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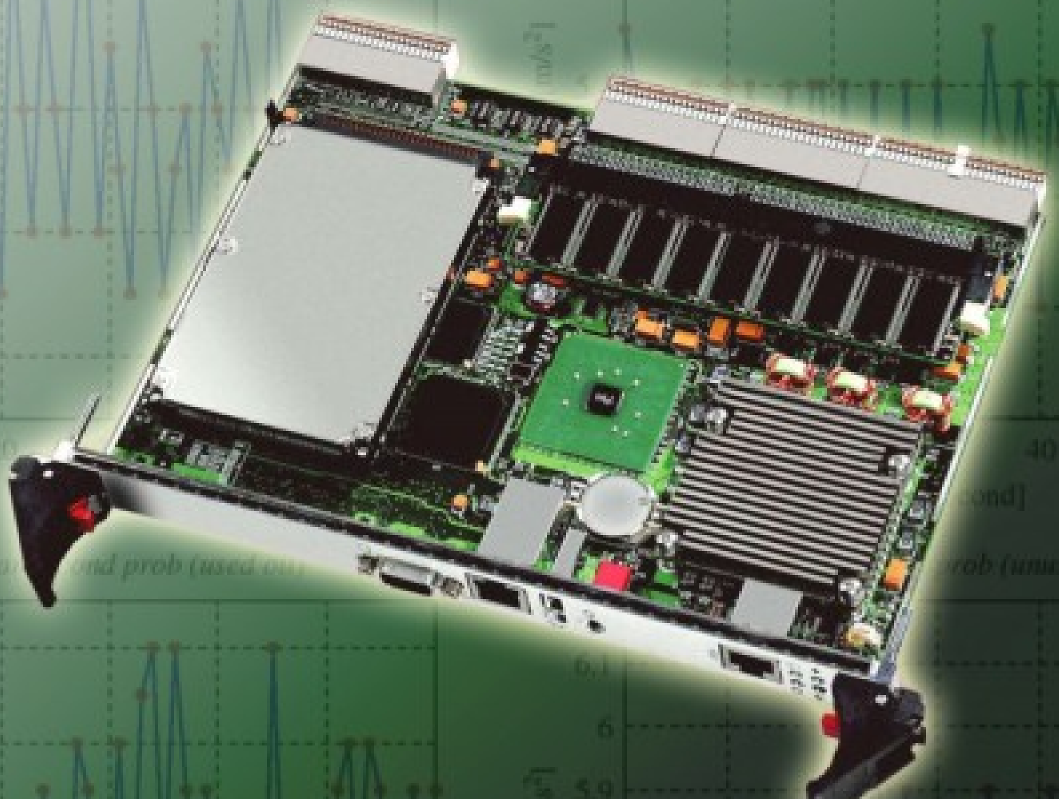

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Research on Attainable Moment Subset of Singular Over-actuated System

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Abstract: Attainable moment subset (AMS) is an important concept in control allocation problem of aircrafts with multiple control surfaces. It is produced by effectors and can be expressed as a mapping of the subset of constrained controls into the moment space. Determination of AMS is useful to calculate the aerodynamic efficiency of combined surfaces and evaluate the performance of the allocation system. But existing AMS calculation methods can only solve the cases that every 3 by 3 matrix formed of the columns of control effectiveness matrix be of full rank. And there is no method for the cases that not satisfy the condition. Focusing on this problem, the geometry structure characteristics of AMS is deeply studied. With the relationships among vertexes, edges and faces of AMS determined, a method is proposed to solve the problem. Simulation results show that the method can solve the AMS of singular over actuated system well. *Copyright © 2012 IFSA.*

Keywords: Flight control, Control allocation, Attainable moment subset, Singular over actuated system.

