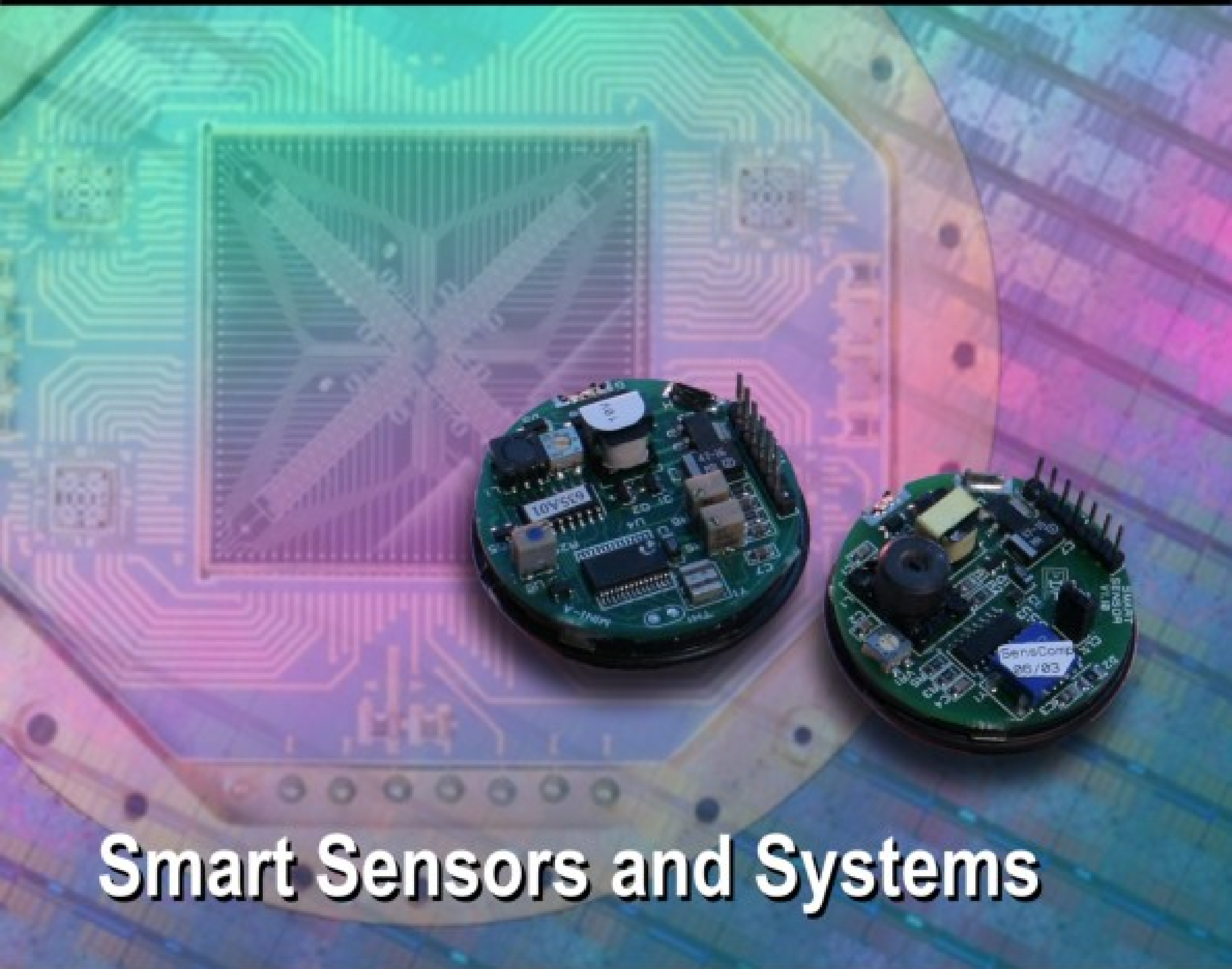
ISSN 1726-5479

SENSORS & TRANSDUCERS

3/09 vol. 102



Smart Sensors and Systems

International Frequency Sensor Association Publishing

IFSA

Sensors & Transducers

Volume 102, Issue 3
March 2009

www.sensorsportal.com

ISSN 1726-5479

Sensors & Transducers Journal (ISSN 1726-5479) is a peer review international journal published monthly online by International Frequency Sensor Association (IFSA).

Sensors & Transducers Journal

Contents

Volume 102
Issue 3
March 2009

www.sensorsportal.com

ISSN 1726-5479

Research Articles

Authors are encouraged to submit article in MS Word (doc) and Acrobat (pdf) formats by e-mail: editor@sensorsportal.com
Please visit journal's webpage with preparation instructions: http://www.sensorsportal.com/HTML/DIGEST/Submition.htm

International Frequency Sensor Association (IFSA).

Sensors & Transducers Journal, Vol. 102, Issue 3, March 2009, pp. 71-83

Sensors & Transducers
ISSN 1726-5479

Discrete Time Sliding Mode Control Using Fast Output Sampling Feedback for Piezoelectric Actuated Structures

[1]L. R. Karl Marx, [2]M. Umapathy, [3]A. Girija, [4]D. Ezhilarasi
[1]Department of Electronics and Communication Engineering
Thiagarajar College of Engineering, Maduari-625 013, India
[2, 3, 4] Department of Instrumentation and Control Engineering
National Institute of Technology, Tiruchirappalli-620 015, India
E-mail: lrkarlmarx@tce.edu, umapathy@nitt.edu

Received: 13 January 2009 /Accepted: 24 March 2009 /Published: 31 March 2009

Abstract: A discrete time Sliding Mode Control (SMC) using fast output sampling feedback is designed for piezoelectric actuated structure to suppress the vibration in the presence of disturbance. Recursive Least Square (RLS) method is used to identify the model. The performance of the controller for vibration suppression and disturbance rejection is demonstrated through simulation and experimental results. The 89S52 microcontroller based hardware is used for experimental implementation. The use of microcontroller provides design flexibility, less component count and low cost solution. *Copyright © 2009 IFSA.*

