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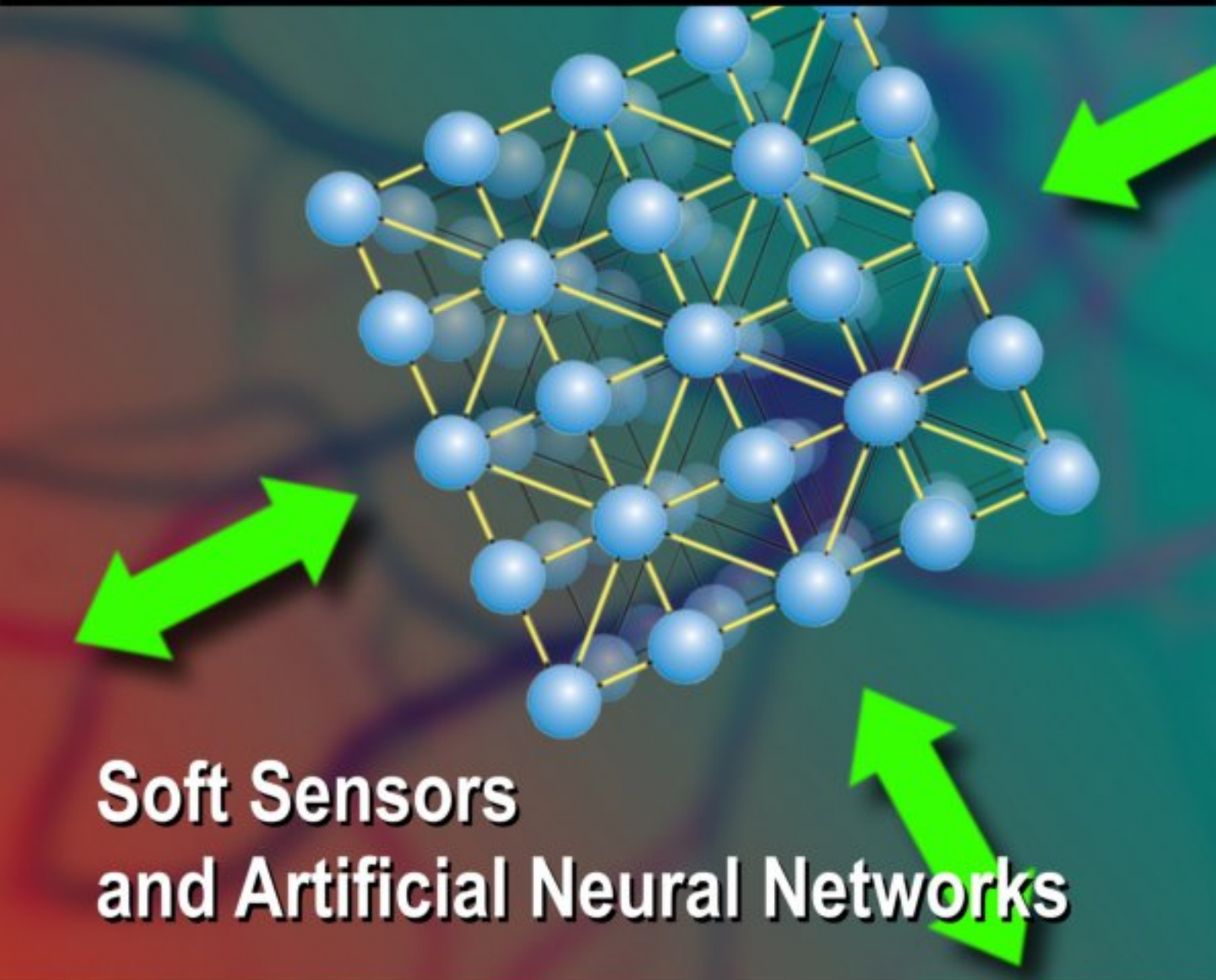

Soft Sensors and Artificial Neural Networks

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Contents

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Design of Adaptive PID Controller Based on Maximum Modulus Theorem for Inverted Pendulum on a Cart

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Abstract: Stabilization of a pendulum in inverted position when the base is moving is one of the challenging tasks in aircraft, automobile and ship industries. So in this paper the system under consideration is inverted pendulum on a cart which has single degree of freedom (angular rotation of pendulum). In industries depending upon the variation in operating conditions controller have to be tuned accordingly. That is depending upon the desired conditions, controller parameters should have flexibility i.e. the principle of adaptive control. This paper in brief discuss about design of a tuning parameter based on Maximum Modulus Theorem so that the required time domain specifications can be achieved. *Copyright © 2012 IFSA.*

Keywords: Inverted pendulum on cart, Maximum Modulus Theorem, Tuning parameter, PID Controller, Time domain specifications.

