Sensors & Transducers, Vol. 171, Issue 5, May 2014, pp. 220-225

Sensors & Transducers

Monofractal Characteristics of Traffic in Wireless Sensor Networks for Smart Grid

Ming-Yue Zhai
North China Electric Power University, Chang Ping, Bei Jing, 102206, China
Tel.: 86-10-61773737
E-mail: mingyue.zhai@gmail.com

Received: 15 May 2014 /Accepted: 30 May 2014 /Published: 31 May 2014

Abstract: Wireless sensor networks (WSNs) have extensive applications in the smart grid in recently years. The time series in WSNs in smart grid should be discussed by analysis method which is non-linear. The method of monofractal is used here. Self-similar Hurst parameter calculated by the algorithm of Rescaled Range Analysis (R/S) and fractal dimension are obtained, which confirm that the time sequences in WSNs for smart grid belong to the fractal sets and have the characteristic of self-similarity. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Wireless sensor networks (WSNs), Hurst exponent, Fractal dimension, Monofractal.

1. Introduction

Smart grid is very hot in recent years, with the development of information technology, communication technology and power system. The smart grid is a network with various components which are mainly responsible for energy generation, power transmission and electricity distribution processes. It has a lot of advantages, which includes high reliability, environmental protection, great economic effects and compatibility. Also the smart grid support usage of new type energy, such as wind energy source, solar energy and so on. However there are a lot of problems which need to be work out, mainly focusing on the area of power transmission.

The wireless sensor networks (WSNs) consist of large micro-nodes, which use the way of wireless to communicate with each other. These micro-nodes consist of sensor, data processing unit and communication module. The information about humidity, temperature, pressure and speed in the environment could be measured by these sensors. Besides these sensors are relatively inexpensive to operate and the cost could be controlled within a certain scope. So the WSNs provide a convenient way to monitor the change of information in the smart grid. That is the reason why WSNs has gotten deep development recently. However many researchers have pay close attention on the energy conservation, route selection and applications of WSNs in smart grid. And there are few topics about the characteristics of the time series in WSNs in smart grid. With the growing number of WSNs deployed in smart grid to monitor the online status, it is necessary to analyze the characteristics of time sequences about traffic in WSNs in smart grid. It presents a new set of challenges and perspective to researchers and developers.

Fractional characteristics about the time sequences in WSNs in smart grid have been discussed in the paper. The time series analyzed here are measured in wireless sensor networks in one transformer substation for smart grid. The traffic has the characteristics of nonlinearity, non-Gaussian and nonstationarity. Monofractal theory is used here [1-5]. Firstly, time sequences based on different sampling frequency and different time period are extracted by Matlab R2010a here. Secondly, Rescaled Range Analysis (R/S) is used here in order to calculate the Hurst value of the time series. Then the Hurst exponents of the time sequences in WSNs based on different sampling frequency and different time slot are figured up. And fractal dimension about the time sequences in WSNs for smart grid has been gotten here. Finally we get the conclusion that the time series in WSNs in smart grid have the self-similarity and long-range dependence [6-8].

Here 1000000 sampling points of the time sequences are analyzed. In order to present the time series clearly, we just choose the time series from 0 ms to 6000 ms which is showed in Fig. 1. It is plotted by time on the horizontal and the number of packets received at the wireless sensor here on the vertical.

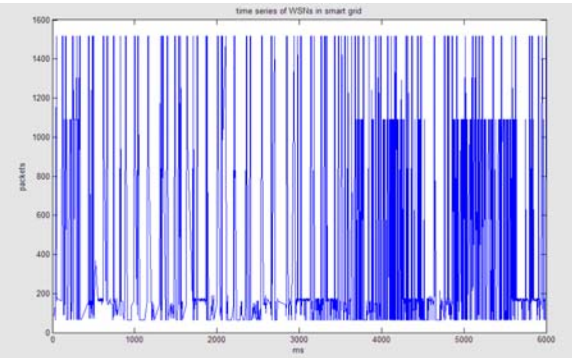

Fig. 1. Time series of WSNs in smart grid.

