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The Optimization Function of Lognormal Distribution on the Airborne Particle Counter Channel

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Abstract: With the increase of fog and haze weather, people pay more and more attention to the pollution of the suspended particulates in the air. And, the development of science and technology requires high cleanliness make the measurement of the aerosol and the improvement of the instrument become more important. In this paper, on the basis of analyzing the typical particle size measurement process by the light scattering method, the statistical model of light scattering particle's random measurement is established. Meanwhile, the theoretical deduction indicates that the statistical distribution function in the light scattering random signal measurements meets a lognormal distribution function with the natural number as the independent variable. The experiment results manifest that the introduction of the lognormal distribution into the counting channel optimization of the measurement instruments 1) Saves the counting channels, 2) Expands the sensor's measurement range by 1/3 and enhances the instrument's ability to distinguish the small particles.

Keywords: Airborne particle counter, Lognormal distribution, Counting channel, Optimization.

1. Introduction

In recent years, as the effect of fog haze weather, environment detection, especially PM2.5 monitoring problem, receives more and more people's attention. Therefore, the research on method and the instrument for analyzing and accurately quantitative measuring pollution is of great significance. So far, people have developed many measuring methods based on different principle [1-4]. Among which, light scattering as a kind of important method, because of fast measuring speed, high precision and good repeatability, as well as got the favor of scientists and engineers. Consequently, it has been widely used in particle parameters measurement [5, 6].

In the detection system based on the principle of light scattering, the particles correspond to the discrete signal group of light scattering random voltage pulse. The system put the amplitude of the pulse signal as a basis for determining the size of particle [7, 8]. The particle morphology's irregular, light intensity's inhomogeneity of photosensitive area and other factors cause different photosensitive area, even if the same particles' signal amplitude and width, which indicates the randomness [9, 10]. So, different particles' counting distribution of signal amplitude (width) has a certain overlap [11], which leads to a certain measurement error.

This article analyzes the process of particle size's measurement based on typical light scattering method, and a statistical model of random measurement is established. Combined with double parameters analysis method of random pulse signal, it is found that the light scattering signal characteristics quantity meet logarithmic normal distribution with natural number as the independent variable and have statistical self-similarity. Based on which, the lognormal distribution is applied to the channel optimization of measuring instrument. The experimental results show that the optimization can save the counting channel, expand the measurement range, and enhance the instrument's ability to distinguish small particle size of particles.

2. Statistical Distribution of Standard Particle Light Scattering Signal

2.1. Optical Particle Counter Measurement System

The experimental device is shown in Fig. 1. Aerosol flows through an optical sensing volume, and the light scattered by a single particle is collected and focused by a mirror in the angular range. And then, the light signal is converted to an electrical pulse by a photo-detector. Through pre-amplification and multichannel collection, the voltage pulse of the particle is obtained by a computer. The experimental apparatus and measuring environment are easy to be realized, and the measuring accuracy could be guaranteed. Besides, the system can obtain enough data with good repeatability. The higher the accuracy of the measuring system in recording signals, the larger sample size is measured in unit time. So the statistical characteristics of random signals will be more obvious.

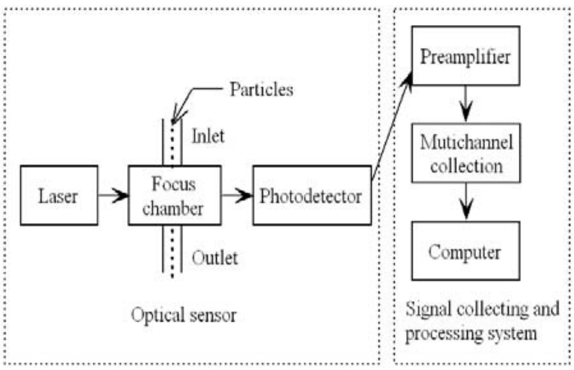

Fig. 1. Schematic diagram of optical particle counter system.

As a result of the discrete degree of standard particle's size is small, the same size's standard particle signal amplitude distribution reflects the nature of the interaction between particles and sensors. Therefore, the statistical distribution of measurement results reflect the randomness of sensor noise, photosensitive area in-homogeneity, the instability of scattered light collecting efficiency, etc. The PG-100 type particle generator of PMS Company is adopted in experiment, and diluted standard particle emulsion is put after spraying into measurement system. And, the large flow semiconductor new airborne particle counter is used to measure standard particles with different size. The mainly technical parameters are listed as follows. A collimated continuous laser beam (808 nm, 50 mW) is used as the light source, and the photosensitive area is about 0.5 mm×1 mm×8 mm. The reversed biased voltage is set to be 12 V, dark current is no more than 10^{-9}A. The sampling rate is 28.3 L/min, the maximal range of voltage is 5 V which is divided into 2048 counting channels. The maximal signal sampling rate is 20 MHz (the corresponding time-accuracy Δτ=0.05 μs), continuously sampled data of signal is up to 1×10^7, a 1000 level ultra-clean room is the measuring environment only for measuring the background noise.