Keywords: Sliding mode control, Output feedback, Piezoelectric, Microcontroller

---

1. Introduction

The active vibration control of flexible structures using piezoelectric as sensor/actuator has been a research area of wide interest during the past two decades [1, 2, 3]. The theory of sliding-mode control (SMC) is based on the concept of changing the structure of the controller in response to the changing state of the system in order to obtain a desired response [4]. In recent years a considerable amount of effort has been expended on digital sliding mode (DSM) controller design [5, 6]. Most of the Variable structure control methods require full-state feedback. A new approach for sliding mode control of discrete time systems using the reaching law approach together with fast output feedback sampling is proposed in [7]. This method does not need the system states for feedback as it makes use of only the output samples for designing the controller. In this paper discrete-time sliding – mode control using fast output sampling feedback controller is designed for the vibration control of piezoelectric actuated cantilever beam. The outline of this paper is as follows: Section 2 describes the experimental setup and in Section 3 model identification is presented. In Section 4 brief review of control laws are presented. Controller design and its implementation are presented in Section 5 and conclusion is given in Section 6.

2. Experimental Setup

Experimental setup shown in Fig. 1 consists of a cantilever beam made up of aluminum; surface bonded with one pair of piezoceramic patches as sensor/actuator at a distance of 10 mm from the fixed end. The piezoceramic bonded on the bottom surface acts as a sensor and one on the top surface acts as an actuator. To apply an excitation input to the structure another piezoceramic patch is bonded on the top surface at a distance of 153 mm from the fixed end. The dimensions and properties of the beam and piezoelectric patches are given in Tables 1 and 2.

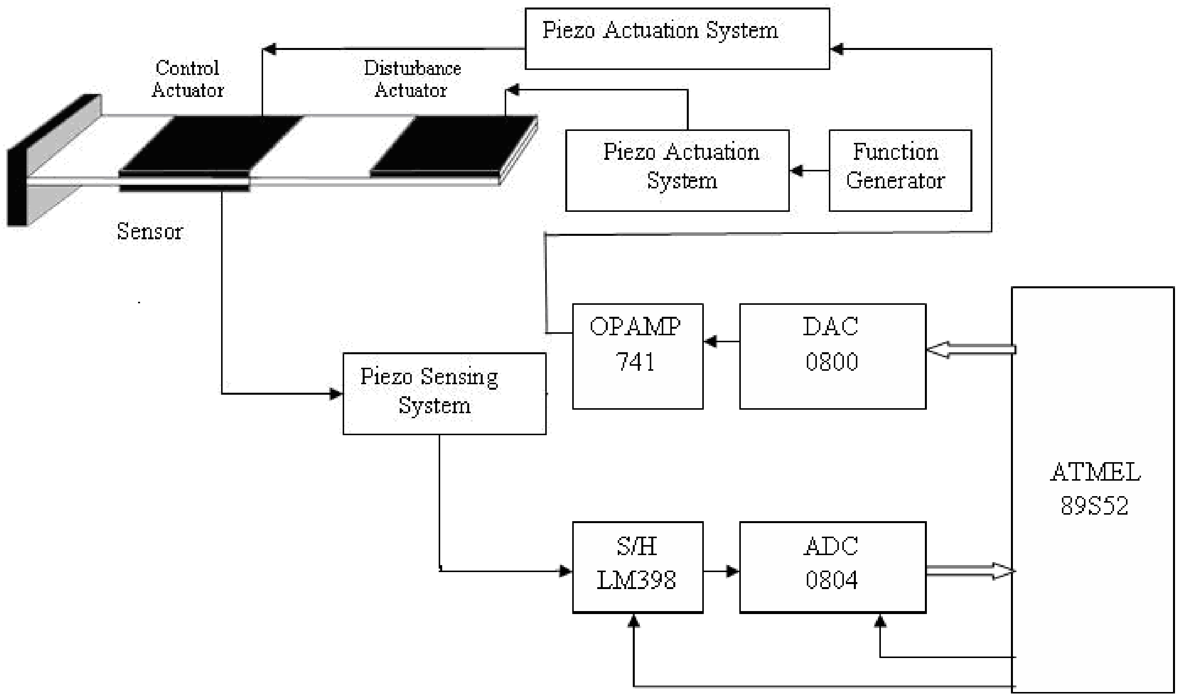


Fig. 1. Schematic Diagram of Experimental Setup.

The sensor output is given to the piezo sensing system which consists of a high quality charge to voltage converting signal conditioning amplifier. The conditioned piezo sensor signal is given as input to the sample and hold (LM 398) device. The output of the sample and hold device is given as the input of an 8 bit Analog to Digital Converter (ADC0804). The ADC output is fed to the port 1 of the ATMEL 89S52 microcontroller, where the control algorithm is fixed as a firmware in the EEPROM of 89S52. The control signal generated from the microcontroller is fed as an input to control actuator through a Digital to Analog Converter (DAC 0800) and piezo actuation system. The excitation signal to the disturbance actuator is applied using a function generator (Agilent 33220) through a piezo actuation system.

Table 1. Properties and dimensions of the Aluminium beam.

Length (m)	l	0.3
Width (m)	b	0.0127
Thickness (m)	t_b	0.0023
Young's modulus (Gpa)	E_b	71
Density (kg/m^3)	ρ_b	2700
First natural frequency (Hz)	f	31.7

Table 2. Properties and dimensions of piezoceramic sensor/actuator.

