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A Method Based on Dial's Algorithm for Multi-time Dynamic Traffic Assignment

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Abstract: Due to static traffic assignment has poor performance in reflecting actual case and dynamic traffic assignment may incurs excessive compute cost, method of multi-time dynamic traffic assignment combining static and dynamic traffic assignment balances factors of precision and cost effectively. A method based on Dial's logit algorithm is proposed in the article to solve the dynamic stochastic user equilibrium problem in dynamic traffic assignment. Before that, a fitting function that can proximately reflect overloaded traffic condition of link is proposed and used to give corresponding model. Numerical example is given to illustrate heuristic procedure of method and to compare results with one of same example solved by other literature's algorithm. Results show that method based on Dial's algorithm is preferable to algorithm from others.

Keywords: Dynamic traffic assignment, Compute cost, Dial's logit algorithm, Dynamic stochastic user equilibrium, Heuristic procedure.

1. Introduction

Static user equilibrium is a state that achieves the user optimal. In this condition, user optimal and user equilibrium are unified, models based on User Equilibrium (UE) principle thus are widely proposed. First and second principle of equilibrium is developed by Wardrop, which emanate from concept of Nash Equilibrium in the game theory. Then Beckmann introduces mathematical extreme value model of equilibrium theory. A hybrid model combining traffic volume distribution with traffic assignment is proposed in 1971, and it solves assignment problems through linear programming. Cesario [1] proposes combination model of traffic volume distribution and traffic assignment which bases on the gravity model at the cost of ignorance of elements effecting traffic state. Basing on the equilibrium theory, Smith [2] develops variational inequality model that improves the theory. Daganzon and Sheffic [3] unifies the conception of stochastic and Wardrop equilibrium and proposes Stochastic User Equilibrium (SUE) model, which can be efficiently solved by the method introduced in Dial [4]. The static traffic assignment, however, can not reflect congestion of traffic on the link, this to some extent prevent assignment results from getting more precise.

Dynamic traffic assignment models have significant employments in the rapidly developed advanced traveler information systems and advanced traffic management systems. Generally, these models can be classified into two categories: the reactive assignment model and the predictive one. The former

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assumes that each traveler will chose the shortest path to get to the destination according to current instantaneous traffic situation. If different paths have been chosen, travelers between the same Origin-Destination (OD) pair at the same time thus may arrive at destination differently. Models proposed by Wie et al. [5] and Ran et al. [6] are belong to this paradigm. The later considers the impact of future traffic situation on path choice behavior, that is, the shortest path is usually determined by the actually experienced travel time. Most of these models aim at satisfying the dynamic user-equilibrium condition, which requires actual travel time used by travelers is equal and minimal. Models belonging to the later one are used in Friesz et al. [7] and Chen and Hsueh [8].

Dynamic traffic assignment is regarded as a new approach to solve the traffic problems and it deals with congestion, dynamic models, however, are more complicated. What is more, existence and uniqueness of the solution are still not absolutely guaranteed. But investigation in Lo and Szeto [9] shows that static traffic assignment and dynamic one can even produce directly opposite results, and this illustrates the importance of adopting advantages from the dynamic traffic assignment for planning and policy evaluation. In the point of view on macro traffic policy, traffic assignment aims to predict distribution of traffic flow on the route network and to get predictive results such as utilization of link and influence from new added OD pair. The policy thus can be made to improve or remove corresponding infrastructure or to control part of flows through traffic regulations. The method in this paper is to precisely have traffic assignment with low compute cost at the extent of macro traffic policy.

To make traffic assignment results approach realistic condition, such a target has attracted an increasing number of study efforts. Basing on shortcomings from the static traffic assignment and defections of high compute cost in the dynamic traffic assignment, new models between the static traffic assignment and the dynamic one are proposed in the past few years. The introduction of new models aims to overcome defect on the static type's poor performance in reflecting actual traffic situations, which may lead to error of policy maker's decision, and on the big complexity and huge compute cost in dynamic type. Link state is defined and then it divides traffic flows into different states to have traffic assignment. Method proposed in Kong and Li [10] has traffic assignment through multipath increment loading. And it also considers the influence of the traffic flow which has been assigned in the network, using an algorithm that figures results of traffic assignment according to variation of impedance in each link. Model based on traffic assignment between static and dynamic one is proposed in Tang [11], and it is solved by Frank-Wolfe algorithm in the literature.