The paper is organized as follows: in Section 2, the algorithm of Rescaled Range Analysis (R/S) is discussed here. Section 3 shows the different Hurst values based on different sampling frequencies of the time sequences in WSNs in smart grid. Section 4 contains the different Hurst values based on different time periods of the time sequences in WSNs in smart grid. Then we compute the fractional dimension about the time sequences in WSNs. Finally the conclusion is summarized that the time sequences of WSNs in Smart grid has characteristics of self-similarity and long-range dependence.

2. Algorithm of Rescaled Range Analysis(R/S)

Hurst exponent is much critical when we discuss the characteristics of the time series using fractal theory. It is put forward by English Hydrologist H. E. Hurst in 1951 at the earliest just as Rescaled Range analysis (R/S) [4-5]. Hurst exponent could be calculated by different analysis, such as Rescaled Range analysis (R/S), Periodogram-based analysis, wavelet-based analysis and variance-time analysis. Comparison with the other three analyses, R/S analysis has been certified to be the most stable. The advantage of the analysis is we do not need to assume independent and random distribution for each variable in the system. It is widely used in various areas, such as financial market and network flow. It is a way of average. When we take use of it, we need large number of sample points to get relative accurate result. Here 1000000 sample points is given for discussing. And Rescaled Range analysis(R/S) is discussed here to calculate the value of Hurst exponent.

R/S analysis can be introduced as follow.

Given a time sequence $\{X_t\}, t = 1, 2, ..., N$ recorded at discrete time over a time span τ, the average influx and the standard deviation over the period τ can be calculated as follows.

$$E(m) = \frac{1}{\tau}\sum_{t=1}^{\tau} x_t \quad (1)$$

$$S(m) = \sqrt{\frac{1}{\tau}\sum_{t=1}^{\tau}\{x_t - E(m)\}^2} \quad (2)$$

Compute $X(t)$ as the accumulated departure of x_t from the mean $E(m)$,

$$X(t,\tau) = \sum_{u=1}^{t}\{x(u) - E(m)\} \quad (3)$$

The range R is defined as the difference between the maximum and minimum accumulated influx X,

$$R(m) = \max x(t,\tau) - \min x(t,\tau)\ , \quad (4)$$

where $1 \le t \le \tau$. So

$$R/S = (c\tau)^H\ , \quad (5)$$

where H is the Hurst exponent. Eq. (5) can be written as follow:

$$\log_{10}(R/S) = H\log_{10}(\tau) + H\log_{10} c\ , \quad (6)$$

$\log_{10}(R/S)$ against $\log_{10}(\tau)$ is plotted, whose slope is the Hurst exponent H.

The scope of Hurst value is from 0 to 1. Different Hurst exponent means different character of the time sequences in WSNs in Smart grid [6]:

H=0.5. The time series obey Gaussian distribution, and the process degrades into Brownian motion. The elements among the time series are random, standalone and uncorrelated with each other.

0<H<0.5. The time sequences of WSNs have the characteristics of non-durability, sometimes referred to as the feature of mean reversion. With Hurst value tends to 0, the non-durability will be more obvious.

0.5<H<1. The time sequence of WSNs have the characteristics of durability, sometimes referred to as the characteristics of long-rang dependence. When H tends to 1, the long-range dependence would be more notable. It means the time series of WSNs in smart gird has the feature of self-similarity.

Here H parameter is obtained by Matlab R2010a which use 1000000 sampling points showed in Fig. 2. Here we can get H=0.9579, certifying that the time series of WSNs in smart grid belong to fractal sets and has the characteristic of self-similarity.

3. Hurst Exponent Based on Different Sampling Frequencies

We have mentioned that 1000000 sampling points of the time series in WSNs for smart grid are used in the paper. Here we decelerate the sampling rate, i.e. expanding the sampling period for the time series. In order to show the detail clearly, Fig. 3 just present the detail of the time series from 0 ms to 6000 ms which is the same as Fig. 1. But the Hurst exponent is calculated by Matlab R2010a based on 1000000 sampling points [7]. Fig. 3 (a) shows the time series the sampling period of which is double its original one. And Fig. 3 (b) (c) (d) show the time series the sampling period of which is five, ten and fifteen times its original one.

Besides Hurst values based on different sampling frequencies of the time sequences in WSNs for smart gird are calculated by the algorithm of R/S analysis. We conduct simulations using Matlab R2010a. The results of simulation are showed as follow in Table 1.