1. Introduction

An Inverted Pendulum on a Cart consists of a pendulum which is attached to the cart (moving base pendulum). The aim of this paper is to design adaptive PID controller based on Maximum Modulus Theorem.

The system inverted pendulum on a cart is considered as a single input single output (SISO) system with force applied to the cart as input and the angular displacement of the pendulum as output.

The only assumption taken is friction is negligible.

2. Inverted Pendulum on a Cart

An inverted pendulum is a pendulum which has its mass above its pivot point (Fig.). It is often implemented with the pivot point mounted on a cart that can move horizontally and may be called a cart and pole. Whereas a normal pendulum is stable when hanging downwards, an inverted pendulum is inherently unstable, and must be actively balanced in order to remain upright, either by applying a torque at the pivot point or by moving the pivot point horizontally as part of a feedback system. A simple demonstration is achieved by balancing an upturned broomstick on the end of one's finger.

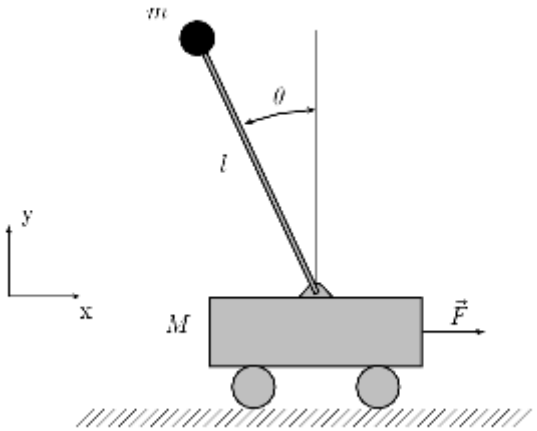

Fig. 1. Inverted Pendulum on a moving cart.

3. System Modeling

The equations of motion can be derived using Lagrange's equations. We refer to the drawing above where Θ(t) is the angle of the pendulum of length l with respect to the vertical direction and the acting forces are gravity and an external force *F* in the x-direction. Define x(t) to be the position of the cart. Lagrangian (L) is given by difference between total Kinetic Energy and total Potential Energy.

$$L = KE_{Total} - PE_{Total} \tag{1}$$

$$L = \frac{1}{2}Mv_1^2 + \frac{1}{2}mv_2^2 - mg\ell\cos\theta \tag{2}$$

,

where v_1 is the velocity of the cart and v_2 is the velocity of the point mass m. v_1 and v_2 can be expressed in terms of x and θ by writing the velocity as the first derivative of the position:

$$v_1^2 = \dot{x}^2 \tag{3}$$

$$v_2^2 = \left(\frac{d}{dt}(x - \ell\sin\theta)\right)^2 + \left(\frac{d}{dt}(\ell\cos\theta)\right)^2 \tag{4}$$

Simplifying the expression for v_2 leads to:

$$v_2^2 = \dot{x}^2 - 2\ell\dot{x}\dot{\theta}\cos\theta + \ell^2\dot{\theta}^2 \tag{5}$$

The Lagrangian is now given by:

$$L = \frac{1}{2}(M+m)\dot{x}^2 - m\ell\dot{x}\dot{\theta}\cos\theta + \frac{1}{2}m\ell^2\dot{\theta}^2 - mg\ell\cos\theta \tag{6}$$

and the equations of motion are:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = F \tag{7}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = 0 \tag{8}$$

substituting L in these equations and simplifying leads to the equations that describe the motion of the inverted pendulum:

$$(M+m)\ddot{x} - m\ell\ddot{\theta}\cos\theta + m\ell\dot{\theta}^2\sin\theta = F \tag{9}$$

$$\ell\ddot{\theta} - g\sin\theta = \ddot{x}\cos\theta \tag{10}$$

These equations are nonlinear, but since the goal of a control system would be to keep the pendulum upright the equations can be linearized around $\theta \approx 0$.