In this paper, a method based on Dial's logit assignment algorithm [4] is proposed in the article to solve the dynamic stochastic user equilibrium problem in multi-time dynamic traffic assignment. The article uses the Dial's algorithm in that it is a probabilistic assignment algorithm which can reflect travelers' actual behavior better. Numerical example from [10] will be figured by method introduced in the paper and its results will be compared to results in [10], in order to show its improvement.

What is note worthy is that method of Kong uses difference between traffic volume of each period bordered upon as increment of traffic volume, which can be expressed with $\Delta q = q_{nm} - q_{nm-1}$ [10]. If $\Delta q \geqq 0$, then traffic flow is loaded through “all or nothing”, else the longest distance path in the network will be searched and there will be a $|\Delta q|$ reduction in its traffic volume. And new impedance value then is updated by Bureau of Public Roods (BPR) model [12].

In the method of Kong, all trips between a fixed Origin-Destination pair are assigned to the link of shortest length path. That approach used in the literature of Kong is so-called “all-or-nothing” (AON) assignment, which assigns all traffic flows on the shortest path. AON assignment is known to contradict actual trip behavior, and the link volumes these traffic assignment models output are to some extent inaccurate to the point of compromising the transportation planner's design decisions, due to the effects of trip volumes on travel time and the trip maker's non-deterministic choice function on route selection.

In order to satisfy the realistic traffic demand and to reflect the traffic assignment conditions in different links at different time, the model divides time into several different periods and then it does have traffic assignment according to the regulation of demand variation and average demand in each period. In the dynamic model, time dividing is usually based on the time-traffic volume curve due to division of time hinges largely on the regulation of variation of traffic volume. The variety regulation of traffic flow in different time can be shown and graphed in Fig. 1.

Stable variation of traffic volume can be regarded as a period, thus each period of time are not same all the time. In the route networks, link travel time is not always in proportion to distance, this is because traffic flow on each link plays an important role in impacting link travel time, the more it becomes large, the more link travel time becomes huge. That is, link travel time is to some extent in proportion to the traffic impedance rather than the distance. Due to the traffic demand volume is a variable, the traffic impedance can thus be treated as variable as well. So, the problem can be generalized as follows: If the impedance is quantified as length of path, with the variation of traffic volume between two given node, the shortest path is changes too. This compels us to update the impedance of link frequently so that large part of traffic flow can be assigned on the new shortest path.

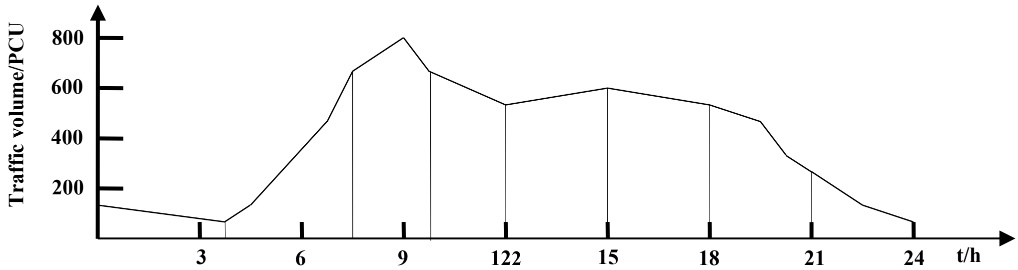

Fig. 1. Variety regulation of traffic flow in different time [10].

2. Method Based on Dial's Algorithm

According to forgoing defection of the Kong's method, method based mainly on Dial's logit assignment algorithm is proposed to deal with such dynamic traffic assignment problem. And then we will give a comparison between Kong's result and writer's. Due to the using of BPR model in the literature of Kong, it is still used as function of impedance in the article.

2.1. The Fitting Function

In order to achieve a better performance in precision and low compute cost than before, point-queue representation is needed to be concerned. Three kinds of functions are widely used, they are: the outflow function, the travel time function, and the mixed function. The outflow function regards outflow of link as a nondecreasing function of the number of vehicles traveling in the link. And it is used to figure cumulative flows in the link so that travel time can be obtained. Outflow function is adopted by Cary et al. [13]. The link travel time is defined from the inflows and outflows. While the travel time function reverses the procedure, that is, it calculates link travel times before outflows. Yang and Meng [14] and Huang and Lam [15] adopt this kind of function in developing their traffic flow models. In mixed function, outflows are determined by exit-flow function and travel time is determined by travel time function. The function is employed in Lam and Huang [16]. In all of these three kinds of functions, outflow is needed weather before or after the calculation of travel time. Introduction of fitting function in this article makes value of outflow dispensable.