1. Introduction

With the improvement of the aircraft performance in maneuverability and survivability, advanced aircraft is usually designed with multiple control surfaces. By cooperating and allocating the redundant aerodynamic surfaces, additional forces and moments are provided to the aircraft to enhance the over-stall maneuver ability and survivability. However, such an aerodynamic layout also leads to a series of problems, such as: coupling of the control surfaces and increased freedom of the control system. In order to solve the above problems and distribute the three-axis moments into the control surfaces reasonably, the control allocation technique is proposed and studied [1-2].

The control allocation technique is defined as the determination of the deflections of control effectors that will generate a given set of desired three-axis moments specified by a pilot through a control stick inside the cockpit. The objective of allocation is to choose a configuration of the effectors to meet a specified objective subject to the saturation constraints of the actuators [3], or make the aircraft tolerant to some faults [4]. Attainable moment subset (AMS) is an important concept in control allocation problem of aircrafts with multiple control surfaces. It is composed by roll moment, pitch moment and yaw moment and can be expressed as a mapping of the subset of constrained controls into the moment space. Different effector combination schemes and allocation methods will produce different AMS. For example, the AMS produced by pseudo-inverse method [5], weighting generalized inverse method [6] and redistributed pseudo-inverse methods [7] are different. So it is very meaningful to calculate the AMS and know how much attainable moment subset (AMS) can be obtained by the specific allocation scheme. In 1992, Durham studied the two-dimension allocation problem and gave an inspection method to solve the boundary of attainable moment subset for the two-dimension allocation problem [8]. In 1994, a method solving the attainable moment subset for the three-dimension allocation problem was proposed by Durham [9], but it’s very complicated. Later he proposed an improved method [10].

The aircraft with multiple control surfaces is an over actuated system. Although the method proposed by Durham can solve the AMS for most over-actuated system, there is a strict requirement to use the method. That is every 3 by 3 sub-matrix of effectiveness matrix should be full of rank. For the cases that do not satisfy the requirement, the method can not be used directly. Unfortunately some modern tailless aircrafts, robots and new style transport aircrafts with more spoilers are singular over actuated system and their control effectiveness matrixes really do not satisfy the condition.

Focusing on the problem, comprehensive mathematic analyses are given to describe the geometry of the AMS, and a method is proposed to solve the AMS of the singular over actuated systems. At last, a simulation is given and the results show that the method proposed by this article can solve the AMS of singular over actuated systems effectively.

2. Description of the Problem

In a conventional aircraft, there are three major control effectors $\boldsymbol{u}(t)$: aileron, elevator and rudder. Flight control systems are usually designed utilizing one control surface for each rotational degree of freedom. Essentially, the aileron is used differentially to produce a rolling moment, the elevator is to generate a pitching moment, and the rudder controls the yawing moment of the aircraft. In a traditional airplane, with three desired moments and three independent control effectors to generate these moments, a unique solution can be found. However, with an increase in the number of redundant control effectors $\boldsymbol{u}(t)$, the problem of allocating these controls to achieve the desired moments becomes non-unique and far more complex.

Consider an m-dimensional control space $\boldsymbol{u}(t) \in R^m$. The controls are constrained to minimum and maximum values, defined by the constrained control subset: $\Omega = \{\boldsymbol{u} \in R^m \mid \underline{u}_i \le u_i \le \overline{u}_i\} \subset R^m$ These controls generate moments $\boldsymbol{v}(t)$ through a linear mapping $\mathbf{B}: R^m \to R^3 (m > 3)$ onto a three dimensional moment space $\boldsymbol{v}(t) \in R^3$. If $\boldsymbol{v}(t)$ is specified, control allocation is the solving of the constrained equation:

$$\mathbf{B}\boldsymbol{u}(t) = \boldsymbol{v}(t), \qquad \mathbf{B} = \begin{bmatrix} \boldsymbol{b}_1 & \boldsymbol{b}_2 & \cdots & \boldsymbol{b}_m \end{bmatrix}, \tag{1}$$

where $\boldsymbol{v}(t) = \begin{bmatrix} C_l & C_m & C_n \end{bmatrix}^T$ is the desired moment; $\boldsymbol{u}(t)$ is the sought which should meet the desired system performance.

