

Experimental Investigation and Comparison of Nonlinear Kalman Filters

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Abstract: One of the most important problems when designing controller is how to deal with all kinds of uncertainties, which, along with the high nonlinearities of most real systems, makes it difficult to guarantee the desired closed loop performance. Recently, nonlinear Kalman-class filter has been extensively researched and several well-known algorithms, including Extended Kalman Filter (EKF), Unscented Kalman Filter (UKF) and Adaptive Unscented Kalman Filter (AUKF), have been reported to be applicable in some cases. In this paper, on the basis of the moving target cooperative observation problem, performances of these nonlinear filter algorithms are analyzed and tested on a multi-flying-robot testbed, and the experimental results are listed to show the advantages and disadvantages of them. *Copyright © 2013 IFSA.*

Keywords: Nonlinear Kalman Filter, Moving target observation, Flying robots.

1. Introduction

Since Kalman Filter was proposed by Kalman R. E. in 1960 [1], linear Kalman filters have been extensively utilized in many real systems. However, when nonlinearities are unavoidable, the performance of linear Kalman filter can only be guaranteed locally, which make many researchers turn their attentions to the topic of nonlinear Kalman filter.

Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF) are two typical nonlinear Kalman-class filters. Among them, EKF is the earliest and most often referred-to one. Its basic idea is to linearize the nonlinear term using Taylor series expansion, and then linear Kalman filter algorithm is directly used with respect to the linearized system model [2]. While in UKF, a set of samples are used to approach the nonlinear system model. It has been proved that UKF algorithm is equivalent to using the

second order Taylor expansion to replace the nonlinear terms, thus it is of much better performance compared to EKF algorithm. It should be noted that the performance of both EKF and UKF rely much on accurate prior noise distribution [3], which is difficult to be obtained in most real applications. Therefore, adaptive Kalman-class filtering algorithms which are able to regulate the noise covariance online have been discussed [4-7]. For example, Jiang [7] proposed an MIT based adaptive UKF (MIT-AUKF), in which the noise covariance is updated adaptively to minimize a cost function and the updated covariance is then fed back into normal UKF to compensate the inaccurate prior information.

Theoretical and experimental comparison study on these three kinds of nonlinear Kalman-class filter, in this paper, are conducted with respect to the problem of moving target Motion Estimation using Mobile Robot (MEMR). MEMR is a typical application and

key problem in the field of robotics [8-9]. Besides, the high nonlinearity in system and measurement model, the uncertainty of moving targets and unpredictable time-varying noise and disturbance, make it fit to test the performance of nonlinear Kalman-class filters.

2. Problem Description

Moving target observation is a typical nonlinear estimation problem can be modeled as follows,

$$\begin{aligned} x_k &= f(x_{k-1}, w_{k-1}) \\ y_k &= h(x_k, n_k) \end{aligned} \quad (1)$$

where $x_k \in R^n$ are states of the moving target (or the relative states between moving target and robot); w_k and n_k are process noise and measurement noise; $y_k \in R^p$ are measurements. The aim of "moving target observation" is to design an algorithm to estimate the real-time system states x_k based on known y_k as accurately as possible.

3. Nonlinear Kalman Filtering Algorithms

Kalman-class filter is usually composed of two steps, called prediction step (or time update) and correction step (or measurement update). These two steps are conducted iteratively and form a closed-loop as in Fig. 1.

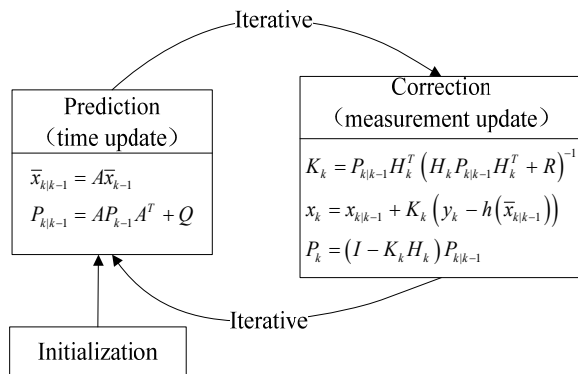


Fig. 1. Basic idea of Kalman filter.

In prediction step, the current states are predicted using the states in last time step and their statistical covariance using system model. While in correction step, based on the measurement, the Kalman gain K_k is updated according to measurement models and the pre-known measurement noise covariance.

