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Efficient Routing in Wireless Sensor Networks with Multiple Sessions

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Abstract: Wireless Sensor Networks (WSNs) are subject to node failures because of limited energy and link unreliability which makes the design of routing protocols in such networks a challenging task. The multipath routing scheme is an optimal alternative to address this problem which splits the traffic across multiple paths instead of routing all the traffic along a single path. However, using more paths introduces more contentions which degrade energy efficiency. The problem becomes even more difficult in the scenario of multiple sessions since different source-destination pairs may pass the same link which makes the flow distribution of each link uncertain. Our goal is to minimize the energy cost and provide the robust transmission by choosing the optimal paths. We first study the problem from a theoretical standpoint by mapping it to the multi-commodity network design problem. Since it is hard to build a global addressing scheme due to the great number of sensor nodes, we propose a Distributed Energy Efficient Routing protocol (D2ER). In D2ER, we employ the transportation method which can optimize the flow distribution with minimal energy consumption. Simulation results demonstrate that our optimal algorithm can save energy drastically. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Energy efficient, Wireless sensor networks, Routing, Multi-commodity, Transportation method.

1. Introduction

WSNs have been used for a wide range of applications such as military surveillance, disaster management and scientific exploration [1]. In WSNs, unpredictable nature of the wireless environment (e.g., channel fading or obstructions) usually leads to path failures and data loss. Empirical result from Berkeley [2] shows that the average packet delivery ratio reduces 5 %-10 % per link in sensor networks. Thus, in traditional single path routings, a source node prefers to select a node with high link quality as its own successor. As shown in Fig. 1, the source node S1 has two links to forward data to the destination D1. The weight of each link is the delivery ratio. Obviously, the source selects the link with higher delivery ratio for reliability. However, the single link makes the path delicate and the excessive energy cost of the nodes with high link quality drops the network lifetime. Recently, multipath routing approaches are widely utilized to address this problem. Different to single path routing methods, multi-path routing approaches split the traffic across multiple paths instead of routing all the traffic along a single path, which makes transmissions more resilient to failures. Moreover, the consumed energy spreads evenly across the nodes within the network, potentially resulting in longer lifetimes. However, the problem becomes difficult when multiple source nodes transmit simultaneously since the flow distribution among all the nodes is uncertain. For example, as shown in Fig. 2, source nodes S1 and S2 have the highest delivery ratio on links to the same relay node 2. To avoid the congestion on node 2, both source nodes should transmit part of packets on the links to node 1 and node 3 respectively. In addition, sensor nodes power is supplied by batteries which are subject to a critical constraint for different applications. Using multipath to sending packets is proved inducing more energy cost since more paths introduces more contentions [3, 4]. Therefore, how to design energy efficient multipath routing protocols to satisfy the performance requirements of multiple source-destination pairs is a crucial issue.

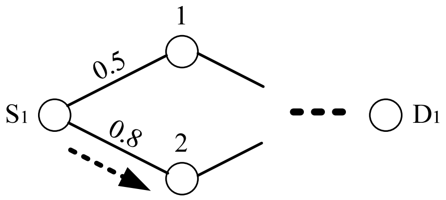

Fig. 1. The single path.

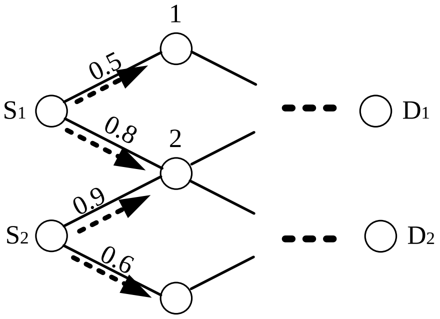

Fig. 2. The multi-path.