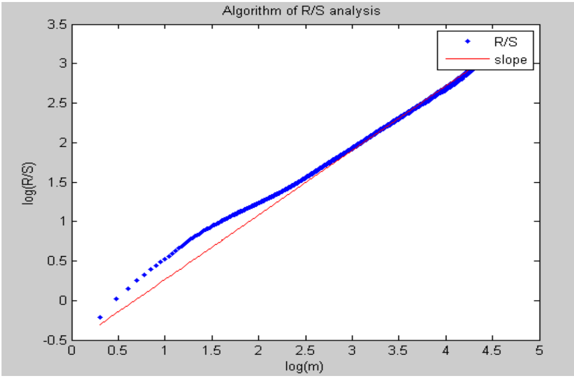

Fig. 2. Result of R/S analysis.

Table 1. Hurst exponent based on different sampling frequencies.

Widening multiple of sampling interval	Hurst exponent	Widening multiple of sampling interval	Hurst exponent
2	0.9490	9	0.9280
3	0.9456	10	0.9201
4	0.9407	11	0.9210
5	0.9384	12	0.9080
6	0.9338	13	0.9176
7	0.9317	14	0.9087
8	0.9297	15	0.8897

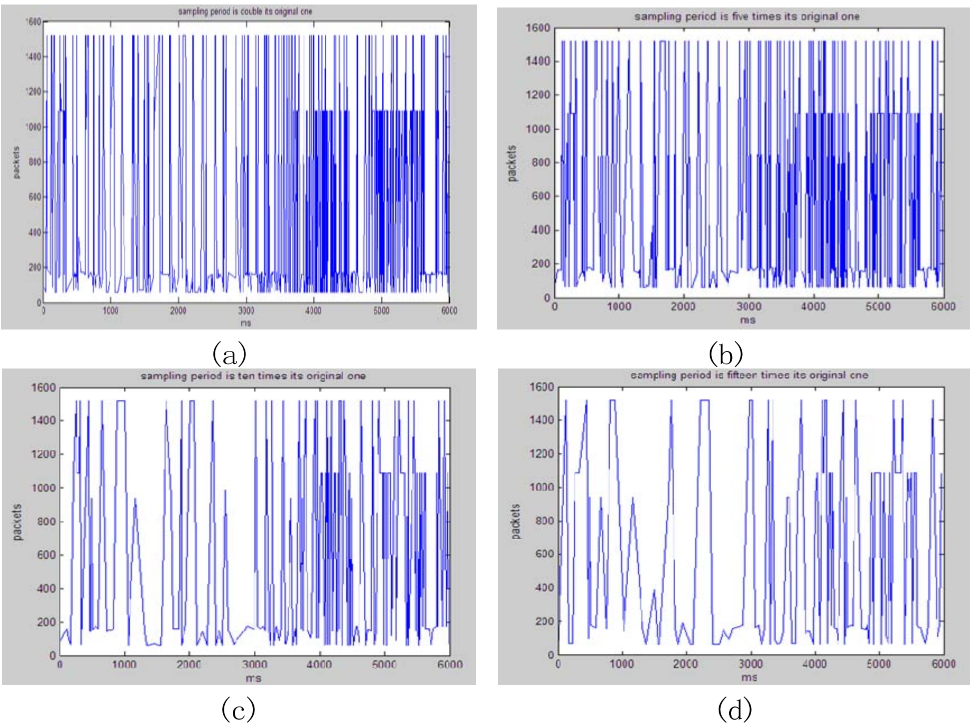

Fig. 3. Time series the sampling period of which is double, five, ten and fifteen times its original one.

From Table 1 and Fig. 4, we can get:

0.5<H<1, confirm that the time series based on different sampling frequencies have the characteristic of self-similarity and long-range dependence. It doesn't affect the self-similarity of the time series, when the widen multiple of sampling interval vary from 2 to 15. Hurst parameter is relative stable when we need to describe the monofractal characteristic of the time sequences in WSNs in smart grid. So although the sampling frequencies are different, the Hurst values vary only from 0.8897 to 0.9490.