The transfer function of the system is obtained as:

$$\frac{\Theta(s)}{F(s)} = \frac{1}{s^2(l(M+m) - ml^2)} \tag{A}$$

The system is double integrating system having two poles at the origin of s plane which is a highly unstable system.

3. Design of Adaptive PID Controller based on Maximum Modulus Theorem

If the plant has a zero at right half of s- plane then the theorem states that:

$$\|W(1 - QP)\|_\infty \geq W\left(\frac{1}{\theta}\right), \tag{11}$$

where $s = \frac{1}{\theta}$ is the zero on the right half of s plane of the plant P, and W is the weighting function.

Minimizing the left hand side of the of the above equation, we get

$$\min\|W(1 - QP)\|_\infty = W\left(\frac{1}{\theta}\right) \tag{12}$$

Let us consider the plant

$$G(s) = \frac{ke^{-\theta s}}{s^2} \tag{13}$$

By Taylor series approximation G(s) can be written as:

$$G(s) = \frac{k(1-\theta s)}{s^2} \tag{14}$$

Solving equation (a) gives the optimal Q which is given by the expression:

$$Q_{lim}(s) = \frac{W(s) - \theta}{W(s)sG(s)} = \frac{s}{K} \tag{15}$$

So Q(s) will result in an improper transfer function given by:

$$Q_{lim}(s) = \frac{s^2}{K} \tag{16}$$

To implement the above transfer function, it should be cascaded with a low pass filter $f_c(s)$

Selection of $f_c(s)$:

$$\lim_{s \to 0}\left(1 - Q(s)P(s)\right) = 0 \tag{17}$$

s tends to zero means t tends to infinity (final value theorem), so that the process results in a constant output.

The process model

$$G(s) = \frac{ke^{-\theta s}}{s^2}$$

is a double integrator.

If a step disturbance is given to it, then it becomes equivalent to:

$$G(s) = \frac{ke^{-\theta s}}{s^3}$$

So the sensitivity function S should posses three zeros at the origin of splane. This rejects the disturbance which is injected at the process input side.

Therefore we select $J_c(s)$ as:

$$f_c(s) = \frac{as^2 + bs + 1}{(\lambda_c s + 1)^4}, \tag{18}$$

where 'λ_c' is the tuning parameter.
By employing low pass filter, the suboptimal Q(s) can be obtained as

$$Q(s) = Q_{lm}(s) f_c(s) \tag{19}$$

To find a and b:

$$\lim_{s\to 0}\frac{d}{ds}(1-QG)=0 \tag{20}$$

$$\lim_{s\to 0}\frac{d}{ds}(1-\frac{s^2(as^2+bs+1)}{k(\lambda_c s+1)^4(1-\theta s)})=0 \tag{21}$$

Solving the above equation, we get:

$$a=6\lambda_c^2+4\lambda_c+\theta^2$$

$$b=4\lambda_c+\theta$$

So Q(s) is obtained as in the form given below:

$$Q(s)=\frac{s^2((6\lambda_c^2+4\lambda_c+\theta^2)s^2+(4\lambda_c+\theta)s+1)}{k(\lambda_c s+1)^4} \tag{22}$$

We have the general expression of Q as given below:

$$Q=\frac{C}{1+PC} \tag{23}$$

So the controller C is given by:

$$C=\frac{Q}{1-QP} \tag{24}$$

Substituting Q and P in C we obtain:

$$C(s)=\frac{1}{k}\frac{(6\lambda_c^2+4\lambda_c\theta+\theta^2)s^2+(4\lambda_c+\theta)s+1}{\lambda_c^4 s^2+(4\lambda_c^3+6\lambda_c^2\theta+4\lambda_c\theta^2+\theta^3)s} \tag{25}$$

This is in the form of PID controller

$$C(s)=k_c(1+\frac{1}{T_i s}+\frac{T_d s}{T_f s+1}) \tag{26}$$

Comparing both the equations, we get

$$T_f=\frac{\lambda_c^4}{4\lambda_c^3+6\lambda_c^2\theta+4\lambda_c\theta^2+\theta^3} \tag{27}$$

$$T_d=\frac{6\lambda_c^2+4\lambda_c\theta+\theta^2}{T_i}-T_f \tag{28}$$

$$T_i=4\lambda_c+\theta-T_f \tag{29}$$

$$k_c=\frac{T_i}{k(4\lambda_c^3+6\lambda_c^2\theta+4\lambda_c\theta^2+\theta^3)} \tag{30}$$

Let l=1m, m=.5 kg, M=10 kg,

Substituting in equation (A),

$$\frac{\Theta(s)}{F(s)} = \frac{.1}{s^2}$$

For this system time delay, Θ=0 s.