Mathematically, the moments produced by control surfaces can be expressed as a mapping of the subset of constrained controls into the moment space. Different effector combination schemes and allocation schemes will produce different attainable moment subsets. The geometry of attainable moment subset (denoted by Φ) is a complex convex polyhedron. According to the property that the parallelism of parallel hyper-planes does not change if they are mapped by a linear mapping, following conclusions can be obtained.

M-dimensional control space Ω has $C_m^2 2^{m-2}$ bounding hyperplanes. These hyperplanes can be divided into C_m^2 groups and every group has 2^{m-2} parallel hyperplanes. Through linear mapping $\mathbf{B}$, the parallelism of the new facets in three-dimension moment space does not change. For every group, most new facets are interior of the AMS and only the two facets that are the farthest away from each other are the two surfaces or parts of the two surfaces. That means that boundary of AMS is comprised by $m(m-1)$ facets, and some of them may be coplanar. In order to solve the AMS of singular over actuated system, we first studied the case that every 3 by 3 sub-matrix of effectiveness matrix is full of rank.

3. AMS of Non-singular Over-actuated System

For three-dimension allocation problem, control space $\Omega = \{\boldsymbol{u} \in R^m \mid \underline{u_i} \le u_i \le \overline{u_i}\} \subset R^m$ is mapped onto the three-dimension moment space through a linear mapping $B: R^m \to R^3$. Φ_3 is convex polyhedron. Considering the complexity of AMS, here we only research the situations that every 3×3 matrix formed of the columns of B is full rank.

Suppose that u_a, u_b, u_c, u_i and u_j are any five control variables of control vector $\boldsymbol{u}$. So the control allocation problem can be rewritten as:

$$\boldsymbol{v} = \begin{bmatrix} b_{1a} & b_{1b} & b_{1c} \\ b_{2a} & b_{2b} & b_{2c} \\ b_{3a} & b_{3b} & b_{3c} \end{bmatrix} \cdot \begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} + \begin{bmatrix} b_{1i} \\ b_{2i} \\ b_{3i} \end{bmatrix} u_i + \begin{bmatrix} b_{1j} \\ b_{2j} \\ b_{3j} \end{bmatrix} u_j + \sum_{k=1}^{m} \begin{bmatrix} b_{1k} \\ b_{2k} \\ b_{3k} \end{bmatrix} \cdot u_k ,$$

where $k \ne a,b,c,i,j$. By analyzing the features of the formula (2), it's not difficult to get following conclusions:

(1) When u_i, u_j, u_k are constant values and u_a, u_b, u_c are control variables and free vary within constrains, obviously Φ_3 is a cube.

(2) When u_j, u_k are constant values and u_a, u_b, u_c, u_i are control variables varying within their constrains, we can consider Φ_3 as a geometry extended from a cube free sliding along the line: $\boldsymbol{v} = \begin{bmatrix} b_{1i} & b_{2i} & b_{3i} \end{bmatrix}^T u_i + \sum_{k=1,k\ne i}^{m} \begin{bmatrix} b_{1k} & b_{2k} & b_{3k} \end{bmatrix}^T \cdot u_k$ and the cube mentioned above is formed by u_a, u_b, u_c free varying in their constrains.

(3) When u_k is constant value and u_a, u_b, u_c, u_i, u_j are control variables and free vary within their constrains, we can consider Φ_3 as a geometry extended from a cube free sliding on the face: $\boldsymbol{v} = \begin{bmatrix} b_{1i} & b_{2i} & b_{3i} \\ b_{1j} & b_{2j} & b_{3j} \end{bmatrix}^T \begin{bmatrix} u_i \\ u_j \end{bmatrix} + \sum_{k=1,k\ne i,j}^{m} \begin{bmatrix} b_{1k} & b_{2k} & b_{3k} \end{bmatrix}^T \cdot u_k$ and the cube mentioned above is formed by u_a, u_b, u_c free varying in their constrains.