Length (m)	l_p	0.0765
Width (m)	b	0.0127
Thickness (m)	t_a	0.005
Young's modulus (Gpa)	E_p	47.62
Density (kg/m^3)	ρ_p	7500
Piezoelectric strain constant (m V^{-1})	d_{31}	-247x10^{-12}
Piezoelectric stress constant (V m N^{-1})	g_{31}	-9x10^{-3}

3. Identification of Smart Structure Dynamics

The unknown parameters of the smart structure dynamics are estimated using online identification method, which is proven to be more universal and feasible than analytical and numerical models for the present system. The recursive least square (RLS) method based on ARX model is used for linear system identification, which is easy to implement and has fast parameter convergence. The ARX model for the system shown in Fig. 1 is given as,

$$\hat{y}(k)+a_1 y(k-1)+\ldots+a_{n_a} y(k-n_a)=b_1 u(k-1)+\ldots+b_{n_b} u(k-n_b)+c_1 r(k-1)+\ldots \\ +c_{n_c} r(k-n_c)+e(k). \tag{1}$$

where u(k) is the input signal, r(k) is the excitation signal, y(k) the piezo sensor output, e(k) is white noise and n_a, n_b and n_c determine the model order.

The unknown parameter and data vector is thus

$$\theta = (a_1, a_2, \ldots, a_{n_a}, \quad b_1, b_2, \ldots, b_{n_b}, \quad c_1, c_2, \ldots, c_{n_c})^T \tag{2}$$

$$\varphi(k) = (-y(k-1), -y(k-2), \ldots, -y(k-n_a), \ u(k-1), u(k-2), \ldots, u(k-n_b), \\ r(k-1), r(k-2), \ldots, r(k-n_c))^T. \tag{3}$$

The estimated model output is

$$\hat{y}(k) = \varphi^T(k)\hat{\theta}(k-1) \tag{4}$$

For the RLS algorithm to update the parameters at each sample time, it is necessary to define an error. The model prediction error, ε(k) is the key variable in the RLS algorithm and is defined as

$$\varepsilon(k) = y(k) - \hat{y}(k) \tag{5}$$

The $\varepsilon(k)$ is used to update the parameter estimate as

$$\hat{\theta}(k) = \hat{\theta}(k-1) + P(k)\varphi(k)\varepsilon(k), \tag{6}$$

where the covariance matrix $P(k)$ is updated using

$$P(k) = P(k-1)\left(1 - \frac{\varphi(k)\varphi^T(k)P(k-1)}{1+\varphi^T(k)P(k-1)\varphi(k)}\right). \tag{7}$$

The initial values of $\hat{\theta}(k)$ and $P(k)$ are chosen to be $\hat{\theta}(0) = 0$ and $P(0) = \alpha I_Z$ where $\alpha=10^4$. The natural frequency of the structure is measured experimentally as 31.7 Hz. To identify the parameters in online, the structure is excited by a sinusoidal signal with first natural frequency and a square wave signal as an input to the control actuator. The sampling time is chosen to provide four measurements per cycle (sampling frequency 120 Hz). The excitation signal, input signal and sensor output are given to MATLAB/Simulink through ADC port of dSPACE 1104 system. The RLS algorithm is implemented by writing a C-file S-function used in MATLAB/Simulink. The estimated parameters for a second order model are shown in Fig. 2 and the estimated parameters converged after 10 seconds.

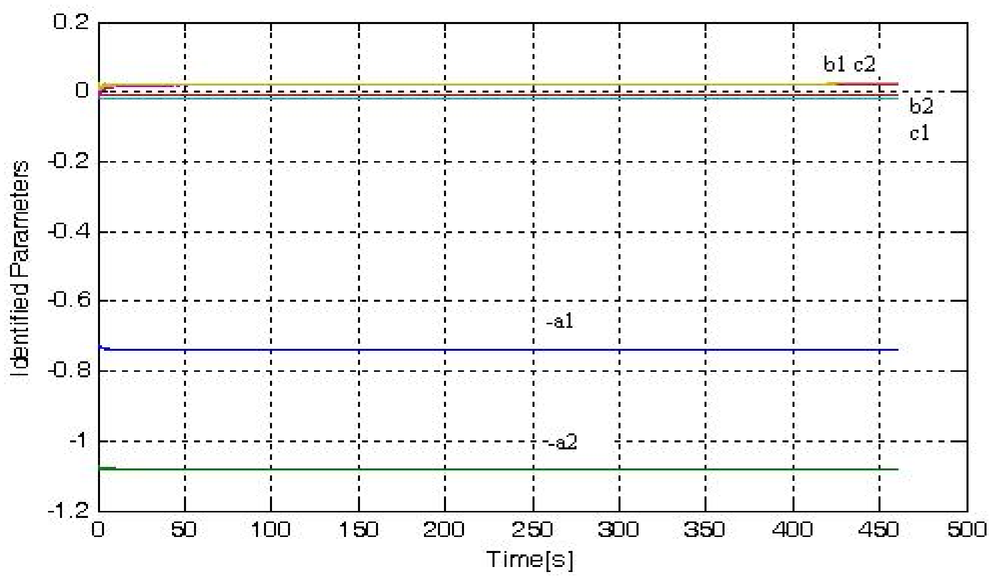


Fig. 2. Estimated parameters of 2nd order smart structure model.

For validating the model, Mean Square Errors (MSE) method is most commonly used and is defined as, $MSE = \frac{1}{N_s}\sum_{k=1}^{N_s}(y(k)-\hat{y}(k))^2$, where N_s is the number of samples used in the identification process.

The MSE values for different model orders are calculated and are given in Table 3.

Table 3. Root-mean square error for different model order.

1st Order Model	2nd Order Model	3rd Order Model	4th Order Model
1.0612	0.0061	0.1057	0.0459

The results indicate that the second order linear dynamics gives the best result among all other orders. A continuous state space model derived from the identified second order ARX model is given as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u + \mathbf{e}r\,;\;\; y = \mathbf{c}^{T}\mathbf{x}\,, \tag{8}$$

where

$$\mathbf{A} = \begin{bmatrix} -83.0583 & 218.2890 \\ -204.9014 & 76.7292 \end{bmatrix},\;\; \mathbf{b} = \begin{bmatrix} -1.4349 \\ -1.708 \end{bmatrix},\;\; \mathbf{e} = \begin{bmatrix} -0.2359 \\ -0.0477 \end{bmatrix},\;\; \mathbf{c}^{T} = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

4. Review of Control Laws

The review of Discrete time quasi-sliding -mode control [8] and Fast Output Sampling feedback control [9, 10] are briefly presented in this section.