Case 1. Let tuning parameter, $\lambda_c = 1$

So,

$$T_f = .25$$
$$T_i = 3.75$$
$$T_d = 1.35$$
$$k_c = 9.375$$

The step response and stability can be shown as given in Fig. 2.

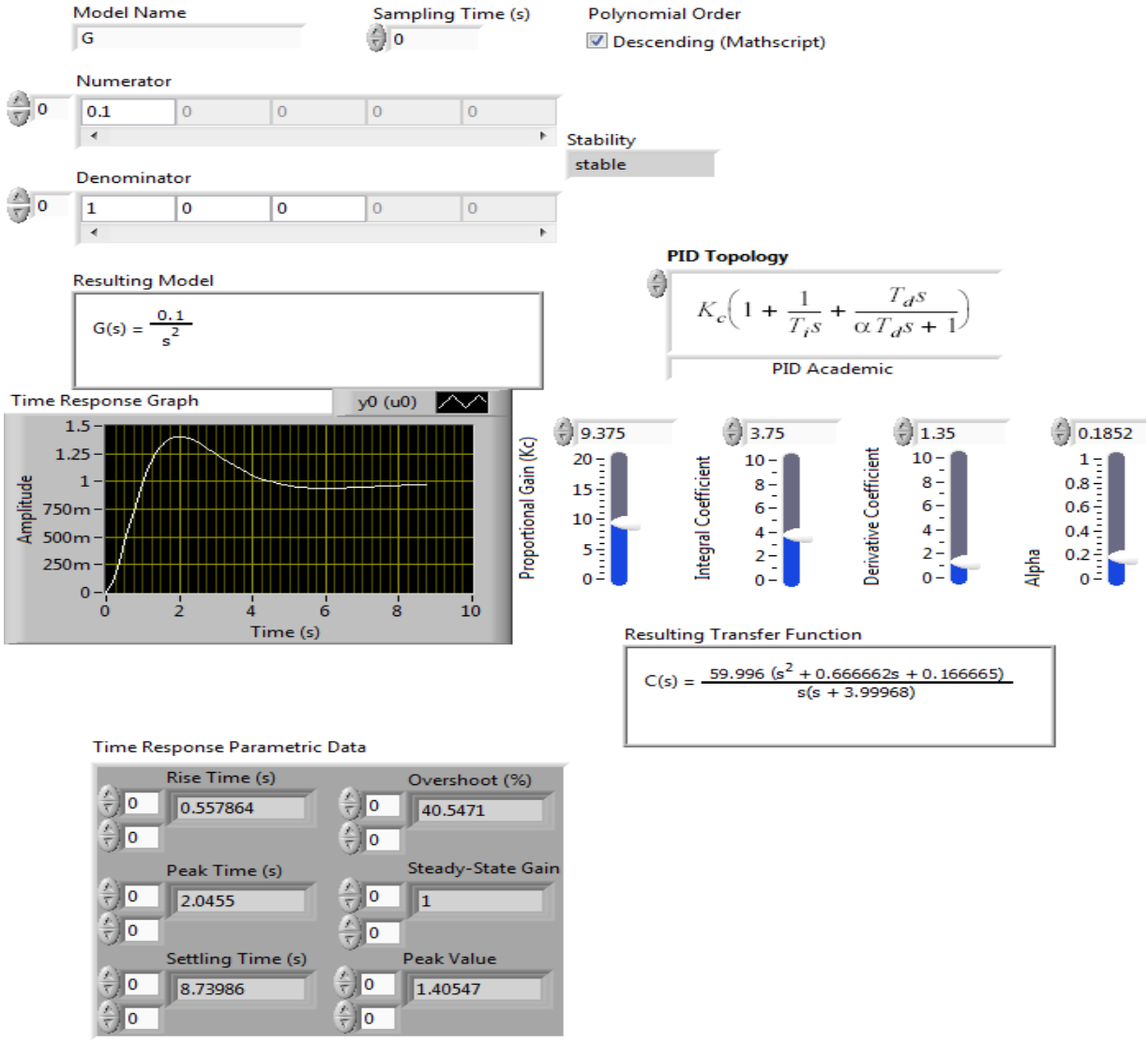

Fig. 2. Step response, time response parametric data, resulting transfer function(controller) and stability analysis for system with controller with tuning parameter =1.