From discussion above, we know that under the requirement that every 3×3 matrix formed of the columns of B is full rank, any 2-D plane P in Φ_3 can only be formed under following conditions that two control variables free change in their constrains and the other $m-2$ control variables are constant. Because the vertices of Φ_3 are mapped by the vertices of Ω, $m-2$ control variables must at their limits $\underline{u}(t)$ or $\overline{u}(t)$ if 2-D plane P is the surface of Φ_3. According to the property that the parallelism of parallel hyper-planes does not change if they are mapped by linear mapping, following conclusions can be obtained.

M-dimensional control space Ω has $C_m^2 2^{m-2}$ bounding surfaces. These facets can be divided into C_m^2 groups and every group has 2^{m-2} parallel surfaces. Through linear mapping $B: R^m \to R^3 (m>3)$, the parallelism of the new facets mapped by every group does not change. However, most new facets of every group are interior of the AMS and only two facets are the boundaries of the AMS. These two facets of AMS are farthest away from each other in every group. So the boundary of AMS is comprised of $m(m-1)$ facets and every facet is a parallelogram. If the two boundaries of AMS in every group can be solved, so the boundary $\partial(\Phi_3)$ of the AMS can be obtained. The method of solution is as follows.

Suppose that facet subset $\xi_{i,j}$ is defined by control variable u_i, u_j varying free within constrains and the other controls being limits:

$$\xi_{i,j} = \{\boldsymbol{v} \mid \boldsymbol{v} = \begin{bmatrix} b_{1i} & b_{2i} & b_{3i} \\ b_{1j} & b_{2j} & b_{3j} \end{bmatrix}^T \begin{bmatrix} u_i \\ u_j \end{bmatrix} + \sum_{k=1}^{m} \begin{bmatrix} b_{1k} & b_{2k} & b_{3k} \end{bmatrix}^T u_k\} \; u_k \in \{\overline{u}_k, \underline{u}_k\}, k \neq i, j \tag{2}$$

There are C_m^2 such facet subsets corresponding to the choice of i and j. Introduce the coordinate rotation transformation $\boldsymbol{v}' = \mathbf{T}\mathbf{B}\boldsymbol{u} = \mathbf{B}'\boldsymbol{u}$, which make the facets of $\xi_{i,j}$ be perpendicular to the new axis C_l'. According to the orthogonality, matrix $\mathbf{B}'$ satisfies:

$$\mathbf{B}'(1,i) = 0 \;\; \mathbf{B}'(1,j) = 0 \tag{3}$$

Suppose matrix $\mathbf{T} = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix}$. Substitute $\mathbf{T}$ into formula.3 and following equation can be derived:

$$\begin{bmatrix} t_{11} \\ t_{12} \end{bmatrix} = -\begin{bmatrix} b_{1i} & b_{2i} \\ b_{1j} & b_{2j} \end{bmatrix}^{-1} \cdot \begin{bmatrix} b_{3i} \\ b_{3j} \end{bmatrix} t_{13} \tag{4}$$

Suppose that the intersection points between facet subset $\xi_{i,j}$ and axis C_l' are $\boldsymbol{v}_k = \begin{bmatrix} Cl_k & 0 & 0 \end{bmatrix}^T, k = 1, 2, \cdots, 2^{m-2}$. Obviously facets which make the Cl_k maximum or minimum are the two surfaces of the Φ_3. Substitute farmula.4 into formula.2, Cl_k is obtained:

$$Cl_k = \frac{t_{13}}{b_{1i}b_{2j} - b_{1j}b_{2i}} \sum_{k=1}^{m} \boldsymbol{\eta}^T \cdot \boldsymbol{b}_k \cdot u_k \tag{5}$$

