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New Exponential Strengthening Buffer Operators and Numerical Simulation

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Abstract: Based on the theory of buffer operators in the grey system, some new exponential strengthening buffer operators are established, and the relevant theorems are proved according to the axiomatic of buffer operators. The problem that there are some contradictions between quantitative analysis and qualitative analysis in pretreatment for vibration data sequences is resolved effectively. An example simulation shows compared with the existing strengthening buffer operators, the kind of new strengthening buffer operators increases the forecast precision of GM (1, 1) remarkably. *Copyright © 2013 IFSA.*

Keywords: Grey system, Buffer operator, Exponent, Simulation, Exponential.

1. Introduction

The grey system theory has been caught great attention by researchers since 1982 and has already been widely used in many fields, such as industry, agriculture, zoology, market economy and so on. GM (1, 1) has been high improved by many scholars from home and abroad. The grey system theory can effectively deal with incomplete and uncertain information system.

Although the data of the objective system is scattered, they always have their overall function, and inevitably contains a certain rule. The key is how to choose the appropriate methods to mining and utilization. Professor Liu Sifeng put forward the concept of impact disturbance buffer operator, and constructs a kind of buffer operator extensively. The weakening buffer operator and strengthening buffer operator are researched deeply by some scholars. The concept of buffer operator is put forward, and some widely used buffer operator is structured by paper [1-5]. The properties of strengthening buffer operator are studied by paper [6-7]. A new strengthening buffer operator $x(k)d_1 = x(k)\left(\frac{x(k)}{x(n)}\right)^{\frac{1}{n-k+1}}$ is put forward by paper [8], the average strengthening buffer operator, weighted average strengthening buffer operator and the geometric mean strengthening buffer operator are structured. At last, electricity consumption per capita is constructed through two order enhancement treatment by this method. It has improved the precision of fitting and prediction accuracy. A new buffer operator $x(k)d_1 = x(k)\frac{x(k)}{\sqrt{x(k)x(n)}}$ is obtained by paper [9], and it is used to simulate to examples. The result shows good prediction accuracy of GM (1, 1) model. Several kinds of variable weight strengthening buffer operator are presented by paper [10], the problem of traditional buffer operator which interact too strongly or too weakly is solved.

2. Several Concepts

Definition 1 Suppose that $X = \{x(1), x(2), ..., x(n)\}$ is a non-negative data sequence, we can obtain that:

1) If for $\forall k = 1, 2, \ldots, n-1$, there exists $x(k) < x(k+1)$, the data sequence X is defined as a monotone increasing sequence;

2) if for $\forall k = 1, 2, \ldots, n-1$, there exists $x(k) > x(k+1)$, the data sequence X is defined as a monotone decreasing sequence;

3) if for $\exists k, k' \in \{1, 2, \ldots, n-1\}$, there exists $x(k) < x(k+1)$, $x(k') < x(k'+1)$ the data sequence X is defined as an oscillatory sequence.

$$\begin{cases} M = \max\{x(k) | k \in \{1, 2, ..., n\}\} \\ m = \min\{x(k) | k \in \{1, 2, ..., n\}\} \end{cases}$$

The value of M-m is called the amplitude of sequence X.

Definition 2 Suppose that X is a non-negative data sequence, D is the operators of X, the data sequence which acts by D is gotten as follows.

$$XD = (x(1)d, x(2)d, \ldots, x(n)d)$$

Then, D is called sequence operators.

The data sequence is acted continuously, two step operator, three step operator, even r step operator are gotten as follows.

$$XD^2, XD^3, ..., XD^r$$

Axiom 1 (Fixed-point axiom) Suppose that X is a non-negative data sequence, D is the sequence operators, then D meets that:

$$x(n)d = x(n)$$

Axiom 2 (Full use of information axiom) Each data $x(k)(k = 1, 2, \ldots, n)$ in system behavior data sequence X should be the whole process fully participate in the operator function.

Axiom 3 (Analytical, normative axiom) Any $x(k)d(k = 1, 2, \ldots, n)$ can be expressed by a unified elementary analytic type $x(1), x(2), \ldots, x(n)$.

The three axioms above are called three axioms of buffer operator. Sequence buffer which satisfies three axioms of buffer operator is defined as buffer operator.