4.1. Discrete Time Quasi-Sliding -Mode Control

The new discrete time quasi-sliding-mode control law eliminates chattering, requires smaller control effort and guarantees essentially better robustness of the system than Gao's reaching law control. Indeed, for the system controlled according to the strategy, the maximum distance of the disturbed system state from the sliding plane is guaranteed to be less than one-half of the same distance in Gao's scheme. Furthermore, for the nominal undisturbed system, the same distance is decreased to zero, rather than to a positive constant. Another feature of the strategy is that it guarantees convergence of the system state to a vicinity of the predetermined, fixed plane in finite time, specified a priori by the designer. Consequently, the algorithm is suitable for practical application, the control effort is reduced, and the steady-state accuracy of the system is improved.

Consider the following discrete-time system:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \Delta\mathbf{A}\mathbf{x}(k) + \mathbf{b}u(k) + \mathbf{f}(k) \\ y(k) &= \mathbf{h}^{T}\mathbf{x}(k) \end{aligned} \tag{9}$$

where **x** is the $n \times 1$ state vector, **A** is an $n \times n$ matrix, **b** and **h** are $n \times 1$, u is the system input, and y is the system output. In this equation, the $n \times n$ matrix $\Delta\mathbf{A}$ represents parameter uncertainties and the $n \times 1$ vector **f** denotes external disturbances. The pair (**A, b**) is controllable and the following relations, usually referred to as matching conditions. Let $s(k) = \mathbf{C}^{T}\mathbf{x}(k)$ with vector $\mathbf{C}^{T}\mathbf{b} \neq 0$ and the resulting quasi-sliding motion is stable. Disturbances and parameter uncertainties are bounded so that the following relation holds:

$$d_{l} \leq d(k) = \mathbf{C}^{T}\Delta\mathbf{A}\mathbf{x}(k) + \mathbf{C}^{T}\mathbf{f}(k) \leq d_{u}\,, \tag{10}$$

where the lower bound d_l and the upper bound d_u are known constants and by defining

$$d_0 = \frac{d_l + d_u}{2} \qquad \delta_d = \frac{d_u - d_l}{2}$$

Now, let us specify how the quasi-sliding mode and the reaching condition are understood in this reaching law approach.

If the motion of the system lies in the vicinity of the sliding hyperplane $s(k) = \mathbf{C}^T x(k) = 0$ such that $|s(k)| \le \varepsilon$,it is defined as quasi - sliding mode where the positive constant ε is called the quasi-sliding-mode band width. The system (9) satisfies the reaching condition of the quasi-sliding mode in the vicinity of the sliding surface iff for any the following conditions are satisfied:

$$\begin{aligned} & s(k) > \varepsilon \Rightarrow -\varepsilon \le s(k+1) < s(k) \\ & s(k) < -\varepsilon \Rightarrow s(k) < s(k+1) \le \varepsilon \\ & |s(k)| \le \varepsilon \Rightarrow |s(k+1)| \le \varepsilon \end{aligned} \tag{11}$$

The reaching law to satisfy equation (10) is given by

$$s(k+1) = d(k) - d_0 + s_d(k+1), \tag{12}$$

where the unknown d(k) is defined by equation (10) and $s_d(k)$ is a priori known function such that the following applies.

If $s(0) > 2\delta_d$, then

$$s_d(0) = S(0)$$

$$s_d(k).s_d(0) \ge 0, \quad \text{for any } k \ge 0$$

$$s_d(k) = 0, \quad \text{for any } k \ge k^*$$

$$|s_d(k+1)| < |s_d(k)| - 2\delta_d \text{ for any } k < k^* \tag{13}$$

Relations (13) state that the time-dependent hyper plane $C^T x(k) = s_d(k)$ monotonically, and in a finite time, converges from its initial position $C^T x(k) = s_d(0) = s(0)$ to the origin of the state space. Furthermore, in each control step, the hyper plane moves by the distance greater than $2\delta_d$ .This, together with (12), imply that the reaching condition is satisfied, even in the case of the worst combination of disturbance, in any two consecutive time steps.

Otherwise, i.e., if $s(0) < 2\delta_d$ then

$$s_d(k) = 0 \text{ for any } k \ge 0 \tag{14}$$

The constant k^* in relations (13), is a positive integer chosen by the designer in order to achieve good tradeoff between the fast convergence rate of the system and the magnitude of the control required to achieve this convergence rate. In other words, $s_d(k)$ is an "approximate measure" of the distance between the state and the eventual surface $s(k) = 0$ By controlling the rate of decay (tuning k^*) the convergence to s(k)=0 is tuned. In order to determine the control u which drives the system in such a way that the reaching law (12) is satisfied, we use (9) to calculate s(k+1):

$$s(k+1) = \mathbf{C}^T\mathbf{A}\mathbf{x}(k) + \mathbf{C}^T\mathbf{\Delta A}\mathbf{x}(k) + \mathbf{C}^T\mathbf{b}u(k) + \mathbf{C}^T\mathbf{f}(k)$$

$$s(k+1) = \mathbf{C}^T\mathbf{A}\mathbf{x}(k) + d(k) + \mathbf{C}^T\mathbf{b}u(k) \tag{15}$$

comparing this equation with the reaching law (12), the expression for control law will be

$$u(k) = -(\mathbf{C}^T\mathbf{b})^{-1}[\mathbf{C}^T\mathbf{A}\mathbf{x}(k) + d_0 - s_{\mathbf{d}}(k+1)] \tag{16}$$

4.2. Fast Output Sampling Feedback

The problem of pole assignment by piecewise constant output feedback with infrequent observation was studied by Chammas and Leondes for linear time invariant systems. They have shown that, by using periodically time-varying piecewise constant output feedback gains, the poles of the discrete time control system could be assigned arbitrarily (with natural restriction as they should be located symmetrically with respect to the real axis). Since the feedback gains are piecewise constant, their methodology could easily be implemented. A fast output sampling control law shown in [9, 10] is explained briefly in this section.