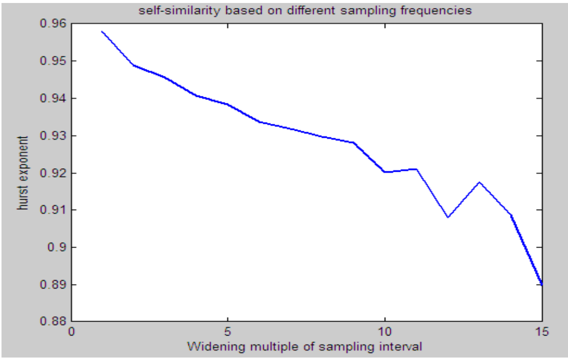

Fig. 4. Husrt exponent based on different sampling frequencies.

With the sampling interval increases, the values of Hurst parameter decrease a little. The biggest value of Hurst parameter is 0.9490 when the widen multiple of sampling interval is 2. And the smallest is 0.8897. But it doesn't affect the characteristic of the time series in WSNs in smart grid. It just because the number of sample size of the time series in WSNs is different. The theoretical error leads to the difference of Hurst exponent.

The conclusion that the time series of WSNs in Smart grid have the characteristic of self-similarity can be summarized here. However, if we want to get more from the various of Hurst parameter, the multi-fractal detrended function analysis (MF-DFA) is needed. Of course the time sequences of WSNs in smart grid here could be discussed by monofractal theory and multi-fractal theory which will get more attention.

4. Hurst Exponent Based on Different Time Slot

Likewise, 1000000 sampling points of the time series in WSNs for smart grid are discussed here. These sampling points are divided into four parts on average according to the time. The first part is from 0 s to 493 s. The second part is from 493 s to1025 s. The third part is from 1025 s to 1420 s. And the last part is from 1420 s to 1760 s. Besides the No. of sampling points in each part are 250000. Then the Hurst value based on different time slot of the traffic in WSNs are computed by Matlab R2010a using R/S analysis showed in Table 2.

Table 2. Hurst exponent based on different time slot.

Time slot	Sample size	Mean value	Standard deviation	Hurst exponent
1	250000	555.8462	520.2193	0.8457
2	250000	611.0856	519.5636	0.9512
3	250000	694.1837	517.6058	0.8784
4	250000	692.6439	506.1158	0.7180

From Table 2, we can get:

The values of Hurst parameter for the four time slots are all bigger than 0.5, i.e. 0.5<H<1. It shows that each part of the time series of WSNs in smart grid has the feature of self-similarity and long-range dependence. They are a part of fractal sets.

The Standard deviation of part 1 is the biggest, which presents variation of the packets number is the maximal relative to the mean value. But the Hurst value doesn't vary very much. It presents the Hurst exponent could not show the volatility of the traffic in WSNs in smart grid. The value of H just can show the characteristics of self-similarity and long-range dependence for time series.

The values of H for each part are different from each other. So the time series are time-varying.

Actually, according to the variation of Hurst exponent, the failover, anomaly and sudden change in the system could be detected. Deep researches should be done if we want to get more features and detail.

5. Fractional Dimension

Fractals, from the Latin fractus (broken, have an irregular form), don't have a precise and unified definition in literature [10]. Anyway, they are defined according to their functions and features. When mathematical objects don't convey the principles of Euclidian's geometry, they are named fractals which have a fractional dimension. It is put forward by Mandelbrot's Manifesto. Besides, fractal is defined by their characteristics of self-similarity across scales. That is why fractal dimension is often calculated as a factor of self-similarity.

Fractal dimension is crucial to describe the features of fractal, and important when we need to know more about the detail. There are various ways to calculate fractal dimension, such as Hausdorff dimension, generalized dimension (i.e. Renyi dimension), box dimension, correlation dimension and so on. Their functions are identical when we estimate whether the time sequences have the characteristic of fractal or not.

Here only the Hausdorff dimension and the box dimension are discussed.

5.1. Hausdorff Dimension

The definition of Hausdorff dimension is put forward by Felix Hausdorff who is a Bonn Mathematician in 1919. Hausdorff dimension, the most commonly used fractal dimension, provides a quantitative distinction among the measure 0 sets. Hausdorff dimension is defined in any metric space [11]. The Hausdorff dimension of fractal geometry all equals to fraction [12]. Here Hausdorff dimension of traffic based on different time slot are computed.