Definition 3 Suppose $X = \{x(1), x(2), ..., x(n)\}$ is a non-negative data sequence, then $XD = (x(1)d, x(2)d,$ is $\ldots, x(n)d)$ called buffer operator.

1) When X is a monotone increasing sequence, D is strengthening buffer operator $\Leftrightarrow x(k)d \le x(k), k = 1, 2, 3, ..., n$.

2) When X is a monotone decreasing sequence, D is strengthening buffer operator $\Leftrightarrow x(k)d \le x(k), k = 1, 2, 3, ..., n$

3) When X is an oscillatory sequence, D is strengthening buffer operator $\Leftrightarrow$

$$\max_{1 \le k \le n}\{x(k)\} \le \max_{1 \le k \le n}\{x(k)d\} \ \min_{1 \le k \le n}\{x(k)\} \ge \min_{1 \le k \le n}\{x(k)d\}$$

From above, the data of monotone increasing sequence under strengthening operator is shrunk. The data of monotonic decay sequence under strengthening buffer operator is expanded.

3. The Construction of a New Index Strengthening Buffer Operator

Theorem 1 Suppose that $X = \{x(1), x(2), ..., x(n)\}$ is a non-negative data sequence, $XD_1 = (x(1)d_1, x(2)d_1, ..., x(n)d_1)$ is defined as buffer sequence. Where

$$x(k)d_1 = x(k)e^{\frac{x(k)-x(n)}{\sqrt{x(k)x(n)}}}, \ k = 1, 2, ..., n$$

D_1 is defined as a strengthening buffer operator when X is a monotone increasing sequence, or a monotone decreasing sequence, or even an oscillatory sequence.

Prove: It is easily gotten that D_1 satisfies three axioms of buffer operator, so that D_1 is buffer operator.

Suppose X is a monotone increasing sequence, thus

$$x(k) - x(n) \le 0, e^{\frac{x(k)-x(n)}{\sqrt{x(k)x(n)}}} \le 1$$

Thus,

$$x(k)d_1 = x(k)e^{\frac{x(k)-x(n)}{\sqrt{x(k)x(n)}}} \le x(k)$$

We can obtain that $x(k)d_1 \le x(k)$, so D_1 is strengthening buffer operator.

Suppose X is a monotone decreasing sequence, thus

$$x(k) - x(n) \ge 0, e^{\frac{x(k)-x(n)}{\sqrt{x(k)x(n)}}} \ge 1$$

Then,

$$x(k)d_1 = x(k)e^{\frac{x(k)-x(n)}{\sqrt{x(k)x(n)}}} \ge x(k)$$

We can obtain that $x(k)d_1 \ge x(k)$, so D_1 is strengthening buffer operator.

Suppose X is an oscillatory sequence, thus

$$x(a) = \max_{1\le k\le n}\left\{x(k)\middle| k = 1,2,...,n\right\}$$

$$x(b) = \min_{1\le k\le n}\left\{x(k)\middle| k = 1,2,...,n\right\}$$

$$x(a)d_1 = x(a)e^{\frac{x(a)-x(n)}{\sqrt{x(a)x(n)}}} \ge 1$$

Thus, $\max_{1\le k\le n}\{x(k)\} \le \max_{1\le k\le n}\{x(k)d_1\}$.

In the same way that $\min_{1\le k\le n}\{x(k)\} \ge \min_{1\le k\le n}\{x(k)d_1\}$ can be obtained, thus D_1 is strengthening buffer operator.

Corollary 1: Two step strengthening buffer operator can be obtained based on strengthening buffer operator D_1 defined in Theorem 1.

$$XD_1^2 = XD_1D_1 = (x(1)d_1^2, x(2)d_1^2,..., x(n)d_1^2)$$

Thus, $x(k)d_1^2 = x(k)d_1 e^{\frac{x(k)d_1 - x(n)d_1}{d_1\sqrt{x(k)x(n)}}}$

It is easily obtained that d_1^2 is two step strengthening buffer operator when X is a monotone increasing sequence, or a monotone decreasing sequence, or even an oscillatory sequence.