Here $\boldsymbol{\eta} = \boldsymbol{b}_i \times \boldsymbol{b}_j$ is normal vector of the plane $\xi_{i,j}$. Every 3 by 3 sub-matrix of effectiveness matrix be full of rank ensures that no value $\boldsymbol{\eta}^T \cdot \boldsymbol{b}_k$ equals to zero. So the values of every control variable u_k can be decided by the signs of $\boldsymbol{\eta}^T \cdot \boldsymbol{b}_k$. For example, four vertices of the surface $\xi_{i,j}$ with maximal Cl_k are:

$$\begin{aligned} \boldsymbol{v}^{\max}{}_1 &= \boldsymbol{a}_{\max} + \boldsymbol{b}_i u_{i,\min} + \boldsymbol{b}_j u_{j,\min} \\ \boldsymbol{v}^{\max}{}_2 &= \boldsymbol{a}_{\max} + \boldsymbol{b}_i u_{i,\min} + \boldsymbol{b}_j u_{j,\max} \\ \boldsymbol{v}^{\max}{}_3 &= \boldsymbol{a}_{\max} + \boldsymbol{b}_i u_{i,\max} + \boldsymbol{b}_j u_{j,\min} \\ \boldsymbol{v}^{\max}{}_4 &= \boldsymbol{a}_{\max} + \boldsymbol{b}_i u_{i,\max} + \boldsymbol{b}_j u_{j,\max} \end{aligned} \qquad \begin{aligned} \boldsymbol{a}_{\max} &= \sum_{k=1,k\neq i,j}^{n} \boldsymbol{\mu}_{k,\max} \\ \boldsymbol{\mu}_{k,\max} &= \begin{cases} \boldsymbol{b}_k u_{k,\max} & if\ \boldsymbol{b}_k{}^T \boldsymbol{\eta}_{ij} > 0 \\ \boldsymbol{b}_k u_{k,\min} & if\ \boldsymbol{b}_k{}^T \boldsymbol{\eta}_{ij} < 0 \end{cases} \end{aligned} \tag{6}$$

Similarly, four vertices of the surface with minimal Cl_k can be calculated too. For every group (i, j), solve its two surfaces and AMS will be solved.

4. AMS of Singular Over-actuated System

For a singular over actuated system the value $\boldsymbol{\eta}^T \cdot \boldsymbol{b}_k$ may be equal to zero. So we can not decide the control u_k according to the sign of $\boldsymbol{\eta}^T \cdot \boldsymbol{b}_k$. If $\boldsymbol{\eta}^T \cdot \boldsymbol{b}_k = 0$, the vector $\boldsymbol{b}_k u_k$ is inside the plane decided by the vector $\boldsymbol{b}_i u_i$ and $\boldsymbol{b}_j u_j$. So u_k is defined as the coplanar control variable of (u_i, u_j), denoted by $u_k^{(i,j)}$. In order to distinguish the coplanar control variables from the common controls, we write the facet subset $\xi_{i,j}$ decided by varying (u_i u_j) freely within constrains and the other controls being limits as followings:

$$\xi_{i,j} = \{\boldsymbol{v} \,|\, \boldsymbol{v} = \boldsymbol{b}_i u_i + \boldsymbol{b}_j u_j + \sum_{k=1}^{p} \boldsymbol{b}_k u_k^{(i,j)} + \sum_{l=1}^{m-p-2} \boldsymbol{b}_l u_l\} \tag{7}$$

Here p is the number of coplanar control variables of (u_i, u_j). Analyzing the above expression, it can be concluded that the coplanar control variables together with (u_i, u_j) decide the shapes of the surfaces and the remaining controls u_l decide the offset between origin and the facet of $\xi_{i,j}$. So the solving of the AMS can be divided into two parts: First, decide the control variables u_l by making the offset maximum or minimum. Second, solve the shapes of the surface and decide u_i, u_j $u^{(i,j)}$ according to the vertices of the surfaces. Denote the offset as $\boldsymbol{a}$, and it equals:

$$\boldsymbol{a} = \sum_{l=1}^{m-p-2} \boldsymbol{b}_l u_l\ ,\ u_l \in \{\overline{u}_l, \underline{u}_l\} \,\&\, u_l \neq u_i, u_j, u^{(i,j)} \tag{8}$$