Case 2. Let tuning parameter $\lambda_c = 2$,

So,

$$T_f = .5$$
$$T_i = 7.5$$
$$T_d = 2.7$$
$$k_c = 2.34375$$

The step response and stability can be shown as given in Fig. 3.

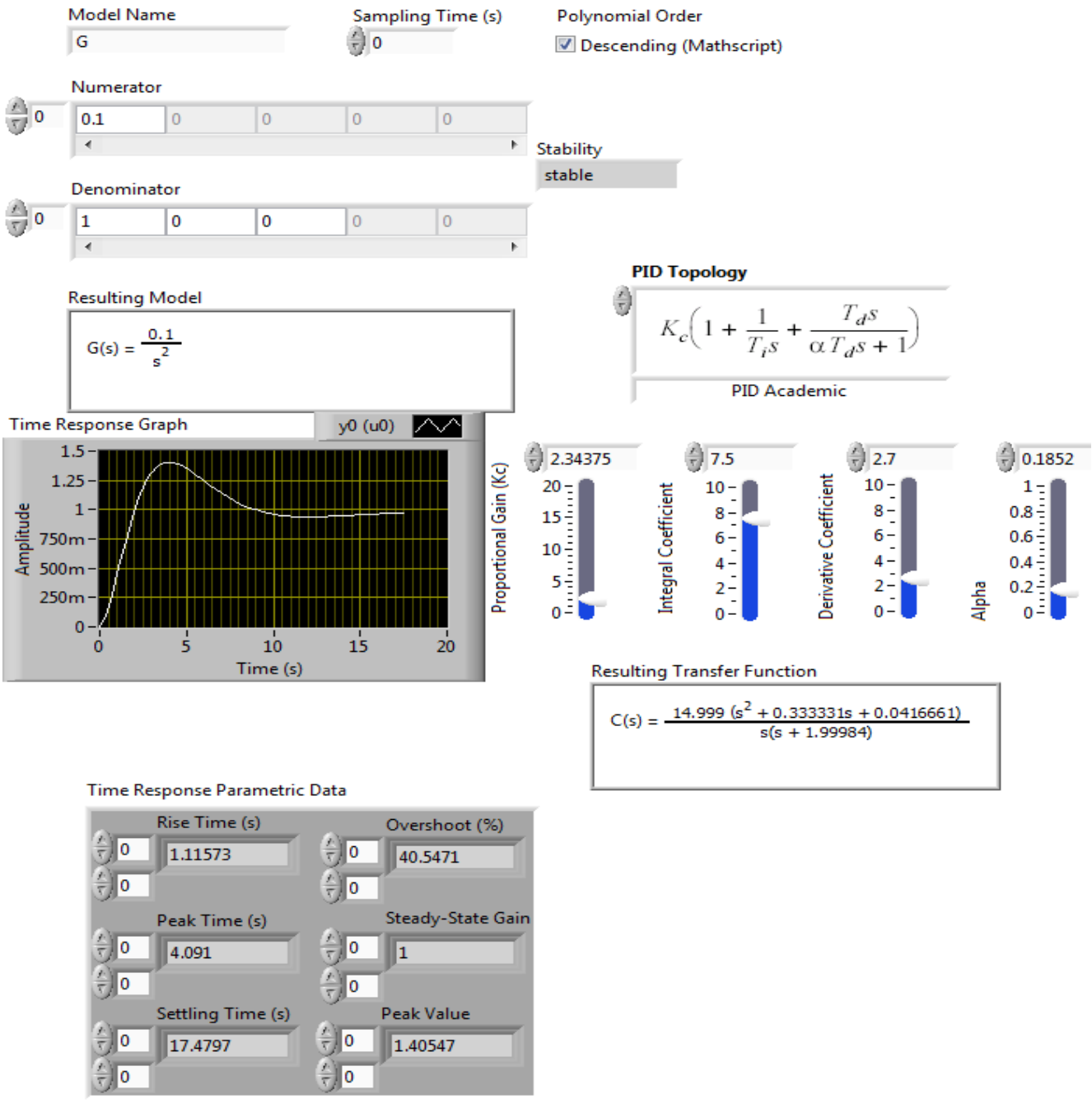

Fig. 3. Step response, time response parametric data, resulting transfer function(controller) and stability analysis for system with controller with tuning parameter =2

The block diagram for above is shown in Fig. 4.