Theorem 2 Suppose that $X = \{x(1), x(2),...., x(n)\}$ is a non-negative data sequence, $XD_2 = (x(1)d_2, x(2)d_2, ..., x(n)d_2)$ is defined as buffer sequence. Where

$$x(k)d_2 = x(k)e^{\frac{(n-k+1)(x(k)-x(n))}{\sum_{i=k}^{n}\sqrt{x(i)x(n)}}}, \quad k = 1,2,...,n$$

D_2 is defined as an arithmetic mean strengthening buffer operator when X is a monotone increasing sequence, or a monotone decreasing sequence, or even an oscillatory sequence.

Prove: It is easily gotten that D_2 satisfies three axioms of buffer operator, so that D_2 is buffer operator.

Suppose X is a monotone increasing sequence, thus

$$x(k) - x(n) \le 0, e^{\frac{(n-k+1)(x(k)-x(n))}{\sum_{i=k}^{n}\sqrt{x(i)x(n)}}} \le 1$$

Thus, $x(k)d_2 = x(k)e^{\frac{(n-k+1)(x(k)-x(n))}{\sum_{i=k}^{n}\sqrt{x(i)x(n)}}} \le x(k)$

We can obtain that $x(k)d_2 \le x(k)$, so D_2 is strengthening buffer operator.

Suppose X is a monotone decreasing sequence, thus

$$x(k) - x(n) \ge 0, e^{\frac{(n-k+1)(x(k)-x(n))}{\sum_{i=k}^{n}\sqrt{x(i)x(n)}}} \ge 1$$

Thus, $x(k)d_2 = x(k)e^{\frac{(n-k+1)(x(k)-x(n))}{\sum_{i=k}^{n}\sqrt{x(i)x(n)}}} \ge x(k)$

We can obtain that $x(k)d_2 \ge x(k)$, so D_2 is strengthening buffer operator.

Suppose X is an oscillatory sequence, thus

$$x(a) = \max_{1\le k\le n}\left\{x(k)\middle| k = 1,2,...,n\right\}$$

$$x(b) = \min_{1\le k\le n}\left\{x(k)\middle| k = 1,2,...,n\right\}$$

$$x(a)d_2 = x(a)e^{\frac{(n-k+1)(x(k)-x(n))}{\sum_{i=k}^{n}\sqrt{x(i)x(n)}}} \ge 1$$

Thus, $\max_{1\le k\le n}\{x(k)\} \le \max_{1\le k\le n}\{x(k)d_2\}$

In the same way that $\min_{1\le k\le n}\{x(k)\} \ge \min_{1\le k\le n}\{x(k)d_2\}$ can be obtained, thus D_2 is strengthening buffer operator.

Corollary 2: Two step strengthening buffer operators can be obtained based on strengthening buffer operator D_2 defined in Theorem 2.

$$XD_2^2 = XD_2D_2 = (x(1)d_2^2, x(2)d_2^2,..., x(n)d_2^2)$$

Thus, $x(k)d_2^2 = x(k)e^{\frac{(n-k+1)(x(k)d_2 - x(n)d_2)}{\sum_{i=k}^{n}d_2\sqrt{x(i)x(n)}}}$

It is easily obtained that d_2^2 is two step strengthening buffer operator when X is a monotone increasing sequence, or a monotone decreasing sequence, or even an oscillatory sequence.

Theorem 3 Suppose that $X = \{x(1), x(2),...., x(n)\}$ is a non-negative data sequence, $XD_3 = (x(1)d_3, x(2)d_3, ..., x(n)d_3)$ is defined as buffer sequence. Where

$$x(k)d_3 = x(k)e^{\frac{(n-k+1)(n+k)(x(k)-x(n))}{2\sum_{i=k}^{n}i\sqrt{x(i)x(n)}}}, \quad k = 1,2,...,n$$

D_3 is defined as a weighted arithmetic mean strengthening buffer operator when X is a monotone increasing sequence, or a monotone decreasing sequence, or even an oscillatory sequence.

Prove: It is easily gotten that D_3 satisfies three axioms of buffer operator, so that D_3 is buffer operator.

Suppose X is a monotone increasing sequence, thus

$$x(k)-x(n)\le 0, e^{\frac{(n-k+1)(n+k)(x(k)-x(n))}{2\sum_{i=k}^{n} i\sqrt{x(i)x(n)}}}\le 1$$

Thus, $x(k)d_3 = x(k)e^{\frac{(n-k+1)(n+k)(x(k)-x(n))}{2\sum_{i=k}^{n} i\sqrt{x(i)x(n)}}}\le x(k)$

We can obtain that $x(k)d_3 \le x(k)$, so D_3 is strengthening buffer operator.