There are 2^{m-p-2} combinations corresponding to the choice of u_l. So the element number of subset $\xi_{i,j}$ is 2^{m-p-2}. Two facets that are the farthest away from each other are two surfaces of AMS. Since $\boldsymbol{\eta}^T \cdot \boldsymbol{b}_l \neq 0$, these $m-p-2$ control variables can also be decided according to the signs of $\boldsymbol{\eta}^T \cdot \boldsymbol{b}_l$. Because the vector $\boldsymbol{b}_i$, $\boldsymbol{b}_j$ and $\boldsymbol{b}_k{}^{(i,j)}$ are coplanar, no matter their control variable u_i, u_j and $u^{(i,j)}$ are at $\overline{u}$ or $\underline{u}$, the produced points $\boldsymbol{v} = \boldsymbol{b}_i u_i + \boldsymbol{b}_j u_j + \sum_{i=1}^{p} \boldsymbol{b}_k u_k^{(i,j)} + \boldsymbol{a}$ are coplanar. There are 2^{p+2} combinations with u_i, u_j and $u^{(i,j)}$ being at $\bar{u}$ or $\underline{u}$. So the number of these points is 2^{p+2}. Some points are interior of the surface, some points are on the edge of the surfaces, and the others are vertices. Our job is to find the vertices and obtain the outline of the surface.

Non-coplanar controls of $(u_i \quad u_j)$ has nothing to do with the shapes of the surfaces, so these controls can be ignored in the process of deciding u_i, u_j and $u^{(i,j)}$. The problem is simplified to solve the convex envelope of following points:

$$\boldsymbol{v} = \boldsymbol{b}_i u_i + \boldsymbol{b}_j u_j + \sum_{i=1}^{p} \boldsymbol{b}_k u_k^{(i,j)}, \ u \in \{\bar{u}, \underline{u}\} \tag{9}$$

In order to solve the problem, we first introduce the coordinate rotation transformation $\boldsymbol{v}' = \mathbf{G}\boldsymbol{v}$ and make the transformed facet parallel to the plane xoy.

$$\mathbf{G} = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ 0 & g_{22} & g_{23} \\ n_1 & n_2 & n_3 \end{bmatrix}$$

Here $\tilde{\boldsymbol{n}} = [n_1 \quad n_2 \quad n_3]^T$ is the unit vector of the vector $\boldsymbol{\eta}$ ($\boldsymbol{\eta} = \boldsymbol{b}_i \times \boldsymbol{b}_j$), other parameters of matrix $\mathbf{G}$ are: $g_{11} = \sqrt{1 - n_3^{\,2}} \quad g_{12} = -n_1 n_2 / g_{11}$

$$g_{13} = -n_1 n_3 / g_{11} \quad g_{22} = -\sqrt{1 - n_2^{\,2} - g_{12}^{\,2}} \quad g_{23} = -(n_2 n_1 + g_{12} g_{13}) / g_{22}$$

Suppose the new points $\boldsymbol{v}'$ produced by the transformation are shown in Fig. 1, specifically $\boldsymbol{v}_1 \cdots \boldsymbol{v}_6$. Our purpose is to find all the vertices of the surface from above points and order them. Obviously the point $\boldsymbol{v}_5$ which has the biggest values in x-axis is one vertex of surface. Then through the method of gradually rotating the z-axis, all the vertices can be found and ordered.

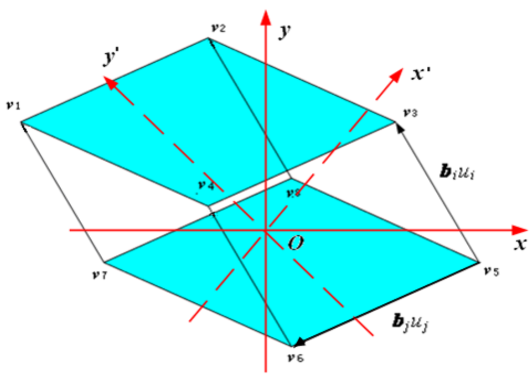

Fig. 1. Process of finding the vertices of the surface.