2.2. The Characteristic Parameter of the Random Signal for Standard Particle

It is well known that the output signal of photo-electricity in airborne particle counter corresponds to a time series $f(t)$. However, the signal in actually experimental measurement could not be continuous, which can be attributed to the experimental results have a certain time accuracy Δt. In other words, $f(t)$ must be written as $f(k \cdot \Delta t)$ which is a natural number sequence. Taking a reference value f_0 to process the experimental results, a subset $V(t) = f(t) - f_0 > 0$ can be gotten. The elements, which have continuous time sequence in subset $V(t)$, compose a pulse structure with the amplitude and width are defined as V and τ respectively, as shown in Fig. 2.

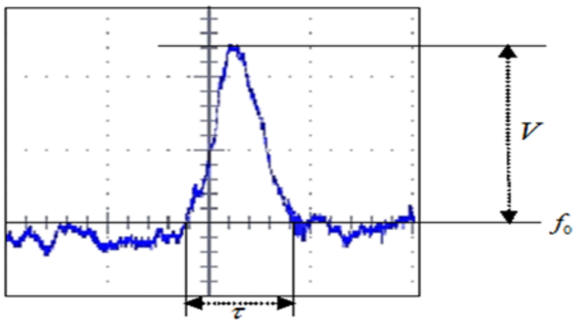

Fig. 2. Sketch map of the pulse structure.

On the basis of the above analysis, a new characteristic subset $A(V,\tau)$ composed by random pulse signal sequences can be used to describe the random signal group $f(t)$ which is dealt with f_0. Owing to the universality of $f(t)$, this signal processing method has universal significance, where Δt and ΔV represent the characteristics of the measurement devices, while V and τ are the characteristic parameters of the pulse signal. Once the parameter group f_0, $\Delta\tau$ and ΔV are given, the physical parameters of the signal subset $\{A(V,\tau)\}$ can be described by the natural numbers. That is, a set of natural numbers (l_τ, l_V) can used to establish the statistical mapping form of the signal subset $\{A(V,\tau)\}$:

$$\begin{aligned} l_\tau &= \left\lceil \frac{\tau_i}{\Delta\tau} - 1 \right\rceil \quad l_\tau \in \left(1,2,\cdots,L_\tau = \frac{\tau_M}{\Delta\tau} - 1\right) \\ l_V &= \left\lceil \frac{V_i}{\Delta V} \right\rceil \quad l_V \in \left(1,2,\cdots,L_V = \frac{V_M}{\Delta V}\right) \end{aligned} \tag{1}$$

According to the size sequence of the observation character values l_τ (or l_V), the characteristic subset and the statistically descriptive form can be established. Let N denotes the total number of $\{A(V,\tau)\}$, while $\Delta N(l_\tau)$ and $\Delta N(l_V)$ denote the pulse numbers of $\{A(V,\tau|\tau \in l_\tau \Delta\tau)\}$ and $\{A(V|V \in l_V \Delta V, \tau)\}$ respectively. Hence, the pulse counting distribution functions of characteristic subset can be expressed as:

$$\begin{aligned} p(l_\tau) &\equiv \frac{\Delta N(l_\tau)}{N} = \frac{N_{l_\tau}}{N} > 0 \\ q(l_V) &\equiv \frac{\Delta N(l_V)}{N} = \frac{N_{l_V}}{N} > 0 \end{aligned} \tag{2}$$

Considering the knowledge of the statistics, the distribution functions $p(l_\tau)$ and $q(l_V)$ will tend to steady values when the sample's number of subsets satisfies $1 \ll \Delta N(l_\tau), and\ \Delta N(l_V) \ll N$ in actual measurement.