In this paper, we consider both reliability and energy constraints in multipath routing. Here, reliability is defined as the packet delivery ratio. Our goal is to find ways for energy efficient route setup and reliable relaying of data from the sensor nodes to the sink so that the lifetime of the network is maximized. To save the energy cost, the links with high delivery ratio are chosen as the forwarding links. The main contributions of this paper include:

- In order to minimize the energy cost and provide the robust transmission, we first study the problem from a theoretical standpoint, by mapping it to the multi-commodity network design problem.
- Since it is hard to build a global addressing scheme for the deployment of a great number of sensor nodes, we propose a decentralized energy efficient routing protocol (D2ER).
- Finally, we give an optimal solution of D2ER routing problem by employing a transportation method.

2. Related Work

Energy efficient routing has always been a hot research topic in wireless networks [5-7]. Some researchers propose energy aware routing schemes by using the residual battery power as a routing metric [8–10]. In [8], the scheme routes data through a path whose nodes have the largest residual energy. An established path is used until a path with more residual energy is discovered. Thus, nodes will not completely exhaust their energy by using different paths, so that the lifetime of the whole system may be prolonged. However, they assume that each link is totally reliable without considering retransmission when computing energy efficient paths. To achieve better energy efficiency in realistic scenarios, the metrics with the capability of handling packet losses in the wireless environment should be considered. Banerjee et al. [9] explores the impact of unreliable links on energy efficient routing and solves the problem of finding minimum energy paths hop-by-hop. In this approach, each node retransmits lost packets as and when necessary. In [10], the authors solve the problem of computing minimum energy paths for reliable transmission in the end-to-end retransmission model where none of the links guarantees any reliability. However, these works mentioned above only consider the scenario of one source-destination pair.

A multipath routing scheme that finds several disjoint paths is presented in [11]. This protocol tends to improve the reliability to node failures. In their work, there are two kinds of multipath schemes are proposed: the classical node disjoint multipath, and a braided multipath that consists of partially disjoint alternate paths. The objective using these braided paths is to keep the overhead of maintaining the multiple paths low. In this proposal, source nodes or intermediate nodes select one path from the available multiple paths to transmit the data to sink based on the quality of links (e.g. delay, throughput). By observing the variations on the network conditions, a node may change its primary path to another one. However, this scheme leads to a drastically increasing energy cost because of frequently switching to or establishing a new primary path in case of unreliable network environments. In addition, in case of node failure, the protocol lacks of an efficient retransmission control mechanism.

MCMP [3] is another multipath routing protocol which provides soft-QoS support in terms of reliability and delay. The end-to-end soft-QoS problem is formulated as a probabilistic programming and then is converted to a deterministic linear programming using an approximation technique. To provide reliability, during the route discovery process, each node chooses one or few of its one-hop neighboring nodes which additively provide the desired reliability along the path towards the sink node. Hence, source node and all the intermediate nodes should forward multiple copies of each data packet over several paths. In [12], the authors propose a multi-path routing to provide QoS routing using load balancing. However, in these algorithms, the imposed data transmission redundancy and the contentions among paths induce more energy cost which is ignored. Furthermore, the proposed protocol only considers the scenario of single source-destination pair. In this paper, we design an energy efficient multipath routing protocol in a multiple source-destination scenario considering the factor of energy consumption.

3. System Model

3.1. Channel Model

Suppose all nodes use the same power for transmission, and the power spectral density from the transmitter is P_t. A widely used model [13] for power propagation gain is

$$g_{ij} = \mu \cdot d_{ij}^{-n}, \tag{1}$$

where n is the path loss factor, μ is the antenna related constant, and d_{ij} is the distance between nodes i and j.

According to Shannon-Hartley theorem, if node i sends data to node j on link i, j with band B, the capacity of link i, j is

$$C_{ij} = B\log_2(1+\frac{g_{ij}P_t}{\beta}), \tag{2}$$

where β is the ambient Gaussian noise density. As we know, to mathematically model the link capacity is imperative in the sense that the aggregate flow rates on each radio link can never exceed this link's capacity, which is an important constraint for routing.