EKF is a linear approximation method which is based on Taylor expansion with only linear terms being used. The system can be described by Eq. (2).

$$\begin{aligned} x_k &= A(x_{k-1})x_{k-1} + w_{k-1}, \\ y_k &= H(x_k)x_k + n_k \end{aligned} \quad (2)$$

where

$$A(x_{k-1}) = \left. \frac{\partial f}{\partial x} \right|_{x=x_{k-1}}, H(x_k) = \left. \frac{\partial h}{\partial x} \right|_{x=x_k}$$

It can be seen in Eq. (2) that EKF reserves first order Taylor expansion of nonlinear models to convert it to linear. That leads to: 1) Local linearization may produce prediction errors, and thus deteriorate the estimation accuracy. 2) Performance of EKF depends much on the noise covariance, which is difficult to be obtained before-hand.

Different from the EKF, UKF uses similar distribution, called Unscented Transfer (UT), to approach the suboptimal filtering performance. The principle of UT is to take the distribution of sample points as the approached distribution of the random variables. According to the prior estimated states means and variances, a set of discrete sample points are generated as sigma points. These points are transferred by the nonlinear models so that the posterior means and variances can be obtained iteratively.

There are analysis showing that UKF has an equal calculation complexity to EKF and equal precision to two-stage EKF [7]. Therefore UKF is considered more suitable for system of stronger nonlinearity. However, the performance of UKF still depends on the initial statistical characteristics of noise, which should be estimated accurately to avoid divergence.

Aiming at the problem that noise covariance is difficult to obtain, adaptive Kalman-class filters have been researched. For example, in preceding study, we proposed an MIT-based adaptive UKF (MIT-AUKF) [7]. In MIT-AUKF algorithm, taking the deviation of the actual and estimated value of innovation as the adaptive index function J_r as Eq. (4), an MIT adaptive rule, i.e., the parameters are updated in the negative gradient direction of the criterion function, is used to regulate the noise covariance online as Eq. (5). Then in each iterative step, the renewal noise variances are substituted into the new UKF so as to improve the estimation performance while the noise is unknown or time-variant.

$$J_r = \text{tr}(\text{diag}(\Delta S_k)^2) = \text{tr}(\text{diag}(S_k - S'_k)^2), \quad (4)$$

$$q_k^m = q_{k-1}^m - \eta_k \frac{\partial J_k}{\partial q_k^m} T_0, \quad (5)$$

where $\text{tr}(\ast)$ is the trace of the matrix(\ast), $\text{diag}(\ast)$ obtains a diagonal matrix, S_k is the mean of innovation in a moving N -size window, q_k^m is the m_{th} diagonal element of Q^n at time k , T_0 is sample time, η_k is the tuning rate of converging, and

$y_k - y'_{k|k-1}$ is the estimation of S'_k , which is defined as follows,

$$S_k = \frac{1}{N} \sum_{i=k-N+1}^k (y_k - y'_{k|k-1})(y_k - y'_{k|k-1})^T,$$

$$S'_k = \sum_{i=0}^{2n} w_i^c (r_{i,k|k-1} - y'_{k|k-1})(r_{i,k|k-1} - y'_{k|k-1})^T + Q^n$$

Reference [7] has a detailed introduction about MIT-AUKF algorithm.

4. Experimental Platform

In this paper, a kind of indoor multi-flying-robots testbed and an extended vision based measurement system on it are used to test performance of the three nonlinear Kalman filters.

4.1. Introduction of the Experimental Platform

The whole multi-flying-robot testbed structure is as Fig. 2. It contains a vertical main shaft and 3 horizontal mechanic arms. Each arm has two passive joints driving the yaw and pitch. A flying robot is installed on the end of each arm. In order to avoid robots crashing, the pitch freedom is limited to rang in $(-15^\circ, 15^\circ)$. On the other end of the arms, adjustable balance weight stacks are used for canceling out the extra load generated by flying robots taking off. Besides, encoders are assembled on each rotary joint to get position information [11-12].

In our experiments, two arms are used, as shown in Fig. 2. Arm-1 carried an LED with 850 nm wavelength as the moving target to be observed. While on Arm-2, an industrial camera is installed vertically to observe the optical target. In order to simplify the image processing, we propose a target-enhance solution to identify the target. In brief, an 850 nm narrow band filter was installed in front of

the camera lens so that the complex background on the images can be removed a lot, and there would only be the target light source on the images as shown in Fig. 3. In that way the focus can be concentrated on the realization and efficiency of the experiments.