Mathematical description of this idea is as follows. Suppose that the points produced by transformation $\mathbf{G}$ are $\boldsymbol{v}_i' = (x_i', y_i', 0)^T \quad i = 1, 2, 3 \cdots 2^p$. Introduce the coordinate rotation transformation:

$$\boldsymbol{v}'' = \mathrm{T}\boldsymbol{v}' , \quad \mathrm{T} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \tag{10}$$

Gradually increase the angle $\phi \in [0, 2\pi]$ and the vertices $\boldsymbol{v}'$ whose images $\boldsymbol{v}'' = T\boldsymbol{v}'$ appear to have the biggest value in x-axis in the process of rotation transformation are the vertices of plane. Denote the vertices by $\boldsymbol{v}_k^{vertice}$, $k = 1, 2, \cdots$. The vertices of the AMS surface $\xi_{i,j}$ can be solved by following expressions. The surface with maximal offset in positive direction:

$$\begin{aligned} &\boldsymbol{v}^{\xi_{i,j}}{}_k = \mathbf{G}^{-1} v_k^{vertice} + a_{\max}, k = 1, 2, 3\cdots \\ &\boldsymbol{a}_{\max} = \sum_{k=1, k\neq i,j}^{n} \boldsymbol{\mu}_{k,\max} \quad , \ \boldsymbol{\mu}_{k,\max} = \begin{cases} \boldsymbol{b}_k u_{k,\max} & if\ \boldsymbol{b}_k{}^T \boldsymbol{\eta}_{ij} > 0 \\ \boldsymbol{b}_k u_{k,\min} & if\ \boldsymbol{b}_k{}^T \boldsymbol{\eta}_{ij} < 0 \end{cases} \end{aligned} \tag{11}$$

The surface with minimal offset in negative direction:

$$\begin{aligned} &\boldsymbol{v}^{\xi_{i,j}}{}_k = \mathbf{G}^{-1} v_k^{vertice} + \boldsymbol{a}_{\min}, k = 1, 2, \cdots \\ &\boldsymbol{a}_{\min} = \sum_{k=1, k\neq i,j}^{n} \boldsymbol{\mu}_{k,\min} \quad , \ \boldsymbol{\mu}_{k,\min} = \begin{cases} \boldsymbol{b}_k u_{k,\max} & if\ \boldsymbol{b}_k{}^T \boldsymbol{\eta}_{ij} < 0 \\ \boldsymbol{b}_k u_{k,\min} & if\ \boldsymbol{b}_k{}^T \boldsymbol{\eta}_{ij} > 0 \end{cases} \end{aligned} \tag{12}$$

5. Simulations

The following example applies above method to solve the AMS. The data in the B matrix is based on an aircraft model which is linearized at a flight condition of 3000m, Mach 0.3 and 12.5° angle-of-attack.

$$\mathbf{B}\boldsymbol{u} = \boldsymbol{v}$$

$$\mathbf{B} = \begin{bmatrix} -0.04382 & 0.04382 & -0.05841 & 0.05841 & 0.01674 & -0.0628 & 0.0628 & 0.0292 & 0.0000 & 0.0100 \\ -0.5331 & -0.5330 & -0.0649 & -0.0648 & 0.0000 & 0.0623 & 0.0623 & 0.0000 & 0.3553 & 0.0000 \\ 0.0110 & -0.0110 & 0.0039 & -0.0039 & -0.0743 & 0.0000 & 0.0000 & 0.0003 & 0.0000 & 0.1485 \end{bmatrix}$$

The controls u_1 and u_2 are right and left horizontal tails, u_3 and u_4 are right and left tailing edge flaps, u_5 and u_6 are the right and left ailerons, u_7 is the joint contribution of rudders and u_8, u_9 and u_{10} are three thrust vectoring controls. The control deflection limits are given in Table 1.