3.2. Network Model

We consider an ad hoc sensor network consisting of a set of $\mathcal{N}$ nodes. We consider a set of $\mathcal{L}$ concurrent multi-hop sessions. Each session $\ell \in \mathcal{L}$ corresponds to a flow f with a source-destination pair $(s(\ell), d(\ell))$ in the network. The traffic demand for each session ℓ is given by $W(\ell)$ (bits). Traffic of a particular session may be split to sub-flows routed over different paths. The choice of these routing paths depends on the underlying schedule of different concurrent transmissions. The objective of our model is to minimize the energy consumption for delivering the $\mathcal{L}$ sessions without violating the interference constraints.

3.3. Expected Transmission Count

In this paper, we investigate the delivery ratio by the Expected Transmission Count (ETX) [14], [15]. ETX is calculated with the forward and reverse delivery ratio of the link. The forward delivery ratio d_f is the measured probability that a data packet successfully arrives at the recipient. Likewise, the reverse delivery ratio d_r refers to the probability that the ACK packet is successfully received. Thus, the probability that a packet is successfully received and acknowledged is $d_f \cdot d_r$. A sender will retransmit a packet that is not successfully acknowledged. Because each transmission attempt can be considered as a Bernoulli trial, ETX can be written as

$$\eta = \frac{1}{d_f \times d_r}, \tag{3}$$

The delivery ratios d_f and d_r can be measured using probe packets. First, we broadcast probes of fixed size in every period. Then, we count the successfully received probes at the recipient node. d_f can be derived from the following formula

$$d_f = \frac{count(t-w,t)}{w/\tau}, \tag{4}$$

where $count(t-w,t)$ is the number of probes received during window w, w/τ is the number of probes that should be received. Similarly, the reverse delivery ratio d_r can be computed at the sender.

4. Energy Efficient Multipath Routing

4.1. Routing

At the network level, a source node may need a number of relay nodes to route the traffic toward its destination node. Clearly, a route having only a single path may be overly restrictive and is not able to take advantage of load balancing. A set of paths (or multi-path) is more flexible to route the traffic from a source node to its destination.

Let Γ_i be the set of nodes that are within the transmission range to node i. Denote $f_{ij}(\ell)$ the data rate on link (i,j) that is attributed to session ℓ, where $i \in \mathcal{N}, j \in \Gamma_i$. The traffic demand for each session ℓ is given by $W(\ell)$ (bits). If node i is the source node of session ℓ, i.e., $i = s(\ell)$, then

$$\sum_{j\in\Gamma_i} f_{ij}(\ell) - \sum_{v\in\Gamma_i} f_{vi}(\ell) = W(\ell), \tag{5}$$

If node i is an intermediate relay node for session ℓ, i.e., $i \neq s(\ell)$ and $i \neq d(\ell)$, then

$$\sum_{j\in\Gamma_i, j\neq s(\ell)} f_{ij}(\ell) - \sum_{v\in\Gamma_i, v\neq d(\ell)} f_{vi}(\ell) = 0, \tag{6}$$

If node i is the destination node of session ℓ, i.e., $i = d(\ell)$, then

$$\sum_{j\in\Gamma_i} f_{ij}(\ell) - \sum_{v\in\Gamma_i} f_{vi}(\ell) = -W(\ell)., \tag{7}$$

In addition to the above flow balance equations at each node i for each session ℓ, the flow rates on each link cannot exceed this link's capacity. We assume all links utilize the same bandwidth B. According to equation (2), the constraint of link (i,j)'s capacity can be formulated as

$$f_{ij}(\ell)\eta_{ij} - B\log_2(1+\frac{g_{ij}P_t}{\beta}) \leq 0, \tag{8}$$

where η_{ij} is the ETX of the link (i,j).