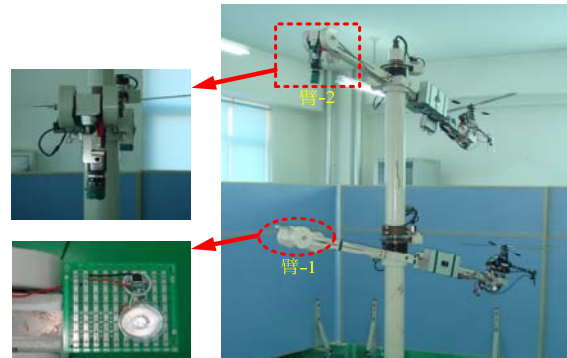


Fig. 2. Multiple Rotated Flying Robots Testbed.

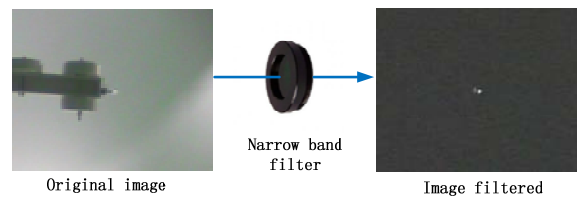


Fig. 3. Effect of Optical Filter.

4.2. Observation Model of Dynamic Target

In this sub-section, the moving target observation model is given out. Firstly, two main coordinates are defined as Fig. 4: 1) Image pixel coordinate system uOv : the left up corner of the image is the origin, u and v are the row and the column of the image; 2) World coordinate system X^w : the bottom center of testbed is origin, z_w is the upward direction of the main shaft, x_w is along the arm when $\beta=0$.

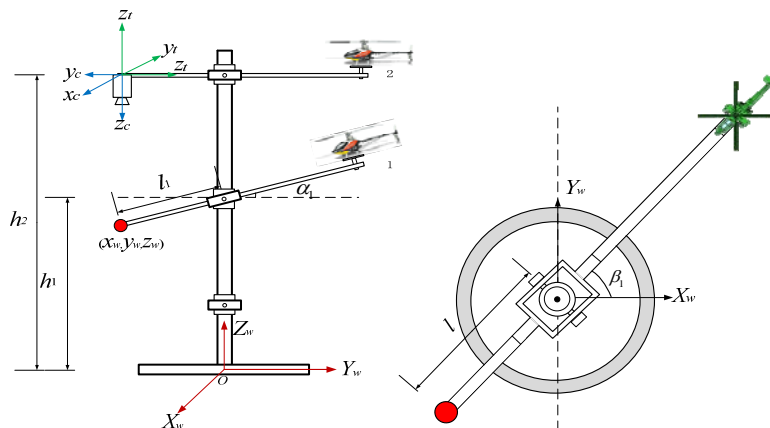


Fig. 4. Definition of Ordinate on the Platform.

In this platform, the position of each robot is denoted uniquely as the angles of α and β which can be measured using the encoders. It means that (α_1, β_1) needs to be estimated. Furthermore, the output of the system is (u, v) , the pixels position on the image of the objective source. Define Q^w as the variance of system state noise w_k , and the system and measurement equation are as follows,

$$\begin{cases} \begin{pmatrix} \beta_{1,k+1} \\ \alpha_{1,k+1} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \beta_{1,k} \\ \alpha_{1,k} \end{pmatrix} + w_k \\ \begin{cases} u = \frac{r_{11}x_w + r_{12}y_w + r_{13}z_w + T_x}{r_{31}x_w + r_{32}y_w + r_{33}z_w + T_z} + u_0 \\ v = \frac{r_{21}x_w + r_{22}y_w + r_{23}z_w + T_x}{r_{31}x_w + r_{32}y_w + r_{33}z_w + T_z} + v_0 \end{cases} \end{cases} \quad (6)$$

where u_0, v_0, f_x, f_y are calibration parameters of camera, r_{ij} and T_i are factors of R and T , as Eq. (7). And (x_w, y_w, z_w) and (α_1, β_1) satisfies Eq. (8).

