

## Workspace Analysis for Parallel Robot

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*Received: 4 April 2013 /Accepted: 14 May 2013 /Published: 30 May 2013*

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**Abstract:** As a completely new-type of robot, the parallel robot possesses a lot of advantages that the serial robot does not, such as high rigidity, great load-carrying capacity, small error, high precision, small self-weight/load ratio, good dynamic behavior and easy control, hence its range is extended in using domain. In order to find workspace of parallel mechanism, the numerical boundary-searching algorithm based on the reverse solution of kinematics and limitation of link length has been introduced. This paper analyses position workspace, orientation workspace of parallel robot of the six degrees of freedom. The result shows: It is a main means to increase and decrease its workspace to change the length of branch of parallel mechanism; The radius of the movement platform has no effect on the size of workspace, but will change position of workspace.

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**Keywords:** Parallel robot, Workspace, Mechanism.

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### 1. Introduction

Parallel robot possess such advantages as the accurate orientation, great load-carrying capacity, easy real-time control, etc., its wide application in a great deal of fields, has already caused the great attention of scholars. The workspace of the robot is a job field of the robot-operating device, and it is one of the important indexes of weighing the performance of the robot. During the design of the parallel robot, the workspace is the important index too, and determines the whole size of the parallel robot. The workspace can be divided into accessible workspace and nimble workspace according to the position characteristic of operating device while working. Accessible workspace is an assembly to accessible whole points of some reference point on operation device and it doesn't consider the location of operation. Nimble workspace is an assembly to

accessible points from any direction of some reference point on operation device. When the reference point operating device is some point of nimble workspace, the operating device can turn around all lines through this point for a whole circle. The nimble workspace is a part of the accessible workspace, connect the organization in parallel to the space, movement platform can't rotate round some point for 360° generally because the structure restriction of space parallel robot, so the parallel robot lathe does not generally have a nimble workspace. Compared with the serial robot, conformation of the workspace of the parallel robot is too complicated; the reason is that the structure of parallel robot is complexity. This paper will set out from the reverse solution to parallel mechanism, and then study the workspace problem of parallel robot by utilizing the numerical boundary-searching algorithm based on the reverse solution of kinematics and limitation of the link length.

## 2. Setting-up of the System of Coordinates

In order to discuss conveniently the factors of impacting the workspace of parallel mechanism and the workspace of the parallel robot further, the system of coordinates shown in Fig. 1 is set up. Firstly it sets up the system of coordinates  $O_G - X_G Y_G Z_G$  fixed on stationary platform (note by abridging for  $\{G\}$ ) and the system of coordinates  $O_H - X_H Y_H Z_H$  fixed on movement platform (note by abridging for  $\{H\}$ ) separately, among them the coordinate origin points  $O_G$  and  $O_H$  lay on the center of the upper and lower platform separately, axle  $Z_G$  and  $Z_H$  are perpendicular to the upper and lower platform separately, the axle  $X_G$  is through the point  $B_1$ , the axle  $X_H$  is through the point  $P_1$ , the axle  $Y_G$  and  $Y_H$  are confirmed by the right principle separately. The location matrix of movement platform relative to stationary coordinates system  $\{G\}$  is expressed with  $T$ , have

$$T = \begin{bmatrix} A_m & Q_m \\ 0 & 1 \end{bmatrix} \quad (1)$$

Among them  $A_m$  is called the location matrix of movement platform relative to stationary coordinates system  $\{G\}$ , if  $A_m = I$ , then two platforms keep parallel, the change of matrix  $A_m$  is called location change;  $Q_m = (x_{O_H}, y_{O_H}, z_{O_H})^T$  is the coordinate of the origin point  $O_H$  of the system of coordinates  $\{H\}$  in the coordinate of the system of coordinates  $\{G\}$ , and it signifies the position of the movement platform, so the change of  $Q_m$  value is called position change.