Suppose X is a monotone decreasing sequence, thus

$$x(k)-x(n)\ge 0, e^{\frac{(n-k+1)(n+k)(x(k)-x(n))}{2\sum_{i=k}^{n} i\sqrt{x(i)x(n)}}}\ge 1$$

Thus, $x(k)d_3 = x(k)e^{\frac{(n-k+1)(n+k)(x(k)-x(n))}{2\sum_{i=k}^{n} i\sqrt{x(i)x(n)}}}\ge x(k)$

We can obtain that $x(k)d_3 \ge x(k)$, so D_3 is strengthening buffer operator.

Suppose X is an oscillatory sequence, thus

$$x(a)=\max_{1\le k\le n}\{x(k)|k=1,2,...,n\}$$
$$x(b)=\min_{1\le k\le n}\{x(k)|k=1,2,...,n\}$$
$$x(a)d_3 = x(a)e^{\frac{(n-k+1)(n+k)(x(k)-x(n))}{2\sum_{i=k}^{n} i\sqrt{x(i)x(n)}}}\ge 1$$

Thus, $\max_{1\le k\le n}\{x(k)\}\le \max_{1\le k\le n}\{x(k)d_3\}$

In the same way that $\min_{1\le k\le n}\{x(k)\}\ge \min_{1\le k\le n}\{x(k)d_3\}$ can be obtained, thus D_3 is strengthening buffer operator.

Corollary 3: Two step strengthening buffer operator can be obtained based on strengthening buffer operator D_3 defined in Theorem 3.

$$XD_3^{\,2} = XD_3D_3 = (x(1)d_3^{\,2}, x(2)d_3^{\,2},\dots,x(n)d_3^{\,2})$$

Thus, $x(k)d_3^{\,2} = x(k)e^{\frac{(n-k+1)(n+k)(x(k)d_3-x(n)d_3)}{2\sum_{i=k}^{n} id_3\sqrt{x(i)x(n)}}}$

It is easily obtained that $d_3^{\,2}$ is two step strengthening buffer operator when X is a monotone increasing sequence, or a monotone decreasing sequence, or even an oscillatory sequence.

Theorem 4 Suppose that $X=\{x(1),x(2),....,x(n)\}$ is a non-negative data sequence, $XD_4=(x(1)d_4, x(2)d_4,\ ...,x(n)d_4)$ is defined as buffer sequence. Where

$$x(k)d_4 = x(k)e^{\frac{(x(k)-x(n))}{\prod_{i=k}^{n}\sqrt{x(i)x(n)}^{\frac{1}{n-k+1}}}}, k=1,2,...,n$$

D_4 is defined as a geometric average strengthening operator when X is a monotone increasing sequence, or a monotone decreasing sequence, or even an oscillatory sequence.

Prove: It is easily gotten that D_4 satisfies three axioms of buffer operator, so that D_4 is buffer operator.

Suppose X is a monotone increasing sequence, thus

$$x(k)-x(n)\le 0, e^{\frac{(x(k)-x(n))}{(\prod_{i=k}^{n}\sqrt{x(i)x(n)})^{\frac{1}{n-k+1}}}}\le 1$$

Thus, $x(k)d_4 = x(k)e^{\frac{(x(k)-x(n))}{(\prod_{i=k}^{n}\sqrt{x(i)x(n)})^{\frac{1}{n-k+1}}}}\le x(k)$

We can obtain that $x(k)d_4 \le x(k)$, so D_4 is strengthening buffer operator.

Suppose X is a monotone decreasing sequence, thus

$$x(k)-x(n)\ge 0, e^{\frac{(x(k)-x(n))}{(\prod_{i=k}^{n}\sqrt{x(i)x(n)})^{\frac{1}{n-k+1}}}}\ge 1$$

Thus, $x(k)d_4 = x(k)e^{\frac{(x(k)-x(n))}{(\prod_{i=k}^{n}\sqrt{x(i)x(n)})^{\frac{1}{n-k+1}}}}\ge x(k)$

We can obtain that $x(k)d_4 \ge x(k)$, so D_4 is strengthening buffer operator.