It has been confirmed that there is some relationship between Hurst exponent H and Hausdorff dimension:=2-H. So the Hausdorff dimension could be calculated indirectly by the Hurst exponent gotten by the R/S analysis before in Table 2. The results are presented in Table 3.

Table 3. Hausdorff dimension of the time series based on different time slot.

Hausdorff dimension	Time period 1	Time period 2	Time period 3	Time period 4
	1.1543	1.0488	1.1216	1.2820

From Table 3, we can find $1 < D_H < 2$ for each part of the time series. It means that dimension for every time slot is greater than topological dimension which equals to 1 here. Also the Hausdorff dimensions for the four parts are nearly the same. So we can get that the time series based on different time slots all belong to fractal sets and have the characteristic of self-similarity and long-range dependence.

5.2. Box Dimension

Box dimension, is also known as box-counting. Among these ways of calculating fractal dimension, Box dimension analysis is relatively ripe and has been largely applied. The mathematical calculation and empirical estimation of box dimension are comparatively easier than others. The research on it could date back to 1930 [13-14].

Box dimension about the time series is computed by the software Fractalfox 2.0 here as Fig. 5.

Here the pattern of Box counting should be selected first when we want to get the value of box dimension about the time series by the software. And the range of the box size should be set between 1 and 100. Then load the time series of WSNs in smart grid based on different time slot. Clicking the button of Current fractal, we could get the value of box dimension presented in Table 4.

From Table 4, we can get $1 < D_0 < 2$ for each time slot. And the box dimension for every time slot is greater than topological dimension which equals to 1 here. It has been certified that the topological of curve equals to 1 and the topological of surface equals to 2. So we can get that the time series of WSNs in smart grid belong to fractal sets.

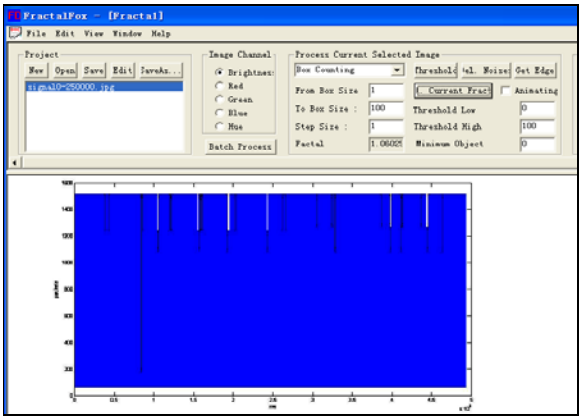

Fig. 5. Box dimension of the traffic in WSNs calculated by Fractalfox 2.0.

Table 4. Box dimension of the time series based on different time slot.

Box dimension	Time period 1	Time period 2	Time period 3	Time period 4
	1.0603	1.0516	1.1020	1.1018

Also we can find that the box dimension of the time series based on different time slot is almost the same, although the shape, amplitude and distribution of the time series based on different time slot are very different. So the fractal feature of the traffic in WSNs can't be described perfectly by box dimension. Box dimension is just a simple indicator to judge if the time series belong to fractal sets or not. Multi-fractal analysis is needed if we want to know more about the characteristics of the time series.

6. Conclusions

The research on the WSNs is active lately, as its large applications in wired and wireless networks. In the paper, the characteristics of time sequences in WSNs for smart grid are discussed. We have known that the time series in WSNs is non-linear, non-Gaussian and non-stationary. So the analysis of nonlinear is needed here. Of course, we could make use of the analysis of wavelet, fractal or chaos if we want. Here fractal theory is analyzed. Fractal theory includes monofractal theory and multi-fractal theory. And Monofractal theory is discussed here.

The values of Hurst exponent based on different frequencies and different time slots are calculated by the analysis of R/S in Section 2, 3 and 4. 0.5<H<1. It means that the time series have the feature of self-similarity. In Section 5, we get the fractal dimension of the time sequence in WSNs. Hausdorff dimension and box dimension are discussed. And we get that 1<D<2 for both the Hausdorff dimension and box dimension. In result, the traffic of WSNs in smart grid belongs to fractal sets and is self-similar.

But it is not enough to apply monofractal theory solely. If deep research is needed, we need to take use of multi-fractal theory to discuss the feature of time sequences of WSNs in smart grid. And we will get more detail and properties about the time series in WSNs for smart grid.