Consider a linear continuous-time state space model:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u;\ \ y = \mathbf{c}^T\mathbf{x} \tag{17}$$

where $\mathbf{x} \in \Re^n, u \in \Re^m, y \in \Re^p$ and $\mathbf{A}, \mathbf{b}$ and $\mathbf{c}^T$ are constant matrices. It is assumed that each $\left(\mathbf{A}, \mathbf{b}, \mathbf{c}^T\right)$ is controllable and observable. A discrete time invariant linear system is obtained by sampling the system (17) at a sampling interval τ. Now assume that output measurements are available from the system (17) at the time instants $t = l\Delta$, where l = 0, 1, 2… and the sampling interval τ is divided into N subintervals with $\Delta = \frac{\tau}{N}$ and N is equal to or greater than the observability index (ν) of the discrete system obtained by sampling the system in equation (17) at the rate $\frac{1}{\Delta}$. The control signal $u(t)$ is then generated according to:

$$u(t) = \begin{bmatrix} L_1 & L_2 & \dots & L_{N-1} \end{bmatrix} \begin{bmatrix} y(k\tau - \tau) \\ y(k\tau - \tau + \Delta) \\ \cdot \\ \cdot \\ \cdot \\ y(k\tau - \Delta) \end{bmatrix} \tag{18}$$

$$u(t) = \mathbf{L}\mathbf{y_k},\ k\tau < t \le (k+1)\tau,$$

Let $(\mathbf{\Phi_\tau}, \mathbf{\Gamma_\tau}, \mathbf{c}^T), (\mathbf{\Phi}, \mathbf{\Gamma}, \mathbf{c}^T)$ be the system obtained by sampling the system $\left(\mathbf{A}, \mathbf{b}, \mathbf{c}^T\right)$ at rate $\frac{1}{\tau}$ and $\frac{1}{\Delta}$. Consider the discrete – time system having at time $t = k\tau$, the input $u_k = u(k\tau)$, state $x_k = x(k\tau)$, the output $\mathbf{y}_k$ is represented as:

$$\mathbf{x}_{k+1} = \mathbf{\Phi}_\tau \mathbf{x}_k + \mathbf{\Gamma}_\tau u_k;\ y_{k+1} = \mathbf{C}_0 x_k + \mathbf{D}_0 u_k, \tag{19}$$

where

$$\mathbf{C}_0 = \begin{bmatrix} \mathbf{c}^T \\ \mathbf{c}^T\mathbf{\Phi} \\ . \\ . \\ . \\ \mathbf{c}^T\mathbf{\Phi}^{N-1} \end{bmatrix};\ \mathbf{D}_0 = \begin{bmatrix} \mathbf{0} \\ \mathbf{c}^T\mathbf{\Gamma} \\ . \\ . \\ . \\ \mathbf{c}^T\sum_{j=0}^{N-2}\mathbf{\Phi}^j\mathbf{\Gamma} \end{bmatrix}.$$

Now design a state feedback gain $\mathbf{F}$ such that $\left(\mathbf{\Phi_\tau} + \mathbf{\Gamma_\tau F}\right)$ has no eigen values at origin and provides the desired closed loop behavior. For this state feedback one can define the fictitious measurement matrix as $\mathbf{C(F,}\mathrm{N}\mathbf{)} = (\mathbf{C_0} + \mathbf{D_0 F})(\mathbf{\Phi_\tau} + \mathbf{\Gamma_\tau F})^{-1}$, which satisfies the fictitious measurement equation $y_k = \mathbf{C}\mathbf{x_k}$, then the feedback control in equation (18) can be interpreted as static output feedback $u_k = \mathbf{L}\mathbf{y_k}$, for the system in equation (19) with the measurement matrix $\mathbf{C}$. To realize the effect of $\mathbf{F}$ through $\mathbf{L}$, it must satisfy:

$$\begin{aligned} \mathbf{x}_{k+1} &= (\mathbf{\Phi_\tau} + \mathbf{\Gamma_\tau F})\mathbf{x}_k = (\mathbf{\Phi_\tau} + \mathbf{\Gamma_\tau LC})\mathbf{x}_k \\ \mathbf{LC} &= \mathbf{F} \end{aligned} \tag{20}$$

For $N \ge v$, $\mathbf{C}$ has full column rank, any state feedback gain can be realized by fast output sampling gain $\mathbf{L}$. If the initial state is unknown, there will be an error, $\Delta u_k = u_k - \mathbf{F}\mathbf{x}_k$ in constructing a control signal under state feedback. Then the closed loop dynamics are governed by:

$$\begin{bmatrix} x_{k+1} \\ \Delta u_{k+1} \end{bmatrix} = \begin{bmatrix} \left(\mathbf{\Phi_\tau} + \mathbf{\Gamma_\tau F}\right) & \mathbf{\Gamma_\tau} \\ \mathbf{0} & \mathbf{LD_0} - \mathbf{FT}_\tau \end{bmatrix} \begin{bmatrix} x_k \\ \Delta u_k \end{bmatrix} \tag{21}$$

The system in equation (11) is stable, if and only if, $\mathbf{F}$ stabilizes $(\mathbf{\Phi_\tau}, \mathbf{\Gamma_\tau})$ and matrix $(\mathbf{LD_0} - \mathbf{F\Gamma_\tau})$ has all its eigen values inside the unit circle.

Let **E**=-**F** $\mathbf{\Gamma_\tau}$.The problem is to assign the eigen values of (**E**+**LD$_0$**) by choice of **L**. However, the gain **L** must satisfy **L*C*=F**. If N is large enough ,**L** can be chosen from a linear variety, and one might pose the problem of designing **L** as an optimization problem, where the spectral radius of **(E**+**LD$_0$)** is minimized over **L** subject to **L*C*=F**.

