hat{D}_{12} \\ \hat{C}_2 & \hat{D}_{21} & 0 \end{bmatrix},$$

where

$$\hat{D}_{11} = -D_{1121}D_{1111}^T(\gamma^2 I - D_{1111}D_{1111}^T)^{-1}D_{1112} - D_{1122}$$
$$\hat{B}_2 = (B_2 + H_{12})\hat{D}_{12}$$
$$\hat{C}_2 = -\hat{D}_{21}(C_2 + F_{12})Z$$
$$\hat{B}_1 = -H_2 + \hat{B}_2 \hat{D}_{12}^{-1} \hat{D}_{11}$$
$$\hat{C}_1 = F_2 Z + \hat{D}_{11} \hat{D}_{21}^{-1} \hat{C}_2$$
$$\hat{A} = A + HC + \hat{B}_2 \hat{D}_{12}^{-1} \hat{C}_1$$
$$Z = (I - \gamma^2 Y_\infty X_\infty)^{-1}$$

where

$$\hat{D}_{12} \in R^{p\times q}, \hat{D}_{21} \in R^{q\times q}$$
$$\hat{D}_{12}\hat{D}_{12}^T = I - D_{1121}(\gamma^2 I - D_{1111}^T D_{1111})^{-1} D_{1121}^T$$
$$\hat{D}_{21}^T \hat{D}_{21} = I - D_{1112}^T(\gamma^2 I - D_{1111}D_{1111}^T)^{-1} D_{1112}$$

Therefore, to meet the assumptions of Theorem 2 standard control problem, as long as solving two algebraic Riccati equation, you can find all of the sub-controller optimal control problems [26-31].

4. Simulation Example

An uncertain network control system is established here, and the realization of the controlled object's state space is:

$$A=\begin{bmatrix}-22 & 0 & 0\\ -164 & -0.43 & 0\\ 0 & -1 & -0.001\end{bmatrix}, B_1=\begin{bmatrix}0\\0\\1\end{bmatrix}, B_2=\begin{bmatrix}4.2\\0\\0\end{bmatrix}$$

$$C_1=\begin{bmatrix}0 & -0.1 & 3.5\\ 0 & 0 & 0\\ -0.3 & -0.01 & 0\end{bmatrix}, C_2=\begin{bmatrix}0 & -1 & 0\end{bmatrix}$$

$$D_{11}=\begin{bmatrix}0.1\\0\\0\end{bmatrix}, D_{12}=\begin{bmatrix}0\\0.01\\0\end{bmatrix}, D_{21}=[1], D_{22}=[0]$$

Through solving Riccati equations by MATLAB robust control toolbox, the controller can be obtained

$$K=\frac{356(s+20)(s+0.43)}{s(s^2+205s+11906)}, \tag{6}$$

By the MTLAB simulation, the controller's frequency feature is shown as Fig. 2:

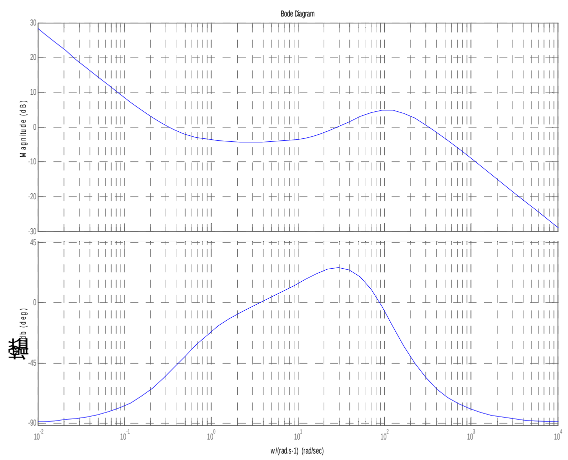

Fig. 2. The frequency feature of the controller.

It's seen that the sink low gain appears at the point w=3.3 rad/s from Fig. 2. The gain can effectively suppress the influence of the interference.