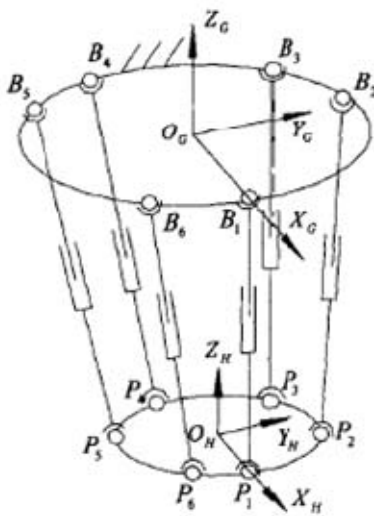


Fig. 1. The system of coordinates of parallel mechanism.

## 3. Factors of Impacting the Workspace of Parallel Mechanism

There are a lot of factors of impacting the workspace of parallel mechanism, the main factors among them are: the influence of pole length, the influence of the hinge point position, the influence of restrained in vice corner of sport and the influence of connecting rod interference, etc.

### 3.1. Influence of Pole Length

Position and posture of platform can be described with posture matrix  $A_m$  of movement platform of component relative to stationary platform and translation vector  $Q_m$  of two systems of coordinates, while giving the location of the platform definitely, the length and direction of every connecting rod can be expressed with the vector  $\vec{l}_i$  ( $i=1,2,\dots,6$ )

$$\vec{l}_i = B_i P_i = A_m \vec{P}_i + Q_m - \vec{B}_i \quad (2)$$

Among them  $\vec{P}_i = \{x_{P_i}, y_{P_i}, 0\}^T$  is a vector of hinge point  $P_i$  ( $i=1,2,\dots,6$ ) of the movement platform relative to system of coordinates  $\{H\}$ ,  $\vec{B}_i = \{x_{B_i}, y_{B_i}, 0\}^T$  is a vector of hinge point  $B_i$  ( $i=1,2,\dots,6$ ) of the stationary platform relative to system of coordinates  $\{G\}$ .

The length  $L_i$  ( $i=1,2,\dots,6$ ) of rod is the modulus of vector  $\vec{l}_i$

$$L_i = \left| \vec{l}_i \right| = \left| A_m \vec{P}_i + Q_m - \vec{B}_i \right| \quad (3)$$

The variety of length of rod has a certain limit, and it should meet the following condition.

$$L_{\min} \leq L_i \leq L_{\max} \quad (4)$$

Among them,  $L_{\min}$  and  $L_{\max}$  express the minimum length and the maximal length of connecting rod, the expansion & contraction value of rod can be written as  $\Delta L = L_{\max} - L_{\min}$ . Obviously, when the length of rod is in the limited position, the platform is in the limited position too. The reference point given on movement platform reaches the border of workspace too.

### 3.2. Influence of the Hinge Point Position

Because the structure of parallel mechanism has nothing in common with each other, there are all kinds of distribution of the hinge point positions of the upper and lower platforms. We can know from Eq.3 that the length of every rod has direct influence on coordinate value of hinge point  $\vec{B}_i$  and  $\vec{P}_i$ , when the hinge point coordinate changes, the length of every rod will change too. So when the limitation length of rod and  $L_{\min}$  and  $L_{\max}$  are certain, every positions of hinge point have various degree influence on workspace.

### 3.3. Influence of Restrained in Vice Corner of Sport

Generally speaking, joint linking the upper and lower platform of parallel mechanism with every branch rod is sphere fit or Hooke joint. Whether the joint is sphere fit or Hooke joint, corner ranges of their real structure have certain restriction.

### 3.4. Influence of Connecting Rod Interference

Generally, the connecting rod has certain size, and the movement of platforms makes the connecting rods interfere each other, which must be avoided. In order to discuss conveniently, supposed that the connecting rod is cylindrical, and its diameter is  $D$ . Discussing the question of interference between the connecting rods, is judging the shortest distance  $d$  between the two space lines that is greater than the diameter or not. If the condition  $d \geq D$  is unsatisfied, then the two connecting rods will be interfered, which will destroy force and balanced condition of the whole mechanism, even can damage the whole mechanism when being serious. Judging the shortest distance between the two space lines is a very complicated problem, which is divided into different situations to judge.