4.2. Formulation

Define ξ as the energy consumption of sending a bit. Let f_{ij} denote the traffic requirement and $f_{ij}\eta_{ij}$ denote the amount of traffic passing through link (i,j). In order to obtain an optimal routing which minimizes the total energy consumption, we define the objective function as

$$\mathbb{R} = \sum_{i\in\mathcal{N}} \sum_{j\in\Gamma_i} f_{ij}\eta_{ij}\xi., \tag{9}$$

Mathematically, the problem can be modeled as follows

$$Minimize\ \mathbb{R}, \tag{10}$$

Subject to:

$$\sum_{j\in\Gamma_i} f_{ij}(\ell) - \sum_{v\in\Gamma_i} f_{vi}(\ell) = W(\ell) \quad (\ell \in \mathcal{L}, i = s(\ell)), \tag{11}$$

$$\sum_{j\in\Gamma_i, j\neq s(\ell)} f_{ij}(\ell) - \sum_{v\in\Gamma_i, v\neq d(\ell)} f_{vi}(\ell) = 0 \quad , \quad (\ell \in \mathcal{L}, i \in \mathcal{N}, i \neq s(\ell), d(\ell)) \tag{12}$$

$$\sum_{j\in\Gamma_i} f_{ij}(\ell) - \sum_{v\in\Gamma_i} f_{vi}(\ell) = -W(\ell) \quad (\ell \in \mathcal{L}, i = d(\ell)), \tag{13}$$

$$f_{ij}(\ell)\eta_{ij} - B\log_2(1+\frac{g_{ij}P_t}{\beta}) \leq 0 \quad , \quad (i \in \mathcal{N}, j \in \Gamma_i, s(\ell) \neq j, d(\ell) \neq i) \tag{14}$$

Here, equation (9) minimizes the total system energy consumption. Equation (10), (11), (12) present the flow conservation constraints. Equation (13) indicates that the total traffic routed through link ℓ cannot exceed the total transport capacity of ℓ.

We note that the above optimization problem is a non-linear programming problem, which is generally an NP-hard problem. Since it is computationally not feasible to obtain the optimal solution to this problem for the deployment of sheer number of sensor nodes, we propose a distributed energy efficient routing protocol in the next section.

5. Distributed Energy Efficient Routing

The objective of our model is to minimize the system energy consumption for delivering the $\mathcal{L}$ sessions. In the distributed routing scheme, a path can be established hop by hop which saves a large amount of computation since per hop information is convenient to acquire. Similarly, we also consider the flow transmission problem hop by hop. At each hop, we employ the transportation method to optimize the flow distribution with minimal energy consumption.

5.1. Formulation

We assume each link corresponds to a start-target pair. The traffic demand for each link is given by f_{ij} (bits). Traffic of a particular session may be split to sub-flows routed over different paths. The choice of these routing paths depends on the underlying schedule of different concurrent transmissions.

Mathematically, to transfer the traffic from starts to their targets with minimum cost, we consider the two nodes sets: the start set $U_s = \{u_s^1, \cdots, u_s^m\}$ and the target set $U_t = \{u_t^1, \cdots, u_t^n\}$. $u_s^i\ (1 \leq i \leq m)$ and $u_t^j\ (1 \leq j \leq n)$ are node representatives, where m is the number of start nodes and n is the number of target nodes in a single hop.

We denote $v(u_s^i)$ as the demand-traffic of u_s^i, and $v(u_t^j)$ as the relay-traffic of u_t^j which measures the relay capability of an intermediate node. Let $P=\{(u_s^1,v(u_s^1)),\cdots,(u_s^m,v(u_s^m))\}$ be the state representation of m start nodes and $Q=\{(u_t^1,v(u_t^1)),\cdots,(u_t^n,v(u_t^n)\}$ be the state representation of n target nodes. So, the problem of transferring the traffic from start nodes to their target nodes is converted into the link matching problem which can be formulated as

$$Q=F(P), \tag{15}$$

The mapping $F=[f_{ij}]$ can transform P into Q (as shown in Fig. 3). Thus, our further work is to find an optimal mapping where all the start nodes in P can find the target nodes in Q so that the total traffic can be transferred with minimum energy consumption.