4. Conclusion

H_∞ robust stability is designed of the NCS with random delay and packet loss through the improved Riccati equations. It only uses a simple mathematical derivation to gain the H∞ sub-optimal control solution. From the simulation example, it's seen that it has a great robust stability and it's simple and effective which has the certain practical value.

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References

[1]. Richard M. Karp and Vijaya Ramachandran, A survey of parallel algorithms for shared-memory machines, Technical Report CSD-88-408, *Department of Electrical Engineering and Computer Science, University of California*, Berkeley, 1988.

[2]. N. C. Beaulieu, An infinite series for the computation of the complementary probability distribution function of a sum of independent random variables and its application to the sum of Rayleigh random variables, *IEEE Transactions on Communications*, Vol. 38, Issue 9, 1990, pp. 1463-1474.

[3]. N. C. Lim, H. C. Freake, C. Brückner, Illuminating zinc in biological systems, *European Journal on Chemistry-A*, Vol. 11, Issue 1, 2005, pp. 38-49.

[4]. T. S. Rappaport, Wireless communications: principles and practice, *Prentice Hall PTR*, New Jersey, 1996.

[5]. R. A. Brooks, A robust layered control system for a mobile robot, *IEEE Journal of Robotics and Automation*, Vol. 2, Issue 1, 1986, pp. 14-23.

[6]. Dong Yue, Qing-Long Hua, and Chen Peng, State feedback controller design of networked control systems, *IEEE Transactions on Circuits and Systems-II: Express Briefs*, Vol. 51, Issue 11, 2004, pp. 640-644.

[7]. Z. Pawlak, Rough sets, *International Journal of Computer and Information Sciences*, Vol. 11, 1982, pp. 34l-356.

[8]. G. C. Walsh, H. Ye, L. G. Bushnell, Stability analysis of networked control systems, *IEEE Transactions on Control Systems Technology*, Vol. 10, Issue 3, 2002, pp. 438-446.

[9]. D. Peaucelle, D. Arzelier, O. Bachelier, et al, A new robust D-stability condition for real convex polytopic uncertainty, *Systems & Control Letters*, Vol. 40, Issue 1, 2000, pp. 21-30.

[10]. P. Kundur, J. Paserba, V. Ajjarapu, et al, Definition and classification of power system stability IEEE/CIGRE joint task force on stability terms and definitions, *IEEE Transactions on Power Systems*, Vol. 19, Issue 3, 2004, pp. 1387-1401.

[11]. R. A. DeCarlo, S. H. Zak, G. P. Matthews, Variable structure control of nonlinear multivariable systems: a tutorial, *Proceedings of the IEEE*, Vol. 76, Issue 3, 1988, pp. 212-232.

[12]. P. P. Khargonekar, I. R. Petersen, K. Zhou, Robust stabilization of uncertain linear systems: quadratic stabilizability and H∞ control theory, *IEEE Transactions on Automatic Control*, Vol. 35, Issue 3, 1990, pp. 356-361.

[13]. D. Huang, K. N. Sing, State feedback control of uncertain networked control systems with random