$$\begin{aligned} R &= \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} = \begin{bmatrix} c\alpha_2 c\beta_2 & c\alpha_2 s\beta_2 & s\alpha_2 \\ -s\beta_2 & c\beta_2 & 0 \\ -s\alpha_2 c\beta_2 & -s\alpha_2 s\beta_2 & c\alpha_2 \end{bmatrix} \\ T &= \begin{pmatrix} (l_2 - h_2 s\alpha_2) c\varphi c\theta - h_2 c\alpha_2 c\varphi s\theta \\ (l_2 - h_2 s\alpha_2) s\varphi c\theta - h_2 c\alpha_2 s\varphi s\theta \\ -(l_2 - h_2 s\alpha_2) s\theta - h_2 c\alpha_2 c\theta \end{pmatrix} \end{aligned} \quad (7)$$

$$x_w = -l_1 c\alpha_1 c\beta_1, y_w = -l_1 c\alpha_1 s\beta_1, z_w = h_1 - l_1 s\alpha_1 \quad (8)$$

where $c\alpha$ indicates $\cos(\alpha)$, $s\alpha$ indicates $\sin(\alpha)$, l_i and h_i are the length and height of manipulator i .

5. Experimental Results and Analysis

In this section, EKF, UKF and AUKF are applied to the moving target observation system to estimate the target's position. The estimated results are then compared with the real position measured by the encoders bonded on the manipulator to evaluate the estimation precision.

Experimental parameters are listed out in Table 1. The target is controlled to move along the trajectory as Eq. (9). The purpose to design such a trajectory is to fully reflect the nonlinearity so that the performances of three nonlinear filters could be best tested.

Table 1. Parameters of platform and vision system.

l_1	l_2	h_1	h_2	u_0	v_0	f_x	f_y
600	700	1275	2185	345	315	618	674

$$\begin{cases} \alpha_1 = 0 \\ \beta_1 = 0 \end{cases} (t \leq t_s), \begin{cases} \alpha_1 = 0 \\ \beta_1 = \varpi(t - t_s) \end{cases} (t > t_s), \quad (9)$$

5.1. Analysis of Noise Characteristics

Noise distribution matters much to performances of filters (even for linear KFs, its optimality can be realized only if the noise characteristics are known). So we firstly analyze the measurement noise statistical characteristics.

The measurement noise (n_u, n_v) can be defined as,

$$n_u = u - u_s, n_v = v - v_s, \quad (10)$$

where (u, v) are the measurement; (u_s, v_s) are the real values based on encoders output.

The noise distribution is as Fig. 5(a) and Fig. 5(b). Furthermore, Curve Fitting Tool Box in Matlab is used to obtain the noise PDF as the red line in Fig. 5(b) (95 % degree of confidence). It can be seen from Fig. 5(b) that n_v can be modeled by a standard Gaussian noise, whose PDF is described by Eq. (11) while n_u is approached by a triple-mixed Gaussian noise, whose PDF is Eq. (12). All the parameters of $p_v(x)$ and $p_u(x)$ are shown in Table 2.

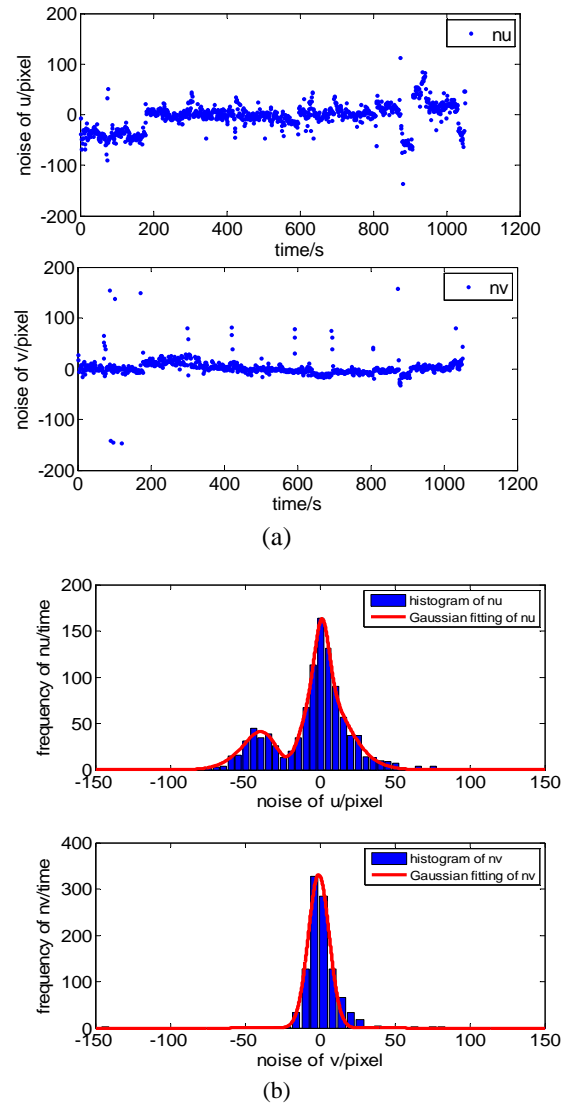


Fig. 5. (a) Noise in time domain; (b) distribution histogram.