We denote $D=[\eta_{ij}]$ as the expected transmit count matrix among nodes in U_s and U_t. Here, the expected transmit count between two nodes u_s^i and u_t^j is $\eta_{ij}=1/\left(d_{f_{ij}}\times d_{r_{ij}}\right)$. In this paper, we assume a unit energy cost, denoted ξ, equals to the energy consumption of transmitting a bit. The entry f_{ij} of F denotes the traffic needed to be transported from u_s^i to u_t^j. So, the optimization problem which minimizes the energy consumption can be expressed as

$$Cost(P,Q,D,F)=Minimize(\sum_{i\in U_s}\sum_{j\in U_t} f_{ij}\eta_{ij}\xi), \tag{16}$$

Subject to:

$$f_{ij}\geq 0 \qquad 1\leq i\leq m, 1\leq j\leq n, \tag{17}$$

$$\sum_{j\in U_t} f_{ij}<v(u_s^i) \qquad 1\leq i\leq m, \tag{18}$$

$$\sum_{i\in U_s} f_{ij}<v(u_t^j) \qquad 1\leq j\leq n, \tag{19}$$

$$\sum_{i\in U_s}\sum_{j\in U_t} f_{ij}=min(\sum_{i\in U_s} v(u_s^i),\sum_{j\in U_t} v(u_t^j)), \tag{20}$$

Equation (16) shows an objective function followed by a set of constraints. As for the first constraint, Equation (17) means that the traffic transported between two nodes u_s^i and u_s^i is no less than 0. Equation (18) denotes that the traffic shipping from a start node is no more than its demand-traffic. Similar situation exists in Equation (19). That means the traffic received from start nodes is no more than the relay-traffic of a target node. As for Equation (20), the total shipping traffic from start nodes to target nodes equals to the smaller one of total demand-traffic and total relay-traffic.

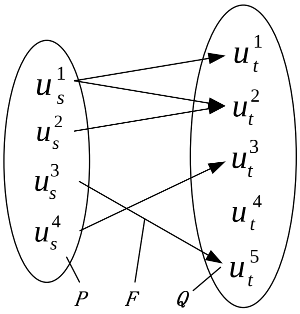

Fig. 3. The Mapping F between P and Q.

5.2. Matching Solution

In this part, we employ a well-known transportation problem [16] to find the optimal matching result between each start node and target node. The solution of the transportation problem is a flow matrix which determines the minimum energy consumption required to transport packets to the target.

Given P, Q and D, we use the efficient transportation method [16] to obtain the matching result among the start nodes and the target nodes by solving the transportation problem. It includes two essential steps: 1) determine the initial basic feasible solution; 2) test for optimization using the stepping stone or MODI method [17]. In the first step, the Vogel's approximation [18] is adopted since it gives an initial basic feasible solution which is much closer to the optimal solution. It reduces the total number of iterations required in the second step to reach an optimal solution considerably. Details of solving the transportation problem can be referred to literature [16].

The solution is denoted as a flow matrix $F=[f_{ij}]$. Any nonzero entry ($f_{ij}>0$) implies that u_s^i matches with u_t^j, and the shipping traffic between u_s^i and u_t^j equals to f_{ij}. Fig. 4 gives an example of the flow matrix. In the example, the flow matrix records the matching result and the traffic load that transported among start and target nodes. The colourful elements denote values greater than 0. In the third row, u_s^3 matches with u_t^2 and u_t^3. In the first column, two start nodes u_s^1 and u_s^2 are matched with u_t^1. It means that these two nodes have the same target node u_t^1.

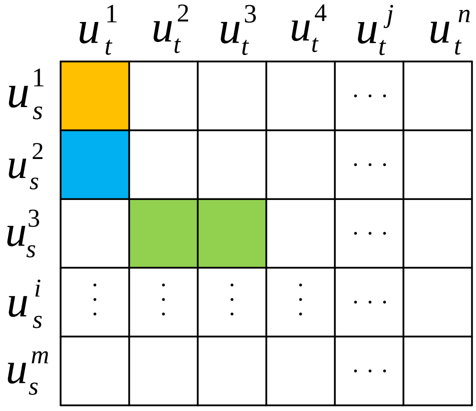

Fig. 4. The solution to our transportation problem: flow matrix. It records the matching result and the traffic load that transported among start and target nodes. The colourful elements denote values greater than 0. In the third row, u_s^3 matches with u_t^2 and u_t^3. In the first column, two start nodes u_s^1 and u_s^2 are matched with u_t^1. It means that these two nodes have the same target node u_t^1.