time delays, *IEEE Transactions on Automatic Control,* Vol. 53, Issue 3, 2008, pp. 829-834.
[14]. Wang Dejin, H2 and $H\infty$ optimal control theory, *Harbin Institute of Technology Press*, Harbin, 2011.
[15]. P. Apkarian, H. D. Tuan, J. Bernussou, Continuous-time analysis, eigenstructure assignment, and H2 synthesis with enhanced linear matrix inequalities (LMI) characterizations, *IEEE Transactions on Automatic Control*, Vol. 46, Issue 12, 2001, pp. 1941-1946.
[16]. Qiu Zhanzhi, Zhang Qingling, Optimal $H\infty$ control of one class of uncertain retardation network control system, *Information and Control*, Vol. 35, Issue 1, 2007, pp. 64-72.
[17]. Shen Tielong, $H\infty$ control theory and application, *Tsinghua University Press*, Beijing, 1996.
[18]. H. M. Jeong, S. Y. Lee, W. H. Shin, et al, Silicon@ porous nitrogen-doped carbon spheres through a bottom-up approach are highly robust lithium-ion battery anodes, *Research Advances*, Vol. 2, Issue 10, 2012, pp. 4311-4317.
[19]. H. Zhang, Y. Shi, A. S. Mehr, Robust $H\infty$ PID control for multivariable networked control systems with disturbance/noise attenuation, *International Journal of Robust and Nonlinear Control*, Vol. 22, Issue 2, 2012, pp. 183-204.
[20]. H. Zhang, Y. Shi, A. S. Mehr, Robust static output feedback control and remote PID design for networked motor systems, *IEEE Transactions on Industrial Electronics*, Vol. 58, Issue 12, 2011, pp. 5396-5405.
[21]. T. Otto, M. Müller, P. Mundra, et al, Colloidal nanocrystals embedded in macrocrystals: robustness, photostability, and color purity, *Nano Letters*, Vol. 12, Issue 10, 2012, pp. 5348-5354.
[22]. J. Wang, W. Li, Z. Li, Fuzzy observer-based robust $H\infty$ guaranteed cost fault-tolerant design for nonlinear NCS, in *Proceedings of Chinese Intelligent Automation Conference,* 2013, pp. 393-402.
[23]. W. Li, H. C. Cao, Robust H sub (proportional to)-guaranteed cost fault-tolerant control for uncertain NCS based on dynamic output feedback, *Computer Science*, Vol. 39, Issue 7, 2012, pp. 305-312.
[24]. F. Yang, H. Zhang, Z. Liu, et al, Delay-dependent resilient-robust stabilisation of uncertain networked control systems with variable sampling intervals, *International Journal of Systems Science*, Vol. 45, Issue 3, 2014, pp. 497-508.
[25]. Z. Han, F. Qiu, R. Eisenberg, et al, Robust photogeneration of H2 in water using semiconductor nanocrystals and a nickel catalyst, *Science*, Vol. 338, Issue 6112, 2012, pp. 1321-1324.
[26]. H. Zhang, Y. Shi, A. Saadat Mehr, Robust energy-to-peak filtering for networked systems with time-varying delays and randomly missing data, *IET Control Theory & Applications*, Vol. 4, Issue 12, 2010, pp. 2921-2936.
[27]. Y. Liu, M. Gibbs, C. L. Perkins, et al, Robust, functional nanocrystal solids by infilling with atomic layer deposition, *Nano Letters*, Vol. 11, Issue 12, 2011, pp. 5349-5355.
[28]. Y. Shi, B. Yu, Robust mixed H2/$H\infty$ control of networked control systems with random time delays in both forward and backward communication links, *Automatica*, Vol. 47, Issue 4, 2011, pp. 754-760.
[29]. M. L. Snyder, B. E. Klein, R. Klein, et al, Abstract P323: arterial stiffness and retinal vessel diameters: the atherosclerosis risk in communities study-neurocognitive study (ARIC-NCS), *Circulation*, Vol. 129, Issue Suppl. 1, 2014, pp. AP323-AP323.
[30]. F. Sambataro, V. S. Mattay, K. Thurin, et al, Altered cerebral response during cognitive control: a potential indicator of genetic liability for schizophrenia, *Neuropsychopharmacology*, Vol. 38, Issue 5, 2013, pp. 846-853.
[31]. I. S. Buick, E. S. Grew, T. Armbruster, et al, Boromullite, Al9BSi2O19, a new mineral from granulite-facies metapelites, Mount Stafford, central Australia: a natural analogue of a synthetic "boron-mullite", *European Journal of Mineralogy*, Vol. 20, Issue 5, 2008, pp. 935-950.