While designing L, a second aspect to be considered is noise sensitivity because of large gain values. It is possible to enhance performance by not insisting that **L*C*=F** be satisfied exactly, but by allowing a slight deviation when this in turn leads to smaller gains. Let ρ_1, ρ_2 and ρ_3 represent upper bounds on the spectral norms of **L**, (**E**+**LD$_0$**) and **L*C*-F**, respectively, the system of LMI's will be

$$\begin{bmatrix} -\rho_1^{\ 2} & \mathbf{L} \\ \mathbf{L}^{\mathbf{T}} & -\mathrm{I} \end{bmatrix} < 0 \tag{22}$$

$$\begin{bmatrix} -\rho_2^2 \mathrm{I} & \mathbf{E} + \mathbf{LD_0} \\ (\mathbf{E} + \mathbf{LD_0})^{\mathrm{T}} & -\mathrm{I} \end{bmatrix} < 0 \tag{23}$$

$$\begin{bmatrix} -\rho_3^{\ 2} \mathrm{I} & \mathbf{L}\boldsymbol{C} - \mathbf{F} \\ (\mathbf{L}\boldsymbol{C} - \mathbf{F})^{\mathrm{T}} & -\mathrm{I} \end{bmatrix} < 0 \tag{24}$$

Here ρ_1 small means low noise sensitivity, and ρ_2 small means fast decay of observation errors. The last inequality represents the constraint that **L** realize an approximation of the state feedback gain F; taking ρ_3=0 gives an exact solution to **L*C*=F** .If suitable bounds ρ_1 and ρ_2 are known; one can keep these bounds fixed and minimize ρ_3 under these constraints. The closed loop stability requires ρ_2<1, i.e., the eigenvalues which determine the error dynamics must lie within the unit disc.

5. Controller Design and Implementation

5.1. Controller Design

Let the discrete state space model of a system in equation (8) is

$$\begin{aligned} &\mathbf{x}(\mathrm{k}+1) = \mathbf{\Phi}\mathbf{x}(\mathrm{k}) + \mathbf{\Gamma}\mathrm{u}(\mathrm{k}) + \mathbf{E}\mathrm{f}(\mathrm{k}) \\ &\mathrm{y}(\mathrm{k}) = \mathbf{C}^{\mathbf{T}}\mathrm{x}(\mathrm{k}) \end{aligned}, \tag{25}$$

where

$$\mathbf{\Phi} = \begin{bmatrix} 0 & 1 \\ -0.9387 & -0.732 \end{bmatrix}, \mathbf{\Gamma} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{E} = \begin{bmatrix} 0.1098 \\ 0.0441 \end{bmatrix}, \mathbf{C}^{\mathrm{T}} = \begin{bmatrix} 0.0014 & \text{-}0.0148 \end{bmatrix}, \mathrm{x}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

For this, the model equation (10) can be written as

$$\mathrm{d}_\mathrm{l} \le \mathrm{d}(\mathrm{k}) = \mathbf{C}^{\mathbf{T}}\mathbf{E}\mathrm{f}(\mathrm{k}) \le \mathrm{d}_\mathrm{u}$$

Since the initial condition is zero, s(0) = 0, so that $s_d(k) = 0,$ and for a sinusoidal disturbance of amplitude 2.5V,

$$\mathrm{d}_0 = \frac{-2.5 + 2.5}{2} = 0\,.$$

Then $s(k+1) = d(k)$ and s (k+1) can be written as

$$s(k+1) = \mathbf{C}^T\mathbf{\Phi x}(k) + \mathbf{C}^T\mathbf{\Gamma}u(k) + \mathbf{C}^T\mathbf{E}f(k).$$

The control law can be derived as

$$u(k) = -(\mathbf{C}^T\mathbf{\Gamma})^{-1}\mathbf{C}^T\mathbf{\Phi x}(k) \tag{26}$$

Using state feedback control $\left(u(k) = \mathbf{Fx}(k)\right)$ designed in FOS feedback control and using the control law (26) the state feedback gain can be derived as

$$F = -(\mathbf{C}^T\mathbf{B})^{-1}\mathbf{C}^T\mathbf{A} \tag{27}$$

Choosing $\mathbf{C}^T = [0.64\ 1]$ **F** is found to be [0.9387 0.0810]

Fast output sampling feedback controller gain **L** is designed for the system in (25) as per the procedure explained in section 4.2 with

$$\tau = 0.01,\ N = 4;\ \Delta = 0.0025.$$

$$\mathbf{L} = \begin{bmatrix}15.5290 & -12.84 & -18.3625 & 0.4291\end{bmatrix}$$

The frequency and amplitude of the disturbance is chosen as 31.7 Hz and 5 Volts peak to peak. The disturbance, control applied, open and closed loop response obtained in simulation are shown in Fig. 3.