Table 2. Parameters for the fitting function.

a_1	70.6	b_1	1.419	c_1	6.475
a_2	-106.9	b_2	-19.75	c_2	15.28
a_3	131	b_3	-12.32	c_3	31.32
a_v	311.5	b_v	-0.8402	c_v	9.85

$$p_v(x) = a_v e^{-(x-b_v)/c_v)^2} \quad (11)$$

$$p_u(x) = a_1 e^{-(x-b_1/c_1)^2} + a_2 e^{-(x-b_2/c_2)^2} + a_3 e^{-(x-b_3/c_3)^2} \quad (12)$$

According to the analysis above, the measurement noise distribution of u is clearly not standard Gaussian. In the following sub-sections, we will test the performance of the three nonlinear Kalman filters (EKF, KF, MIT-AUKF) when the noise is as Eq. (11) and (12).

Supposing the noises of n_u and n_v are uncorrelated, that is, the off-diagonal elements of the covariance matrix are all zeros, and q_1 and q_2 , the diagonal elements of measurement noise covariance matrix Q^n , could be approached using the following equations,

$$q_1 = \sum_{k=1}^n (n_k^u - n_0^u)^2 / (n-1) = 495$$

$$q_2 = \sum_{k=1}^n (n_k^v - n_0^v)^2 / (n-1) = 57 \quad (13)$$

5.2 Experimental Results

The parameters of EKF algorithm can be designed as

$$(\alpha, \beta)^T = (0, 0.8727)^T$$

$$Q^w = \text{diag}([0.01, 0.1]^T), \quad (14)$$

The estimation results using EKF are given in Fig. 6 (a). From Fig. 6(a), it can be seen that estimation of β using EKF is bias, and estimation of α is even not convergent. The main reasons on the phenomenon can be explained from the following two aspects: Firstly, because of local linearization, EKF uses only the first order of Taylor expansion, which results in possible prediction errors. Furthermore, the noises distribution in the experiments are not Gaussian distributed, which destroy the basic assumption of EKF.

The parameter of UKF is as follows. The estimation results are shown in Fig. 6(b). From Fig. 6(b), we can conclude that the estimation performance of UKF is clearly better than that of EKF. However, the estimation result of α is still bias. This is mainly because the noise distribution in the experiment is complicated, while the UKF algorithm

has no adaptive ability with respect to the possible varying of the noise covariance.

The estimation results of MIT-AUKF are shown in Fig. 6(c). From Fig. 6(c), it can be seen that for both angles, the estimated values are unbiased, and high precision of the estimation can be ensured. This presents the strong adaptability of MIT-AUKF to noise.

5.3 Comparison of EKF, UKF and AUKF

In order to quantitatively compare the estimation performances of these three different algorithms, the following evaluating index are defined,

$$\mu = \sum_{i=1}^n (\bar{x}_i - x_i)$$

$$\sigma = \sqrt{\sum_{i=1}^n (\bar{x}_i - x_i)^2 / (n-1)} \quad (15)$$

$$t = t_x / t_{EKF} (t_x = t_{EKF}, t_{UKF}, t_{AUKF})$$

where μ, σ is the mean and square of estimation error; t is the average execution time for each iterative step. The comparison results are shown in Fig. 6(d) and Table 3.

From Fig. 7(d), we can conclude that the estimation errors of AUKF are smaller than that of UKF, and both of them are better than EKF. Table 3 gives out the similar results. About the executive time, EKF needs the least time because it needs actually run a linear Kalman filter program. Determined by the sampling strategy, although UKF does not need to calculate the complex Jacobian matrices, but the computation of sigma points and the unscented transfer will increase the processing workload, thus UKF runs a little more slowly than EKF algorithm. Finally, because of the additional adaptive computation, AUKF takes more time than both UKF and EKF.

Table 3. Statistical performance comparison.