Suppose X is an oscillatory sequence, thus

$$x(a)=\max_{1\le k\le n}\{x(k)|k=1,2,...,n\}$$
$$x(b)=\min_{1\le k\le n}\{x(k)|k=1,2,...,n\}$$
$$x(a)d_4 = x(a)e^{\frac{(x(k)-x(n))}{(\prod_{i=k}^{n}\sqrt{x(i)x(n)})^{\frac{1}{n-k+1}}}}\ge 1$$

Thus, $\max_{1\le k\le n}\{x(k)\}\le \max_{1\le k\le n}\{x(k)d_4\}$

In the same way that $\min_{1\le k\le n}\{x(k)\}\ge \min_{1\le k\le n}\{x(k)d_4\}$ can be obtained, thus D_4 is strengthening buffer operator.

Corollary 4: Two step strengthening buffer operators can be obtained based on strengthening buffer operator D_4 defined in Theorem 4.

$$XD_4^{\,2} = XD_4D_4 = (x(1)d_4^{\,2}, x(2)d_4^{\,2},...,x(n)d_4^{\,2})$$

Thus, $x(k)d_4{}^2 = x(k)d_4 e^{\frac{(x(k)d_4 - x(n)d_4)}{\prod_{i=k}^{n}(d_4\sqrt{x(i)x(n)})^{\frac{1}{n-k+1}}}}$

It is easily obtained that $d_4{}^2$ is two step strengthening buffer operator when X is a monotone increasing sequence, or a monotone decreasing sequence, or even an oscillatory sequence.

4. Example

We take the number of one city's mobile phone users (unit: million) as an example to validate the strengthening buffer operator in GM (1, 1) proposed by this paper. We select the number of 1996-2003 mobile phone $X = (65.20\ 67.16\ 70.61\ 75.66\ 83.22\ 93.21\ 103.93\ 116.92)$ as the original data. Where the number of 1996-2001 is used as modeling data, the number of 2002-2003 is used as simulation test data. Average annual growth rates of the user number of mobile telephone were 3.11 %, 5.15 %, 7.16 %, 10.01 %, 12.01 % from the original data in1996-2001. Thus it can be seen, the growth of original data sequence before half part slows than the latter part. Using the data to predict is difficult to believe. Analysis of this situation, from 2000, in order to expand the amount of mobile phone users, China telecommunication enterprises launched a series of services that stimulated the demand for mobile phone. In order to make a reasonable forecast of the number of mobile phone users, the original data should be strengthen firstly.

The original data which was treated with the one step strengthening buffer operator proposed by this paper firstly establishes prediction model as shown in Table 1.

Table 1. Model GM (1, 1) produced by different strengthening butter operators.

Seq	GM (1, 1) model	Predict		Relative error	
		2002	2003	2002	2003
X	$x(1996+k) = 741.1041e^{0.0845k}$ -675.9041	99.70	108.49	4.07	7.21
XD_1	$x(1996+k)=463.2526e^{0.1173k}$ -417.7323	103.65	116.54	0.27	0.32
XD_2	$x(1996+k)=240.6445e^{0.1679k}$ -193.9488	101.83	120.45	2.01	3.02
XD_3	$x(1996+k)= 100.4510e^{0.2639k}$ -65.9642	103.19	124.36	0.71	6.36
XD_4	$x(1996+k)= 775.5209e^{0.0845k}$ -710.3213	104.28	113.47	0.33	2.95

The relative error which the original data sequence modeling under strengthening buffer operator D_1, D_2, D_3, D_4 is much smaller as shown in Table 1.

The prediction relative error is the best under D_1. The predict values of mobile phone were 103.6454, 116.5447 for 2002 and 2003, which approach the actual values 103.93, 116.92. Relative errors were only 0.27 % and 0.32 %, prediction accuracy is very good. Compared with paper [9], the practical application results show the effectiveness of the proposed approach.

5. Conclusions

Based on the theory of buffer operators in the grey system, some new exponential strengthening buffer operators are established, and the relevant theorems are proved according to the axiomatic of buffer operators. The problem that there are some contradictions between quantitative analysis and qualitative analysis in pretreatment for vibration data sequences is resolved effectively. An example simulation shows compared with the existing strengthening buffer operators, the kind of new strengthening buffer operators increases the forecast precision of GM (1, 1) remarkably.

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