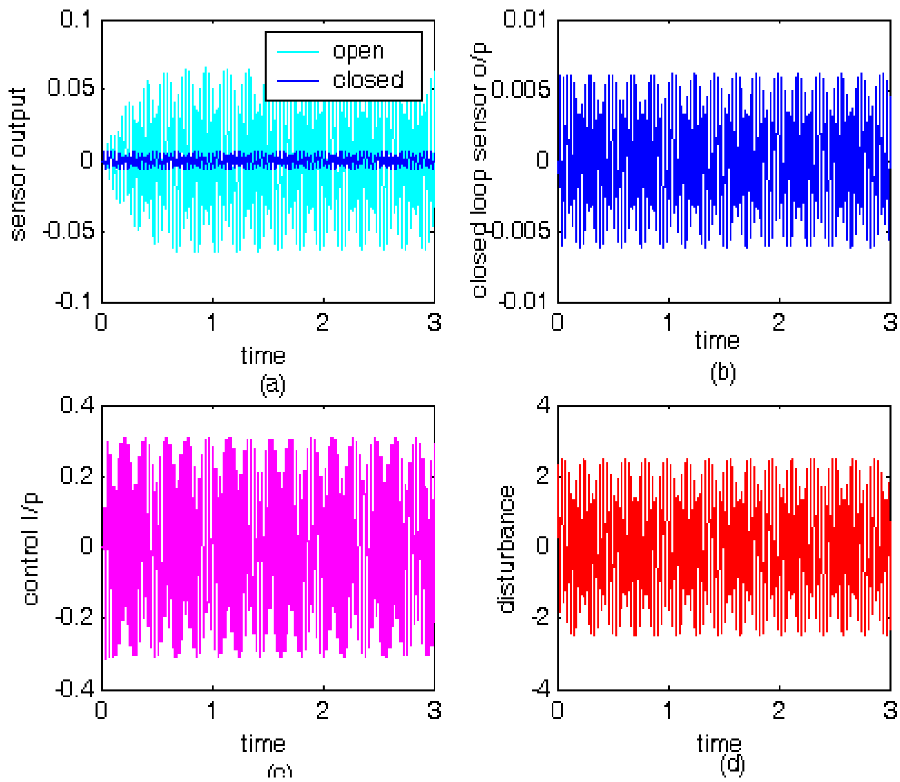


Fig. 3. Simulation response for excitation at first mode frequency: (a) Open and closed loop responses for sinusoidal disturbances, (b) Closed loop response, (c) Control i/p, (d) Disturbance i/p.

5.2. Implementation Using Microcontroller

The experimental set up to measure and control the vibration of a cantilever beam using microcontroller is shown in Fig. 4. A sinusoidal excitation at first natural frequency with the amplitude of 5V peak to peak is applied to the disturbance actuator which makes the beam to vibrate at resonance. The piezoelectric sensor output is applied to a sample and hold circuit (S/H). The S/H circuit is designed with LM 398 and the microcontroller port 3.0 enables it for 7 microseconds as a sampling time, then the A/D conversion is initiated. During the conversion time, sampling is disabled and Hold signal is activated. This process continues for the successive samples.

The sampled analog signals are fed to the input of 8 bit ADC 0804, which is operating with a clock frequency of 606 kHz. The resistance and capacitance (RC) values are chosen as 10 k and 150 pf respectively. Use of poor tolerance components will shift the operating clock frequency. At the end of conversion (EOC) interrupt signal, the output of ADC is made available to microcontroller port 1 and it is disabled during the conversion time to avoid the error data interface. The ADC works with ± 5 volts and has a resolution of 8 bits.

A firmware in hex code was developed and tested through 89S52 simulator has been fused in to the EEPROM of the microcontroller. This firmware controls the overall function of the controller. The two timers, timer 0&1 of the 89S52 are initiated with 1 (τ) and 0.025 (Δ) millisecond durations respectively. At every Δ period the sensor output is interfaced to the A/D converter through S/H device. The converted sensor output at each Δ period is stored in RAM location of 89S52. In this case, the each τ interval consists of four Δ intervals. The stored values are multiplied with their respective controller gain as per the equation (18), a unique 8 bit controller output signal is computed and stored in RAM. Then the microcontroller waits and checks for the timer 0 to complete the τ duration. After the end of τ, the controller output is applied into the D/A converter through port 0 of 89S52. This process continues for the successive τ period.

The DAC 0800 gives an analog signal for an applied digital code from the microcontroller. Since the DAC provides a current output with phase inversion, an OP-AMP 741 is used to convert the current output into voltage with the phase compensation. The output of the OP-AMP is applied to the piezoelectric control actuator.

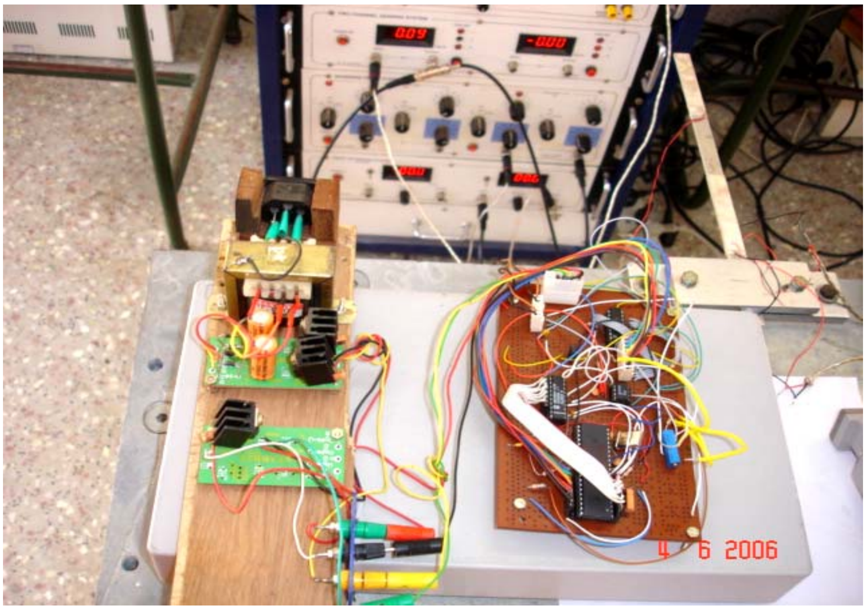


Fig. 4. Experimental set up with microcontroller interface.

The excitation signal, open loop response, closed loop response with discrete time sliding mode control using fast output feedback control and control signal acquired are shown in Fig. 5.