Table 1. Control Position Limits.

Num	Limits (Rad)	Num	Limits (Rad)
u1	-0.4189 ~ 0.1833	u6	-0.1396 ~ 0.7854
u2	-0.4189 ~ 0.1833	u7	-0.1396 ~ 0.7854
u3	-0.5236 ~ 0.5236	u8	-0.5236 ~ 0.5236
u4	-0.5236 ~ 0.5236	u9	-0.5236 ~ 0.5236
u5	-0.5236 ~ 0.5236	u10	-0.5236 ~ 0.5236

The aircraft described above is an over actuated system with ten effectors to generate three-axis moments. By calculating the rank of every 3 by 3 sub matrix of $\mathbf{B}$, it is concluded that the system is a singular overacted system with 4 group coplanar controls. They are $u_1u_2u_9$, $u_6u_7u_9$, $u_3u_4u_9$ and $u_5u_8u_{10}$.

Applying the method proposed by this paper to calculate the AMS of the singular over actuated system, results show that AMS of the aircraft is a complex convex polyhedron constructed with 78 convex polygons. Its geometry structure is shown in Fig. 2. Six of them are hexagons filled with colors and the others are parallelograms. For example two hexagons decided by coplanar controls u_1, u_2, u_9 are listed in Table 2.

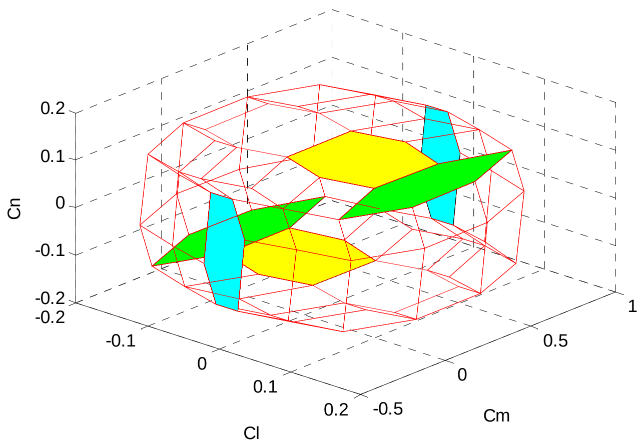

Fig. 2. Geometry structure of AMS.

Table 2. Vertices of Hexagons Decided by (u1, u2, u9).

Face 1	Coordinates	Face 2	Coordinates
v_1	(0.157,-0.020, 0.106)	v_1	(-0.105,-0.020, -0.119)
v_2	(0.131,-0.341, 0.113)	v_2	(-0.131,-0.341, -0.113)
v_3	(0.105,-0.020, 0.119)	v_3	(-0.157,-0.020, -0.106)
v_4	(0.105, 0.352, 0.119)	v_4	(-0.157, 0.352, -0.106)
v_5	(0.131, 0.673, 0.113)	v_5	(0.131, 0.673, -0.113)
v_6	(0.157, 0.352, 0.106)	v_6	(-0.105, 0.352, -0.119)

The volume of the AMS is 0.894. With AMS of the aircraft obtained, we can calculate maximal attainable moment in every vector direction. Especially, the cross points between three-axis and the AMS in both direction respectively are C_l : -0.173～0.173, C_m : -0.4668～0.7984 and C_n : -0.1263～0.1263.

6. Conclusions

Determination of AMS is very important to the solving of the allocation problem. It can help designers to evaluate the maneuverability of the aircraft and obtain the aerodynamic efficiency of the combined effectors. Focusing on the solving of the AMS, this paper studied its geometry structure and boundary. For the cases that not every 3 by 3 matrix formed of the columns of control effectiveness matrix be of full rank, an improved method is proposed to solve the AMS. Simulation results show that the method can solve the AMS of singular over actuated system well.

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Guide for Contributors

Aims and Scope

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