We assume that value of assignment traffic flow in each link is equal to its outflow volume when assignment value is lower than capacity. In the condition that assignment value is equal to or larger than capacity of link, the writer gives a fitting function that can approximately describe relationships between inflow and remaining traffic flow of link, and the fitting function is expressed with:

$$x = v - \frac{c^2}{v}, \tag{1}$$

where x is the remaining traffic flow of link, v denotes total of link, and c denotes link capacity.

In the course of traffic assignment, congestion will occur when value of traffic volume surpasses the capacity of link part of volume thus could not pass through the link instantly. In this situation, due to interaction between vehicles on the link existed, value of remains traffic volume is usually larger than margin between traffic inflow and capacity of link. (1) is just about the function that figures results of remaining flow in the link approaching actual value.

Fig. 2 shows graph of (1) which reflects relationship between total flow and remaining traffic flow of link in a condition that value of capacity is given 720.

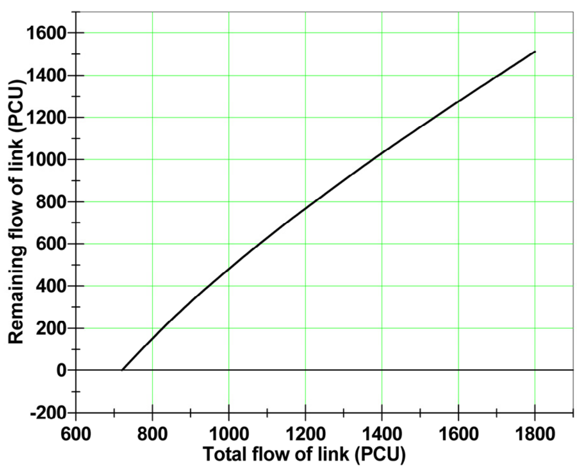

Fig. 2. Image of function between remaining flow of link and total flow of link (the range from 720 to 1800).

2.2. The Model

In order to give expression of conservation of dynamic flow and to give mathematical description for BPR model, here we state our main assumptions:

1) All vehicles passes through the link clearly before the next period in a condition that traffic flow in each link is lower than its capacity.

2) No overtaking and drawback will occur in each link.

Because there is no formulation of outflow for BPR, and with introduction of fitting function, expression of conservation of dynamic flow mentioned before can thus be substituted with:

$$S_a(i) = r_a^{i-1}(S_a(i-1)) + x_a^i(S_a(i-1)), \quad (2)$$

where $S_a(i)$ denotes total flow of link in current period; $r_a^{i-1}(S_a(i-1))$ denotes remaining flow in the former period, which is calculated with $S_a(i)$ in former period; $x_a^i(S_a(i-1))$ is traffic assignment volume of link figured according to impedance determined by $S_a(i-1)$ (i.e. input flow of link a at period t_i).

Basing on Logit SUE model [17], the dynamic model of BPR model thus can be depicted as follows:

$$\min f(h_i, x_i) = \frac{1}{\theta}\sum_{k\in K} h_k(t_i)\ln h_k(t_i) + \sum_{a\in A}\int_{S_a(t_{i-1})}^{S_a(t_i)} t_a(w)dw \quad (3)$$

$$\begin{cases} x_a(t_i) = \sum_{k\in K} h_k(t_i)\delta_a^k \\ S_a(i) = r_a^{i-1}(S_a(i-1)) + x_a^i(S_a(i-1)) \\ \sum_{k\in K} h_k(t_i) = q(t_i) \\ r_a^i = S_a(i) - \dfrac{c^2}{S_a(i)} \end{cases}, \quad (4)$$

where $A(k)$ is the set of all links originated from node k, $B(k)$ is the set of all links terminated at node k, $q_{rs}(t_i)$ is the flow originated from node r and terminated at node s at period t_i, $t_a(q)$ is the time needed for flow q to pass through link a, x_a^i is the input flow of link a at period t_i, $S_a(t_i)$ is the total flow of link at period t_i, c is the capacity of link, $h_k(t_i)$ is the flux of path k at period t_i, $q(t_i)$ is the traffic demand in the network at period t_i, δ_a^k is the weather link a on the path k, $\delta_a^k = \begin{cases} 1 & yes \\ 0 & else \end{cases}$ and r_a^i is the remaining flow in the link.