2.3. Standard Particle Light Scattering Signal Statistical Distribution Law

The statistical distribution functions of aerosol's pulse signals $q(l_V)$ and $p(l_\tau)$ are the projection that measure results are l_V and l_τ in the same environment, which means that they must have the same nature. So, if the nature of the random effect is not changed, the type of the one-dimensional statistical distribution function related this nature will be unchanged. Based on this, we can conclude that one-dimensional distribution functions $q(l_V)$ and $p(l_\tau)$ have the similarity because they reflect the common characteristics of the random effect. According to the theory and calculation analysis, reasonable control the influence of random factors on the measurement results in the process of measuring can improve the performance of the sensor. By using the improved new laser airborne particle counter sensor, the statistical distributions of random scattering pulse signal produced by several kinds of standard particle (PSL) with the diameters are 0.38 μm, 0.499 μm, 0.6 μm, 0.8 μm and 0.99 μm are measured. The amplitude counting distributions are shown in Fig. 3 by the dotted line, and the x axis is counting channel value, while the y axis is statistical probability value.

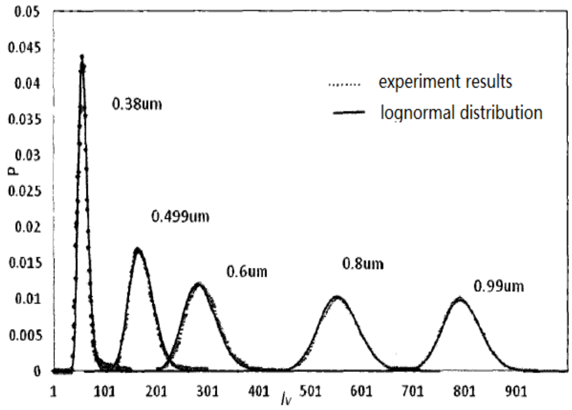

Fig. 3. Measured results and fitting distributions of standard particles.

As shown, the amplitude counting distributions have center asymmetry obviously. Besides, the nonlinearly similar characteristic between the different function sequences is quite well. According to the conclusions of the particle size, many research results shows that the lognormal distribution is a statistical distribution form with better effect for describing the particle parameters [12-14]. Therefore, we choose a statistical function with the form of log-normal distribution to fit the measured data. The fitting results can be seen as the sequence of solid lines in Fig. 3, and the corresponding equation satisfies:

$$q(l_V) = \frac{1}{\Omega \sigma_{\ln V} l_V} e^{-\frac{1}{2}\left(\frac{\ln l_V - \mu_{\ln V}}{\sigma_{\ln V}}\right)^2}, \tag{3}$$

where Ω is the normalized coefficient with a usual value $\sqrt{2\pi}$. It is easy to see, 5 standard particle pulse amplitude distribution curves meet logarithmic normal distribution, the fitting parameters such as Table 1 [15, 16].

Because the standard particles are sphere with the same material and surface structure, as well as the similar optical properties, the random effect of the measurement sensors for different size particles is similar. Hence, the geometric mean of the logarithmic normal distribution's fitting parameters is consistent with the certificated average diameter of standard particles.

Table 1. Statistic parameter values of the standard particle.

Sample	Total counting number N	Geometric mean $\mu_{\ln V}$	Geometric standard deviation $\sigma_{\ln V}$	Peak position l_V	Correlation coefficient *r*
0.38 μm	14802	4.54	0.25	57	0.93
0.499 μm	21010	5.14	0.24	154	0.97
0.6 μm	17886	5.60	0.22	280	0.98
0.8 μm	34062	6.32	0.28	553	0.98
0.99 μm	30223	7.20	0.25	790	0.94

3. The Nonlinear Transformation Statistical Invariance of Lognormal Distribution

The above experimental results show that the standard particle's random scattering pulse signal characteristic parameters obeys logarithmic normal distribution with the natural number as independent variable. It can be seen from Table 1, the fitting correlation coefficient is very high and the fitting effect is very good. Due to the similar random factors of different objects in the process of measurement, the pulse signal for the light scattering has statistical self-similarity law. For all characteristics statistical regularity obeys the lognormal distribution, a transformation rule can be found between the independent variables in theory, which makes experimental results of different characteristic parameters converge in geometry. Therefore, lognormal distribution should have self-similar properties of nonlinear transform unchanged. According to the differential mean value theorem, we can obtain [17]:

$$\begin{aligned} p_X(l_X)\triangle l_X &= \frac{\triangle l_X}{\Omega_X l_X \sigma_{X,\ln}} e^{-\frac{\left(\ln\frac{l_X}{l_{X,\mu}}\right)^2}{2\sigma_{X,\ln}^2}} \\ &= p_Y(l_Y)\triangle l_Y \\ &= \frac{e^{-\frac{\left(\ln\frac{l_Y}{l_{Y,\mu}}\right)^2}{2\sigma_{Y,\ln}^2}}}{\Omega_Y}\triangle\left\{\frac{1}{\sigma_{Y,\ln}}\ln\frac{l_Y}{l_{Y,\mu}}\right\} \end{aligned} \quad (4)$$