Index	μ_α	μ_β	σ_α	σ_β	t_n
EKF	0.1373	2.8909	0.1068	1.6717	1.000
UKF	0.0017	2.8059	0.0085	1.6689	3.460
AUKF	0.0003	2.7074	0.0033	1.6679	15.75

6. Conclusions

In this paper, taking the moving target cooperative observation problem as an example, performances of three kind of nonlinear Kalman filters, including Extended Kalman Filter (EKF), Unscented Kalman Filter (UKF), and MIT based Unscented Kalman Filter (MIT-AUKF) are analyzed, compared and tested on a multi-flying-robot testbed, which has a typical non-Gaussian measurement noise.

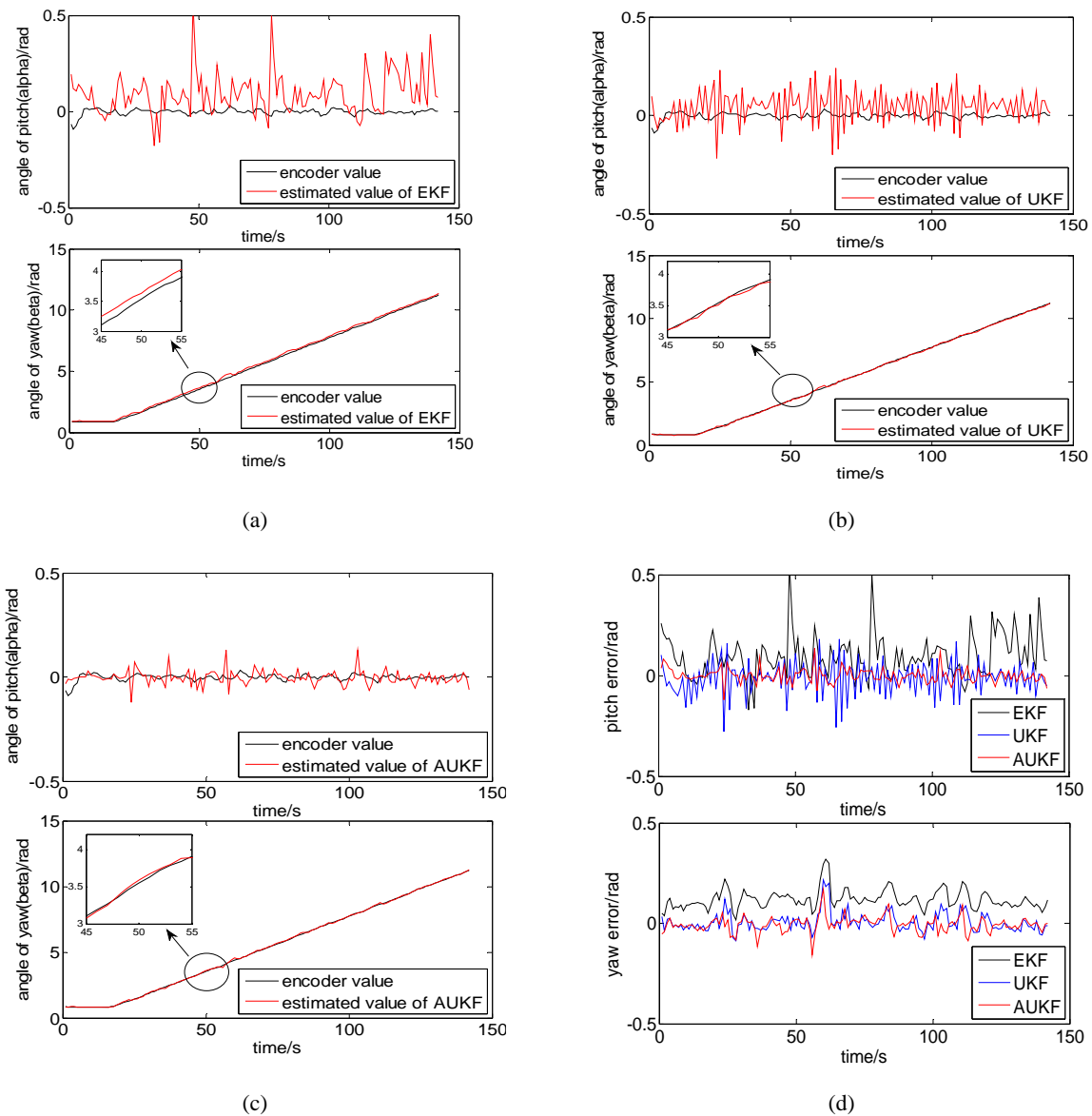


Fig. 6. (a) Estimation by EKF; (b) Estimation by UKF; (c) Estimation by AUKF; (d) Error Comparison.