2.3. The Algorithm

According to the Dial's theory, it stochastically diverts trips to alternate paths, but trips are not explicitly assigned to routs. Instead, they are probabilistically diverted at each node encountered to the competing links entering the node. In multistep dynamic traffic assignment, due to time has been divided into several periods, traffic demand is divided into several groups correspondingly as well. The method introduced in the article uses Dial's algorithm to solve traffic assignment in each step and updates traffic impedance to prepare for the assignment in the next step. We need to know the initial impedance in order to get the first step assignment result. Then the remaining flow can be calculated according to fitting function mentioned before so that iterative procedures could carry on.

Method based on Dial's algorithm in solving dynamic traffic assignment problem can be illustrated as procedures as follows:

Step 0 (Initialization): Divide time into several periods according to stability of variation of traffic volume and then get the traffic demand q(ti) in each period. The initial impedance of network is given.

Step 1: Use Dial's algorithm to get the result of assignment in period ti according to traffic demand and impedance in Step 0 and calculate remaining traffic flow and total flow of each link at period ti.

Step 2: According to the preceding step, the traffic impedance can be figured through BPR model, and the use Dial's algorithm to assign traffic volume. If link in the network is defined as efficiency link, then mark it with Δ. And if all periods' assignment have been finished, the dynamic traffic assignment is complete, else turn to Step 3.

Step 3: Take the result of assignment into equation (2) and take its figuration result into fitting function (1) to calculate total flow of link, turn to Step 2.

3. Numerical Example

3.1. Example and its Results Solved by Kong's Method

Network, as Fig. 3 shows, gives values of traveling time and capacity in each link.

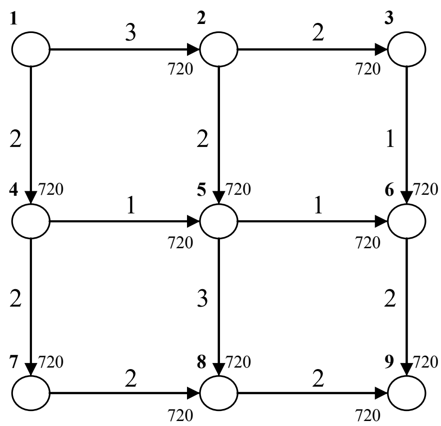

Fig. 3. Network and traveling time [10].

Table 1 gives traffic flow at different time. All travelers in each period, according to traffic condition before, will select traveling path the content with from origin node 1 to destination 9. All values of capacity are given 720, and traffic volume to be assigned into network I has been divided into 10 groups. Results of traffic flow and transportation impedance in each link at different time figured by Kong's method is shown as Table 2-3.

Table 1. The traffic flow of different time [100].

Period i	Traffic volume (PCU)	Period i	Traffic volume (PCU)
1	700	6	1150
2	500	7	1250
3	1050	8	1350
4	1250	9	1000
5	1500	10	750

Table 2. Kong's results of traffic flow and transportation impedance in each link at different time (Part 1) [10].

Period	1-2		1-4		2-3		2-5		3-6		4-5	
	I	F	I	F	I	F	I	F	I	F	I	F
0	3.0	0	2.0	0	2.0	0	2.0	0	1.0	0	1.0	0
1	3.0	0	2.4	750	2.0	0	2.0	0	1.0	0	1.2	750
2	3.0	0	2.1	500	2.0	0	2.0	0	1.0	0	1.0	500
3	3.0	0	3.4	1050	2.0	0	2.0	0	1.0	0	1.7	1050
4	3.0	200	3.4	1050	2.0	0	2.0	200	1.0	0	1.7	1050
5	3.1	450	3.4	1050	2.0	0	2.0	450	1.0	0	1.7	1050
6	3.0	100	3.4	1050	2.0	0	2.0	100	1.0	0	1.7	1050
7	3.0	200	3.4	1050	2.0	100	2.0	100	1.0	100	1.7	1050
8	3.0	300	3.4	1050	2.0	200	2.0	100	1.0	200	1.7	1050
9	3.0	300	2.3	700	2.0	200	2.0	100	1.0	200	1.1	700
10	3.0	50	2.3	700	2.0	50	2.0	0	1.0	50	1.1	700
	-	160	-	895	-	55	-	105	-	55	-	895
Note: I denotes traffic impedance, F denotes traffic flow in the link.												