As expressed by Eq. (4), the independent variable of different characteristics parameters statistical distribution function can get the transformation relationship as:

$$l_Y = (bl_X)^{\frac{\sigma_{Y,\ln}}{\sigma_{X,\ln}}} \quad (5)$$

Obviously, the two independent variables, which have independent significance in the equation, could not take natural number sequence at the same time. When an argument is natural number sequence, another one is the non-integer count channel under the way of non-uniform division, which shows the nonlinear properties in the transformation. Thus, there can be thought a non-uniform channel divided way approach, so the different characteristics statistical distribution function curves coincide.

In further, transforming Eq. (5) to Eq. (4) by introducing scaling coefficient $Z = \Omega_X/\Omega_Y$ corresponding the transformation, we can have:

$$\begin{aligned} p_Y(l_Y)\triangle l_Y &= \frac{\triangle l_Y}{\Omega_Y l_Y \sigma_{Y,\ln}} e^{-\frac{\left(\ln\frac{l_Y}{l_{Y,\mu}}\right)^2}{2\sigma_{Y,\ln}^2}} \\ &= \frac{\Omega_X}{\Omega_Y}\cdot\frac{\triangle l_X}{\Omega_X l_X \sigma_{X,\ln}} e^{-\frac{(\ln l_X - \ln l_{X,\mu})^2}{2\sigma_{X,\ln}^2}} \\ &= Z\cdot p_X(l_X)\triangle l_X \end{aligned}, \quad (6)$$

where the expression of b in the domain is:

$$b = e^{\frac{\sigma_{X,\ln}}{\sigma_{Y,\ln}}\ln l_{Y,\mu} - \ln l_{X,\mu}} = \frac{l_{Y,\mu}^{\frac{\sigma_{X,\ln}}{\sigma_{Y,\ln}}}}{l_X} \quad (7)$$

Thus, with two independent characteristic parameters amplitude V and width τ, when the statistical distribution function of different physical characteristic parameters has a lognormal distribution form, as:

$$\frac{V}{\Delta V}\cdot\frac{1}{l_{V,\mu}} = \left(\frac{\tau}{\Delta\tau}\cdot\frac{1}{l_{\tau,\mu}}\right)^{\frac{\sigma_{\ln V}}{\sigma_{\ln\tau}}}, \quad (8)$$

where $\alpha = \sigma_{\ln V}/\sigma_{\ln\tau}$ is the fractal dimension of characteristic parameter.

As a result, we get the statistical fractal self-similar nonlinear transformation formula of the different random independent physical characteristics. And, it is also known that the fractal dimension is non-integer and only related to statistical discrete degree. From the above deduction, it can be seen that the statistical self-similar fractal nonlinear transformation relationship between different physical characteristics is only related to statistical parameters $l_{V,\mu}, l_{\tau,\mu}, \sigma_{\ln V}, \sigma_{\ln \tau}$, which reflects the constraint relationship between the overall nature and the internal statistical parameters of the random collection. Besides, the above deduction also reflects the equivalent position of different physical characteristics in the description of randomness for the random collection.

4. Airborne Particle Counter Counting Channel Optimization

Standard particle measurement corresponds to the sensor calibration process. When laser airborne particle counter is calibrated, standard particle voltage of the peak position is usually used to represent particle size. Then, we can gain the voltage corresponds to different size and achieve the effect of divided channel to count the particles' number. Fortunately, based on the research of the standard particle signal statistical distribution, looking for more and better optimal calibration method plays guiding role in improving sensors counting efficiency and particle size resolution.

From measurement results in Fig. 3, it can be found that this counter can separate different size particles completely, but there are defects of two aspects. First, the aerosol particle counter has a total of 2048 channels, but we only need about 900 channels to complete the calibration process of 5 kinds of standard particles, the rest of the channels are not used. Second, the particles of small size have the fewer counting channels, big size particles have the more counting channels. However, for the ultraclean environment of mainly used measuring instruments, people pay more attention to the distributions of small size particles.