The experimental results show that: 1) The MIT-AUKF presents the best estimation performance. This is mainly because it can adaptively regulate the noise covariance online based on the realities. The estimation performance of UKF is better than that of EKF because the latter use only first-Taylor-expansion to approach the nonlinear term in system model. 2) In the aspect to executive time, the EKF algorithm is better than both UKF and MIT-AUKF. Although MIT-AUKF presents good estimation performance, it takes most time because of the computational burden from both unscented transfer and adaptive scheme.

References

[1]. R. E. Kalman, A new approach to linear filtering and prediction problems, *Transactions of the*

ASME--Journal of Basic Engineering, Vol. 82, Issue 1, 1960, pp. 35-45.

- [2]. Ito K., Xiong K., Gaussian filters for nonlinear filtering problems, *IEEE Transactions on Automatic Control*, Vol. 5, Issue 5, 2000, pp. 910-927.
- [3]. S. J. Julier, J. K. Uhlmann, H. F. Durrant-Whyte, A new approach for the nonlinear transformation of means and covariance in filters and estimators, *IEEE Transaction on Automatic Control*, Vol. 45, Issue 3, 2000, pp. 477-482.
- [4]. R. Rahbari, B. W. Leach, J. Dillon, C. W. De Silva, Adaptive tuning of a Kalman filter using the fuzzy integral for an intelligent navigation system, *Proceedings of the 2002 IEEE International Symposium on Intelligent Control*, Vancouver, Canada, 27-30 October 2002, pp. 252-257.
- [5]. D. Lee, K. T. Alfriend, Adaptive sigma point filtering for state and parameter estimation, *AIAA/AAS Astrodynamics Specialist Conference and Exhibit*, RI, USA, 16-19 August 2004, pp. 1-20.


- [6]. D. Loebis, R. Sutton, J. Chudley, W. Naeem, Adaptive tuning of a Kalman filter via fuzzy logic for an intelligent AUV navigation system, *Control Engineering Practice*, Vol. 12, Issue 12, 2004, pp. 1531-1539.
- [7]. Z. Jiang, Q. Song, Y. Q. He, J. D. Han, A novel adaptive unscented Kalman filter for nonlinear estimation, in *Proceedings of the IEEE Conference on Decision and Control*, New Orleans, LA, USA, 12-14 Dec. 2007, pp. 4293-4298.
- [8]. J. C. Wang, T. M. Wang, Y. Yang, L. Hu, Robot hand-Eye calibration using unscented Kalman, *Robot*, Vol. 33, Issue 5, 2011, pp. 621-627.
- [9]. M. Wu, J. Ma, J. W. Tian, J. Liu, An adaptive UKF algorithm and its application to geomagnetic navigation, *Information and Control*, Vol. 40, Issue 4, 2011, pp. 558-562.
- [10]. N. J. Gordon, D. J. Salmond, A. F. M. Smith, A novel approach to nonlinear/non-Gaussian Bayesian state estimation, *Radar and Signal Processing, IEE Proceedings F*, Vol. 140, Issue 2, 1993, pp. 107-113.
- [11]. F. Gu, Z. Wang, Y. Q. He, J. H. Han, Y. C. Wang, Theoretical and experimental study of uncertain set based moving target localization using multiple robots, *IEEE International Conference on Robotics and Biomimetics*, Karon Beach, Phuket, 7-11 Dec. 2011, pp. 1646-1651.
- [12]. F. Gu, Z. Wang, Y. Q. He, J. D. Han, Y. C. Wang, Experimental study of vision sensor based multiple robots active cooperative observation using multi-RFRs testbed, *IEEE International Conference on Robotics and Biomimetics*, Karon Beach, Phuket, 7-11 Dec. 2011, pp. 2417-2418.

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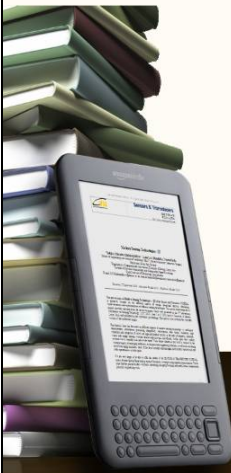
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