The transportation method is guaranteed to converge. Transportation problem is a special type of linear programming, and it is proved that the problem has an optimal solution. Also, transportation method belongs to the simplex method. In the literature [17], Dantzig demonstrated that the simplex method can find the optimal solution (if exists) in a finite number of iterations. Therefore, our method can find the optimal solution and converges in limited iteration steps.

6. Performance Evaluation

In this section, we investigate the energy consumption of a single-hop scenario in terms of number of start nodes and the traffic load of start nodes. As a comparison, we adopt three approaches: Random, Greedy, and our proposed optimal algorithm. The random algorithm selects the links randomly and the greedy algorithm is a naive approach, in which each node chooses the links with the best delivery ratio for transmission.

6.1. Number of Start Nodes

We assume that the traffic loads of start nodes and target nodes are both uniformly distributed over [1~10] Mbps. We set the number of target nodes with a fixed value which equals to 4 and show the improvement of the proposed algorithm with respect to the number of start nodes. It can be seen from Fig. 5 that as the number of start nodes rises, the energy consumption is greatly increased. We also see that our optimal algorithm outperforms the greedy and the random algorithms and reduces the energy consumption by up to 28 % and 35 % as the number of start nodes rises.

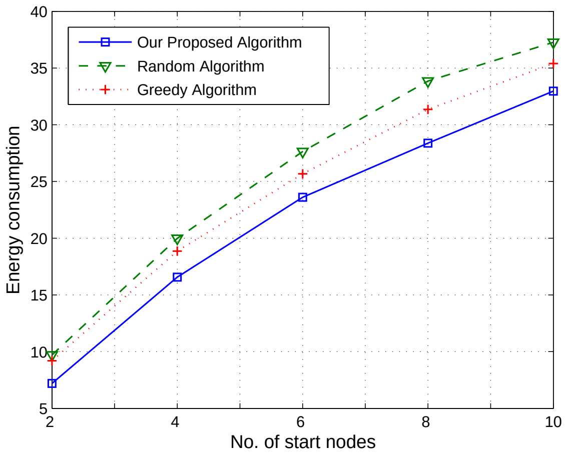

Fig. 5. Energy consumption vs. number of start nodes.

6.2. Average Throughput

We compare the energy consumption of three approaches with respect to the traffic load of start nodes. We set the number of start nodes and target nodes with fixed value which equals to 6 and 4 respectively. Fig. 6 demonstrates that the energy consumption is greatly increased as the traffic load of start nodes rises. It is shown that the energy consumption of our optimal approach is lower than that of the greedy and the random algorithms by up to 16 % and 24 % respectively.

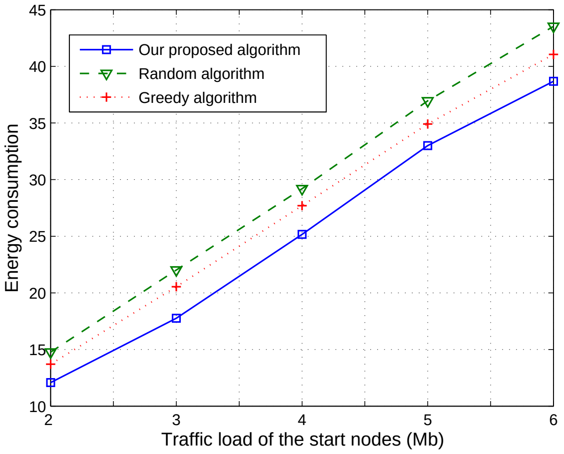

Fig. 6. Energy consumption vs. traffic load of start nodes.

7. Conclusion

In WSNs, multipath routing strategies have been proposed to improve the performance of reliability and energy balance. However, using multipath to sending packets leads to more energy cost since more paths introduces more contentions. How to design an energy efficient multipath routing protocol in multiple source-destination pairs scenario is even more difficult. In this paper, we consider both reliability and energy constraints in multiple source-destination pairs scenario to find the optimal energy efficient and reliable relaying of data from the sensor nodes to the sink so that the energy cost is saved drastically. Simulation results show the effectiveness of our approach.