Acknowledgements

The paper is supported in part by the Natural Science Foundation of Beijing under the Grant No. 4122073.

References

[1]. Mandelbrot B. B., The Fractal Geometry of Nature, *W. H. Freeman & Company*, New York, USA, 1983.

[2]. Peng Gong Wang, Efficient uplink scheduling policy for variable bit rate traffic in IEEE 802.16 BWA systems, *International Journal of Communications Systems*, 25, 6, 2012, pp. 734-748.

[3]. Mandelbrot B. B., Wallis J. R., Some long-run properties of geophysical records, *Water Resources Research*, 5, 4, 1969, pp. 321-340.

[4]. Al-Qassas R. S., Ould-Khaoua M., Performance comparison of end-to-end and on-the-spot traffic-aware techniques, *International Journal of Communications Systems*, 26, 1, 2013, pp. 13-33.

[5]. Tao Shaohua, Properties of Self-similarity Networks, *Journal of Computers*, 5, 10, 2010, pp. 1582-1589.

[6]. J. Mauricio O. Matos, Elineudo P. de Moura, etc., Rescaled range analysis and detrended fluctuation analysis study of cast irons ultrasonic backscattered signals, *Chaos, Solitons and Fractals*, 19, 2004, pp. 55-60.

[7]. Li Wan, Rescaled Range Analysis and Mineralization Information Extraction, *Computing, Control and Industrial Engineer (CCIE)*, 2010, pp. 210-213.

[8]. Wang Taijun, Fractal characteristics analysis of wireless networks traffic based Hurst parameter, in *Proceedings of the International Conference on Communications, Circuits and System (ICCCAS)*, 2009, pp. 173-176.

[9]. Kaliammal N., Gurusamy G., Performance analysis of multicast routing and wavelength assignment protocol with dynamic traffic grooming in WDM networks, *International Journal of Communications Systems*, 26, 2, 2013, pp. 198-211.

[10]. Vassilakis V. G., Moscholios I. D., Logothetis M. D., The extended connection-dependent threshold model for call-level performance analysis of multi-rate loss systems under the bandwidth reservation policy, *International Journal of Communications Systems*, 25, 7, 2012, pp. 849-873.

[11]. Ledesma Sergio, Hurst parameter transition detection on self-similarity network traffic, in *Proceedings of the 12th World Multi-Conference on Systemics, Cybernetics and Informatics, Jointly with the 14th International Conference on Information Systems Analysis and Synthesis (WMSCI ISAS)*, 2008, pp. 71-76.

[12]. Peng Han, The research of WSN architecture for smart grid utility, *Advanced Materials Research*, 546-547, 2012, pp. 266-271.

[13]. Wei Zhen-Hai, Research on strength of fractal structural soil, *Yantu Lixue/Rock and Soil Mechanics*, 33, 3, 2012, pp. 695-701, 702.

[14]. F. Hausdorff, Dimension und äuβeres Maβ, *Math. Ann.*, 79, 1919, pp. 157-179.

[15]. Barreira L., Valls C., Hausdorff dimension and nonlinear relations between frequencies of digits, *Open Systems & Information Dynamics*, 19, 3, 2012, p. 1250018.

[16]. Hitchcock John M., Hausdorff dimension and oracle constructions, *Theoretical Computer Science*, 355, 3, 2006, pp. 382-388.

[17]. Veleva S., Kacarska M., Davcev D., Box-dimension as a correlation measure for date mining of power socket sensor date, in *Proceedings of the IEEE International Conference on Smart Measurements for Grids (SMFG)*, 2011, pp. 88-94.

[18]. Elezovic N., Zupanovic V., Zubrinic D., Box dimension of trajectories of some discrete dynamical systems, *Chaos, Solitons and Fractals*, 34, 2, 2007, pp. 244-252.

[19]. Tao Shao-Hua, Zhang Xiang-Qun, Research on self-similarity characteristic evolution model of complex network, *Computer Engineering*, 38, 1, 2012, pp. 197-198.

[20]. Tseng S. M., Wang Y. C., Throughput of DS-CDMA/unslotted ALOHA radio networks with Markovian arrival processes, *International Journal of Communications Systems*, 26, 3, 2013, pp. 369-379.