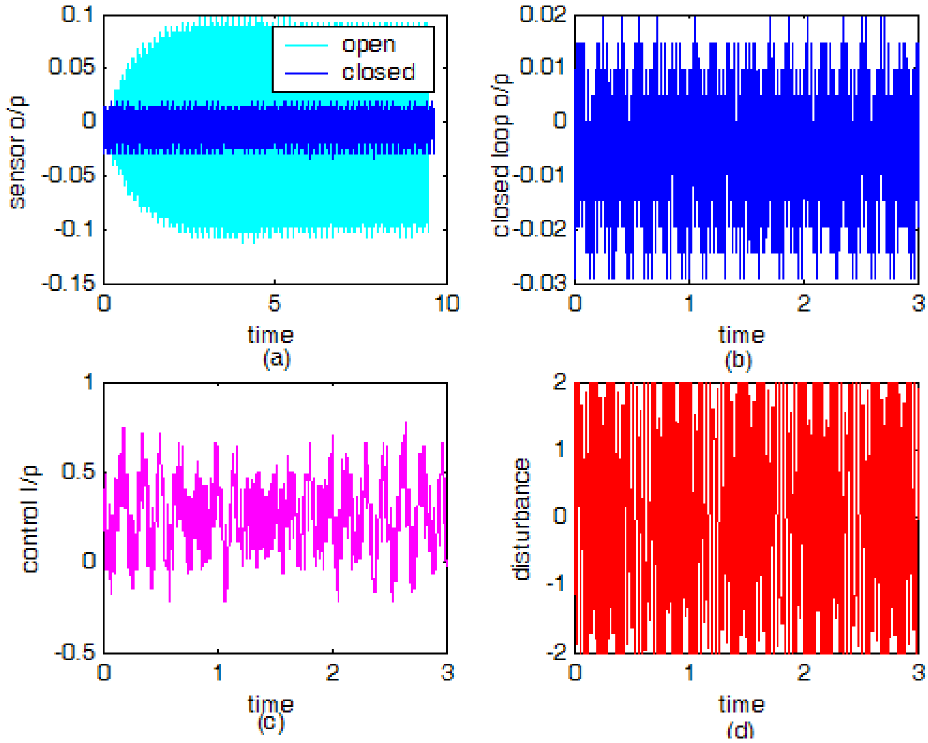


Fig. 5. Real time responses for excitation at first mode frequency: (a) Open and closed loop responses for sinusoidal disturbances, (b) Closed loop response, (c) Control i/p, (d) Disturbance i/p.

6. Conclusion

The accurate model for the beam structure incorporated with piezoceramic patches is experimentally established using online ARX RLS system identification approach. The sliding-mode control (SMC) of discrete-time systems using the reaching law approach together with the fast output sampling (FOS) feedback technique is designed and implemented. To demonstrate the performance of the controller the beam is made to vibrate at resonance (at first natural frequency) and the vibration reduction is observed. The key advantage of this approach is that it neither requires the states of the system for feedback nor an observer/estimator. Only the output of the system, which is available, is used to generate the control action .By proper choice of the parameters in the reaching law, the dynamic response of the closed-loop system can be improved. Also this methodology is more practical and easy to implement. The experimental and simulation results show the effectiveness of this method.

References

[1]. Inderjit Chopra, Review of state of art of smart structures and integrated systems, *AIAA Journal,* 40, 11, 2002, pp. 2145-2187.

[2]. Der-An Wang, Yii-Mei Huang, Application of discrete-time variable structure control in the vibration reduction of a flexible structure, *Journal of Sound and Vibration*, 261, 2003, pp. 483-501.

[3]. D. Ezhilarasi, M. Umapathy and B. Bandyopadhyay, Design and experimental evaluation of piecewise output feedback control for structural vibration suppression, *Smart Materials and Structures*, 15, 2006, pp. 1927-1938.

[4]. V. I. Utkin, Variable structure systems with sliding mode, *IEEE Trans. Automat. Control*, Vol. AC-22, No. 2, 1977, pp. 212-222.
[5]. Sarpturk, S. Z., Istefanopulos, Y., and Kaynak, O., On the stability of discrete-time sliding mode systems, *IEEE Trans. Autom. Control*, 32, 10, 1987, pp. 930-932.
[6]. K, Furuta, Sliding mode control of a discrete system, *Syst. Contr: Lett*., Vol. 14, 1990, pp. 145-152.
[7]. Chakravarthini Saaj, M., Bandyopadhyay, B., and Unbehauen, H., A new algorithm for discrete-time sliding mode control using fast output sampling feedback, *IEEE Trans on Industrial Electronics*., 49, 3, 2002, pp. 518-523.
[8]. Andrzej Bartoszewiez, Discrete-Time Quasi-Sliding Mode Control Strategies, *IEEE Transactions on Industrial Electronics*, Vol. 45, August 1998, pp. 633-637.
[9]. H. Werner, Multimodel robust control by fast output sampling - An LMI approach, *Automatica,* 34 , 12*,* 1998, pp. 1625-1630.
[10].H. Werner and K. Furuta, Simultaneous stabilization based on output measurements, *Kybernetika,* 31, 4, 1995, pp. 395-411.

Guide for Contributors

Aims and Scope

Sensors & Transducers Journal (ISSN 1726-5479) provides an advanced forum for the science and technology of physical, chemical sensors and biosensors. It publishes state-of-the-art reviews, regular research and application specific papers, short notes, letters to Editor and sensors related books reviews as well as academic, practical and commercial information of interest to its readership. Because it is an open access, peer review international journal, papers rapidly published in *Sensors & Transducers Journal* will receive a very high publicity. The journal is published monthly as twelve issues per annual by International Frequency Association (IFSA). In additional, some special sponsored and conference issues published annually.

Topics Covered

Contributions are invited on all aspects of research, development and application of the science and technology of sensors, transducers and sensor instrumentations. Topics include, but are not restricted to:

- Physical, chemical and biosensors;
- Digital, frequency, period, duty-cycle, time interval, PWM, pulse number output sensors and transducers;
- Theory, principles, effects, design, standardization and modeling;
- Smart sensors and systems;
- Sensor instrumentation;
- Virtual instruments;
- Sensors interfaces, buses and networks;
- Signal processing;
- Frequency (period, duty-cycle)-to-digital converters, ADC;
- Technologies and materials;
- Nanosensors;
- Microsystems;
- Applications.

Submission of papers

Articles should be written in English. Authors are invited to submit by e-mail editor@sensorsportal.com 6-14 pages article (including abstract, illustrations (color or grayscale), photos and references) in both: MS Word (doc) and Acrobat (pdf) formats. Detailed preparation instructions, paper example and template of manuscript are available from the journal's webpage: http://www.sensorsportal.com/HTML/DIGEST/Submition.htm Authors must follow the instructions strictly when submitting their manuscripts.

Advertising Information