## 4. Numerical Limited Boundary-searching Algorithm of Parallel Mechanism's Workspace

The Eq. 4 has expressed the length restraint conditions of parallel mechanism, and  $L_{\min}$  and  $L_{\max}$  in Eq. 4 are the length of the branch rod under the shortest and longest journey respectively. So the length of the group rod corresponding to position matrix of every point can be got by using the reverse solution to kinematics of parallel mechanism, so long as it accords with Eq. 4, the position point can be judged to be in workspace,

otherwise outside workspace. If all border points can be hunted out and consist of a surface, then these points between the surface can consist of the workspace of parallel mechanism, which is the principle of the numerical limited boundary-searching algorithm of parallel mechanism's workspace.

Fig. 2 is sketch map of numerical limited boundary-searching algorithm of parallel mechanism's workspace. Firstly along axle  $z$  making several sections that can divide workspace into  $K$  parts, then limited-border curves in all sections are hunted out one by one, so it is very apt to confirm workspaces of parallel mechanism according to these curves. As for  $m$  axle sectional border curve, firstly a foot-path  $r\_step$  along radial direction further forward in section can be got, then search from up to down along  $z$  axle direction until searching out the first limited-border point, then the second limited-border point, and write down them. Using C++Builder language of programming in this paper can get workspace of various kinds of situations. Using cross section perpendicular to  $z$  axle can get the section certainly, and then getting limit border curve of cross section gets workspace. In the whole course of searching, the little step length and the more accurate, but data amounts leap.

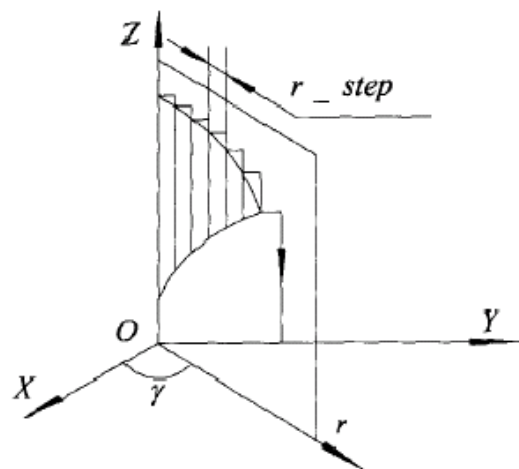


Fig. 2. The sketch map of the numerical limited boundary-searching algorithm of parallel mechanism's workspace.

## 5. Workspace Analysis of the Parallel Robot

### 5.1. Position Workspace

To a group of parallel mechanism with regular mechanism parameter, when  $A_m = I$ , namely stationary platform parallel to movement platform all the time, the workspace of parallel mechanism got is the position workspace. To the robot illustrated in Fig.

1, when the value of  $L_{\min} = 730\text{ mm}$ , and the value of  $L_{\max} = 1130\text{ mm}$ , the position workspace tried to get in the  $X - Z$  section is shown as curve 3 in Fig. 3. When the length of rod of mechanism is changed, Fig. 3 is the position workspace tried to get in the  $X - Z$  section of three groups of length of rod shown in Table 1. We can see from the Fig. 3: The change of mechanism's branch's length has greater influence on the change of the workspace of mechanism. The conclusion can be drawn from the computer emulation and summarizing related documents: It is a main means to increase and decrease the workspace to change the length of branch of parallel mechanism, this conclusion has directive significance to the confirmation of parameter of the robot mechanism.

While changing the radius of the platform, with the change of the hinge point position on the workspace of parallel robot too. Fig. 4 is the position workspace in the  $X - Z$  section of three groups of radius of rod shown in Table 2. The conclusion can be drawn from the Fig. 4: The radius of the activity platform has no effect on the size of the workspace, but will change the position of the workspace.