Table 3. Kong's Results of Traffic Flow and Transportation Impedance in each Link at Different Time (Part 2) [10].

Period	4-7		5-8		5-6		6-9		7-8		8-9	
	I	F	I	F	I	F	I	F	I	F	I	F
0	2.0	0	2.0	0	1.0	0	2.0	0	2.0	0	2.0	0
1	2.0	0	2.0	0	1.2	750	2.4	750	2.0	0	2.0	0
2	2.0	0	2.0	0	1.0	500	2.1	500	2.0	0	2.0	0
3	2.0	0	2.0	0	1.7	1050	3.4	1050	2.0	0	2.0	0
4	2.0	0	2.0	200	1.7	1050	3.4	1050	2.0	0	2.0	200
5	2.0	0	2.0	450	1.7	1050	3.4	1050	2.0	0	2.0	450
6	2.0	0	2.0	450	1.1	700	2.3	700	2.0	0	2.0	450
7	2.0	0	2.0	450	1.1	700	2.5	800	2.0	0	2.0	450
8	2.0	0	2.0	450	1.1	700	2.7	900	2.0	0	2.0	450
9	2.0	0	2.0	100	1.1	700	2.7	900	2.0	0	2.0	100
10	2.0	0	2.0	0	1.1	750	2.4	750	2.0	0	2.0	0
	-	0	-	210	-	795	-	845	-	0	-	210
Note: I denotes traffic impedance, F denotes traffic flow in the link.												

3.2. Optimization Strategy

Here, heuristic procedure based on Dial's algorithm will be used to solve example in literature of Kong. Before that, some of variables in heuristic procedure is needed to be illustrated. Illustrations of variable can be shown in Fig. 4.

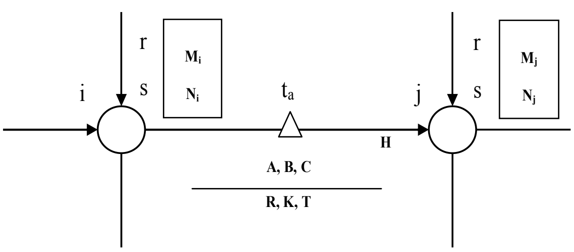

Fig. 4. Network and traveling time.

t_a denotes the time needed to pass through link a, hence it can be used as impedance of link; H denotes capacity of link; M and N denote the shortest distance from origin node respectively in forward pass and backward pass; A、B and C denote link weight, probability of selection and assignment volume respectively; R denotes remaining flow of link during the former period; K denotes remaining flow of link during current period; and T denotes total flow in the link during current. In the act of calculation at each stage, variable K plays an important role, which is used to figure T of current stage and next stage. The impedance of link bases on total flow of link. If link is defined as efficient link then mark it with Δ to avoid mistakes during the assignment. Let parameter θ in Dial's algorithm equal to 1, and the results of traffic flow and impedance in each link at different time by method based on Dial's method is shown in Table 4 and Table 5.

Table 4. Results of method based on Dial's algorithm in traffic flow and transportation impedance in each link at different time (Part 1).

Period	1-2		1-4		2-3		2-5		3-6		4-5	
	I	F	I	F	I	F	I	F	I	F	I	F
0	3.0	0	2.0	0	2.0	0	2.0	0	1.0	0	1.0	0
1	3.0	0	2.0	750	2.0	0	2.0	0	1.0	0	1.0	750
2	3.0	85	2.0	415	2.0	0	2.0	85	1.0	0	1.18	366
3	3.0	126	2.0	924	2.0	0	2.0	126	1.0	0	1.02	836
4	3.0	466	2.0	784	2.0	198	2.0	268	1.0	198	1.27	670
5	3.08	935	2.0	935	2.0	377	2.01	558	1.0	377	1.34	470
6	4.28	448	4.8	702	2.02	121	2.11	327	1.01	121	1.2	480
7	3.79	1250	2.0	0	2.0	591	2.01	659	1.0	591	1.06	0
8	10.42	0	2.14	1350	2.14	0	2.21	0	1.07	0	1.0	985
9	5.39	1000	25.1	0	2.0	500	2.0	500	1.0	500	1.53	0
10	14.48	435	16.18	315	2.07	0	2.07	435	1.04	0	1.02	277
	-	474	-	580	-	178	-	295	-	178	-	478
Note: I denotes traffic impedance, F denotes traffic flow in the link.												