Therefore, based on the nonlinear transform statistical invariance of the standard particle counting distribution curve, and the principle of guarantee adjacent size particles' resolution higher than 90 %, the sensor measurement channels for nonlinear are adjusted. Hereby, the magnification times of the small particle increase, and those of large size particles decreases. But no matter big or small size particles all can be accurately measured. For example, we can do nonlinear transformation based on power function in counting channel of Fig. 3 as Fig. 4.

Measured results and fitting distributions of standard particles by using the counting channel after the nonlinear transformation based on power function are shown in Fig. 5, the fitting parameters are such as those of Table 2 [15, 16]. Compared with Fig. 3, it can be found that different size particles' resolution remains the same after nonlinear transformation. Meanwhile, the statistical distribution of the standard particles is still in a form of logarithmic normal distribution. And, the statistical results of channel range originally in the range [1,901] now only 600 channels can be expressed, which means that we can get a wider range of particle size measurement within the inherent channel scope of the measuring instrument. That is to say, nonlinear transformation can increase measurement range of the instrument by 1/3.

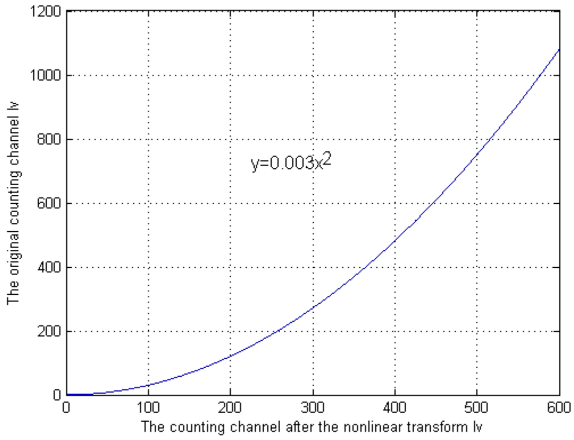

Fig. 4. The nonlinear transformation relation based on power function of counting channel.

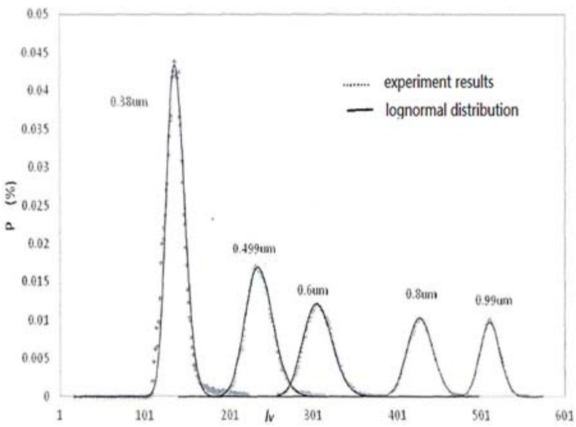

Fig. 5. Measured results and fitting distributions of standard particles after the nonlinear transformation.

Comparing Fig. 3 and Fig. 5, the form of particle statistical distributions has not any change after nonlinear transformation of counting channels, due to the properties of nonlinear transform invariant for lognormal distribution. Besides, the small particles' distribution curves move to the direction of big channels after transformation. The involved results manifest that this effects further improve the lower limit of the instrument as well as complete the distribution curves of the small particles and get more measurement details. In other words, this transformation can enhance the instrument's ability to distinguish small particle size.

Table 2. Statistic parameter values of the standard particle after the nonlinear transformation.

Sample	Total counting number N	Geometric mean $\mu_{\ln V}$	Geometric standard deviation $\sigma_{\ln V}$	Peak position l_V	Correlation coefficient r
0.38 μm	14802	4.54	0.25	138	0.92
0.499 μm	21010	5.14	0.24	227	0.94
0.6 μm	17886	5.60	0.22	306	0.97
0.8 μm	34062	6.32	0.28	429	0.97
0.99 μm	30223	7.20	0.25	513	0.96

5. Conclusion

The results with the airborne particle as the standard particle in counter are studied by theoretical analysis and experimental measurement. Experimental results show that the statistical distributions of light scattering random signal obey logarithmic normal distribution that regard natural number as the independent variable, under the condition of large sample and high precision. Further theoretical analysis results show that the logarithmic normal distribution has a good nonlinear transform statistical self-similarity. Based on this, the lognormal distribution is applied to the measuring instrument counting channels' optimization. The results manifest that the optimization can not only expand the measurement range of sensor by 1/3, and also promote the lower limit of the instrument, which can improve the instrument's ability to distinguish small particle size.