**Table 1.** Mechanical parameters of different length of branch [mm].

No.	Radius of stationary platform	Radius of movement platform	Scope of expansion & contraction value of rod length
1.	800	200	730-930
2.	800	200	730-1030
3.	800	200	730-1130

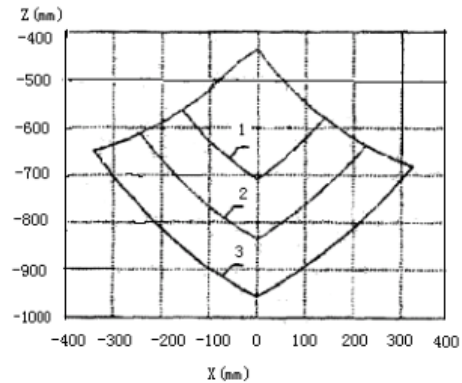
**Table 2.** Mechanical parameters of different radius of platform [mm].

No.	Radius of stationary platform	Radius of movement platform	Scope of expansion & contraction value of rod length
1.	800	150	730-1130
2.	800	200	730-1130
3.	800	250	730-1130

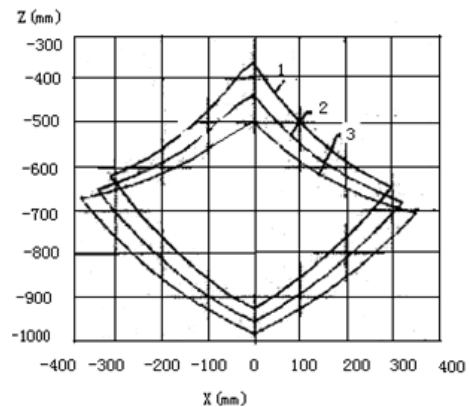
### 5.2. Orientation Workspace

when  $A_m \neq I$ , the workspace of parallel mechanism expresses the orientation workspace. The definition of the orientation workspace is when the center  $O_H$  of movement platform is in a certain position of regular system of coordinates  $O_G - X_G Y_G Z_G$ , normal vector  $n$  of movement platform becomes the angle  $\theta$  relative to

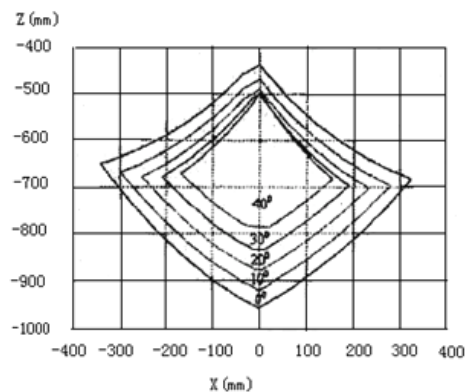
stationary coordinate parallel to axle  $Z'$  (its starting point is through  $P$  point), and normal line  $n'$  revolves round axle  $Z'$  one circle. If the rod length of dispersed points got by reverse solution of kinematics drop on scalable range, this point  $P$  is within the orientation space  $\theta$ , otherwise is outside. Namely position matrix  $Q_m$  relative to each point is constant, and its orientation is changed during the dispersed course of winding a certain angle  $\theta$  with axle  $Z'$ . Fig. 5 is namely the orientation workspace under different angles of the third group data listed of Table 1.



**Fig. 3.** Position workspace of parallel mechanism and impact of the length of rod on it.



**Fig. 4.** Position workspace of parallel mechanism and impact of movement platforms on it.



**Fig. 5.** Orientation workspace of parallel mechanism.



## 6. Conclusions

This paper has pointed out some factors that influence workspace of the parallel robot, and introduced the numerical boundary-searching algorithm of workspace on the basis of the reverse solution of kinematics of parallel mechanism especially. It has a lot of characteristic, such as simple algorithms, clearing physics meaning and easy programming. These relevant conclusions getting from this algorithm have practical value to the design of the parallel robot. The result shows: It is a main means to increase and decrease the workspace to change the length of branch of parallel mechanism; The radius of the activity platform has no effect on the size of the workspace, but will change the position of the workspace.

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