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References

[1]. J. Yick, B. Mukherjee and D. Ghosal, Wireless sensor network survey, *Computer Networks*, Vol. 52, No. 12, 2008, pp. 2292-2330.

[2]. N. Ota, D. Hooks, P. Wright, D. Ausiander and T. Peffer, Poster abstract: wireless sensor networks characterization-application to demand response energy pricing, in *Proceedings of the ACM Conference on Embedded Networked Sensor Systems (SenSys'03)*, November 2003, pp. 334-335.

[3]. X. Huang and Y. Fang, Multiconstrained QoS multipath routing in wireless sensor networks, *Wireless Networks*, Vol. 14, No. 4, 2007, pp. 465-478.

[4]. K. Sha, J. Gehlot and R. Greve, Multipath routing techniques in wireless sensor networks: a survey, *Wireless Personal Communications*, Vol. 70, No. 2, 2013, pp. 807-829.

[5]. S. K. Singh, M. P. Singh, D. K. Singh, A survey of energy-efficient hierarchical cluster-based routing in wireless sensor networks, *Journal of Advanced Networking and Applications*, Vol. 2, No. 2, 2010, pp. 570-580.

[6]. B. Yahya and J. Ben-Othman, REER: robust and energy efficient multipath routing protocol for wireless sensor networks, in *Proceedings of the IEEE Global Telecommunications Conference (GLOBECOM'09)*, December 2009, pp. 1-7.

[7]. M. Radi, B. Dezfouli, K. A. Bakar and M. Lee, Multipath routing in wireless sensor networks: survey and research challenges, *Sensors*, Vol. 12, No. 1, 2012, pp. 650-685.

[8]. J. Chang and L. Tassiulas, Energy conserving routing in wireless adhoc networks, in *Proceedings of the International Conference on Computer Communications (INFOCOM'00)*, March 2000, pp. 22-31.

[9]. S. Banerjee and A. Misra, Minimum energy paths for reliable communication in multi-hop wireless networks, in *Proceedings of the ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc'02)*, June 2002, pp. 146-156.

[10]. Q. F. Dong, S. Banerjee, M. Adler and A. Misra, Minimum energy reliable paths using un-reliable wireless links, in *Proceedings of the ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc'05)*, May 2005.

[11]. D. Ganesan, R. Govindan, S. Shenker, and D. Estrin, Highly-resilient, energy-efficient multipath routing in wireless sensor networks, in *Proceedings of the 2nd ACM International Symposium on Mobile ad hoc Networking & Computing (MobiHoc'01)*, 2001, pp. 11-25.

[12]. S. Venkatasubramanian and N. P. Gopalan, Multi-path QoS routing protocol for load balancing in MANET, *International Journal of Networking & Parallel Computing*, Vol. 1, 2012, pp. 18-27.

[13]. A. Goldsmith, Wireless communications, *Cambridge University Press*, Cambridge, NY, 2005.

[14]. D. Couto, D. Aguayo, J. Bicket and R. Morris, A high-throughput path metric for multi-hop wireless routing, *Wireless Networks*, Vol. 11, July 2005, pp. 419-434.

[15]. R. Draves, J. Padhye and B. Zill, Routing in multi-radio, multi-hop wireless mesh networks, in *Proceedings of the Annual International Conference on Mobile Computing and Networking (MobiCom'04)*, September 2004, pp. 114-128.

[16]. F. S. Hillier and G. J. Lieberman, Introduction to operations research, *McGraw-Hill Science/ Engineering/Math*, 2005.

[17]. G. Dantzig and B. George, Linear programming and extensions, *Princeton University Press*, NJ, 1963.

[18]. N. V. Reinfeld and W. R. Vogel, Mathematical programming, *Prentice Hall*, Englewood Cliffs, New Jersey, 1958.