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References

[1]. David C. Doughty, Steven C. Hill, Automated aerosol Raman spectrometer for semi-continuous sampling of atmospheric aerosol, *Journal of Quantitative Spectroscopy and Radiative Transfer*, Vol. 188, Issue 4, 2017, pp. 103-117.

[2]. Evgeny Kolesnikov, Gopalu Karunakaran, Anna Godymchuk, *et al*., Investigation of discharged aerosol nanoparticles during chemical precipitation and spray pyrolysis for developing safety measures in the nano research laboratory, *Ecotoxicology and Environmental Safety*, Vol. 139, Issue 5, 2017, pp. 116-123.

[3]. Benjamin Figgis, Ahmed Ennaoui, Bing Guo, Outdoor soiling microscope for measuring particle deposition and resuspension, *Solar Energy,* Vol. 137, Issue 12, 2016, pp. 158-164.

[4]. Lei Ding, Jin Bi Zhang, Hai Yang Zheng, A method of simultaneously measuring particle shape parameter and aerodynamic size, *Atmospheric Environment*, Vol. 139, Issue 8, 2016, pp. 87-97.

[5]. Gu Fang, Yang Juan, Bian Baomin, *et al*., Aerosol mass density algorithm based on average mass, *Acta Optica Sinica*, Vol. 27, Issue 9, 2007, pp. 1706-1710 (in Chinese).

[6]. Tian'en Wang, Jianqi Shen, Chengjun Lin, Iterative algorithm based on a combination of vector similarity *measure* and B-spline functions for particle analysis in forward scattering, *Optics & Laser Technology*, Vol. 91, Issue 7, 2017, pp. 13-21.

[7]. Stavros Amanatidis, M. Matti Maricq, Leonidas Ntziachristos, Measuring number, mass, and size of exhaust particles with diffusion chargers: The dual Pegasor Particle Sensor, *Journal of Aerosol Science*, Vol. 92, Issue 4, 2016, pp. 1-15.

[8]. Teimour Tajdari, Mohd Fua'ad Rahmat, Norhaliza Abdul Wahab, New technique to measure particle size using electrostatic sensor, *Journal of Electrostatics*, Vol. 72, Issue 2, 2014, pp. 120-128.

[9]. Yangang Lius, Peter H. Daum, The Effect of Refractive Index on Size Distributions and Light Scattering Coefficients Derived from Optical Particle Counters, *Journal of Aerosol Science*, Vol. 31, Issue 8, 2000, pp. 945-957.

[10]. Yang, Juan, Chen, Yun-Yun. The Aerosol Mass Measurement Based on the Two-parameter Mathematical Model in Optical Sensor, *Sensors & Transducers*, Vol. 175, Issue 7, July 2014, pp. 220-225.

[11]. R. Jaenicke, The optical particle counter: cross-sensitivity and coincidence, *Aerosol Science*, Vol. 30, Issue 7, 1972, pp. 95-111.

[12]. Lee K. W., Lee Y J, Han D. S., The Log-Normal Size Distribution Theory for Brownian Coagulation in the Low Knudsen Number Regime, *Journal of Colloid and Interface Science*, Vol. 188, Issue 4, 1997, pp. 486.

[13]. L. B. Kiss, J. Soderlund, G. A. Niklasson, C. G. Granqvist, The real origin of lognormal size distributions of nanoparticles in vapor growth processes, *Nanostructured Materials,* Vol. 12, 1999, pp. 327-332.

[14]. Kyu-Tae Park, Dongho Park, Sang-Gu Lee, Design and performance test of a multi-channel diffusion charger for real-time measurements of submicron aerosol particles having a unimodal log-normal size distribution, *Journal of Aerosol Science*, Vol. 40, Issue 10, 2009, pp. 858-867.

[15]. Yan Zhen Gang, Bian Bao Min, Wang Shou Yu, Analysis of random laser scattering pulse signals with lognormal distribution, *Chinese Physics B,* Vol. 22, Issue 6, 2013, pp. 246-256.

[16]. Yan Zhen Gang, Lin Ying Lu, Yang Juan, The distribution functions of characteristic quantities for

random noise signal of photodetector, *Acta Physica Sinica*, Vol. 61, Issue 20, 2012, pp. 200502 (in Chinese).

[17]. Yang Juan, Bian Baomin, Yan Zhengang, The Model of Two Parameters Statistical Fractal for Aerosol Scattering Pulse Signal, *Acta Physica Sinica*, Vol. 60, Issue 1, 2011, pp. 010508-1 – 010508-6 (in Chinese).