Table 5. Results of method based on Dial's algorithm in traffic flow and transportation impedance in each link at different time (Part 2).

Period	4-7		5-8		5-6		6-9		7-8		8-9	
	I	F	I	F	I	F	I	F	I	F	I	F
0	2.0	0	2.0	0	1.0	0	2.0	0	2.0	0	2.0	0
1	2.0	0	2.0	203	1.2	547	2.0	547	2.0	0	2.0	203
2	2.0	49	2.0	136	1.05	315	2.1	315	2.0	49	2.0	185
3	2.0	88	2.0	265	1.01	697	2.01	697	2.0	88	2.0	353
4	2.0	114	2.01	324	1.13	614	2.26	812	2.0	114	2.02	438
5	2.0	95	2.01	397	1.08	631	2.49	1008	2.0	95	2.04	492
6	2.0	222	2.03	807	1.09	0	4.17	121	2.0	222	2.07	1029
7	2.0	0	2.47	72	1.0	587	2.62	1178	2.0	0	3.25	72
8	2.0	365	2.0	985	1.07	0	6.8	0	2.0	365	2.14	1350
9	2.02	0	3.05	0	1.0	500	3.52	1000	2.02	0	5.71	0
10	2.0	88	2.04	662	1.04	0	9.33	0	2.0	88	2.97	750
	-	102	-	385	-	389	-	567	-	102	-	487
Note: I denotes traffic impedance, F denotes traffic flow in the link.												

3.3. Results

In comparison with Table 2 and Table 3, the advance of utility of link can be found in Table 4 and Table 5, which reflects traveler's behavior efficiency. That is, not all travelers selecting the shortest path in the realistic condition. Besides, results of Table 4 and Table 5 also take congestion effect into consideration. This makes method results close to actual case. In Table 4 and Table 5, every traffic flow of link 1-2, 1-4, 4-5, 6-9 and 8-9 are larger than other links', which means links with higher traffic flow may be overloaded and those links will thus be defined as “critical links”. Critical links can be compared to bottleneck of network, and through capacity of network hinges largely on critical links. Hence it is of overriding significance to improve capacity of critical links.

4. Conclusion

Method based on Dial's algorithm has superiority, comparing with Kong's method, in approaching the actual case. Such a difference between two methods stems from shortcomings of AON assignment that is used in Kong's method.

The first is instability, different out puts may be derived significantly by insignificant change in input. As a result, it will be difficult to predict the precise estimates of link times, that is, the instability seriously hinders utility of model. The second season is that the AON assignment to some extent can not reflect actual behavior precisely. Although method introduced in Kong ameliorates AON assignment creatively by overcoming its defect of no capacity restraint, it is still unable to allow for realistic random variation of route selection among individual trip makers. While defection of capacity restraint in Dial's algorithm is not reflected due to its usage in the dynamic paradigm. The third one, due to all trips are assumed to use the shortest path according to AON assignment, total vehicle hours are biased. Thus planner may always overestimate the value of his design because AON measure is often used as a macro-evaluator for planner.

Differing from method of single static traffic assignment which loads traffic flow into network just for once, the article uses technique like “increment load” to divide time into several periods according to variation of traveling volume, and it takes capacity restraint and traffic impedance into account to make assignment results approach the realistic case as far as possible. For the property of hybrid method, when the time is more precisely divided, dynamic performance of hybrid method will approach pure dynamic one. What is more, the proposition of using Dial's algorithm to solve traffic assignment for dynamic paradigm significantly decreases compute cost, especially in route enumeration. Hence method proposed in the article is instrumental in structuring advanced traveler information systems and advanced traffic management systems.

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