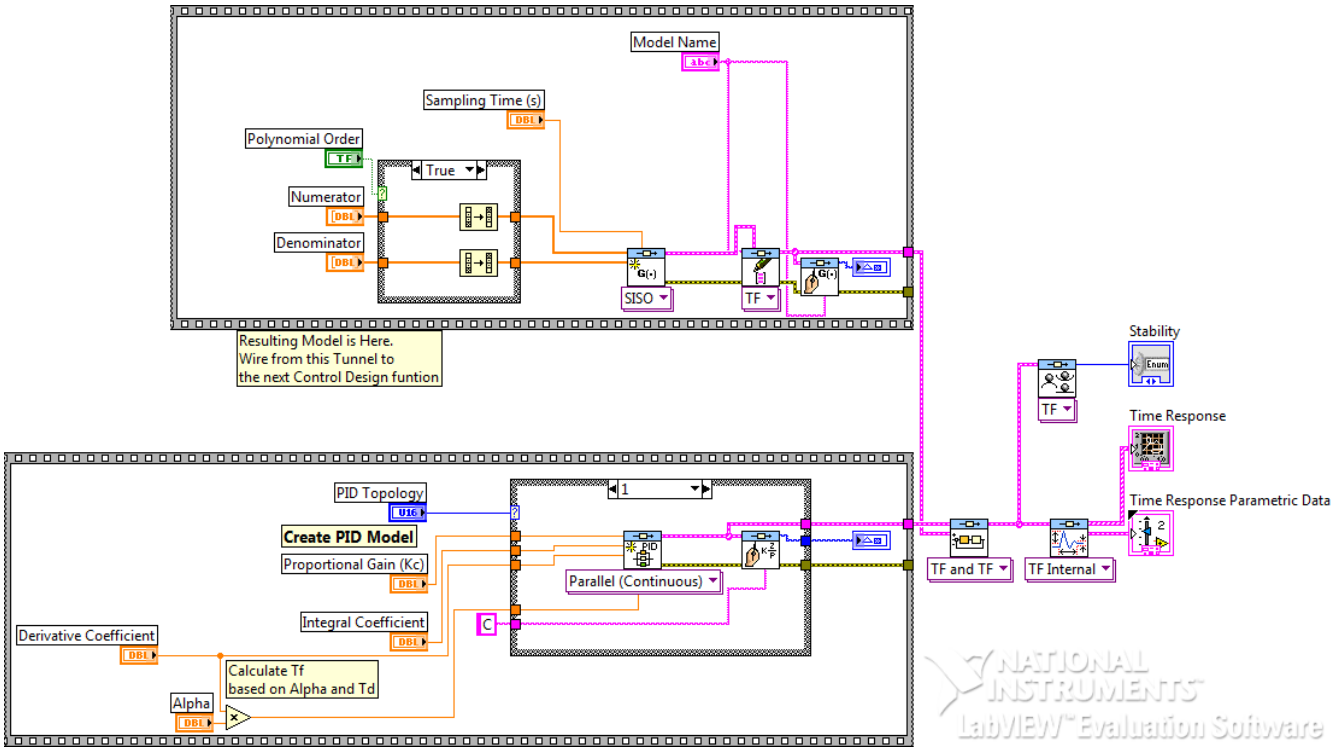

Fig. 4. Block diagram for Step response, time response parametric data, resulting transfer function (controller) and stability analysis for system with controller with different tuning parameters

So the inference is that by varying the tuning parameter, the system with controller can be maintained under stability conditions provided, there is a variation in rise time, peak time and settling time. If tuning parameter is increased these three parameters increases and vice versa. This can be referred to as an adaptive tuning method that is it is a PID controller tuning method by the variation of single tuning parameter.

4. Conclusions

In this paper the system considered is inverted pendulum on a cart. Without considering friction, system modeling is performed. From that it is clear that the system under consideration is a double integrating system which is highly unstable. For designing an adaptive PID controller by variation of only one tuning parameter for the system, Maximum Modulus Theorem is taken into account. From the theorem tuning parameter λ is found. By variation of tuning parameter, different PID values are obtained. So depending upon the requirement in time response parameters such as rise time, peak time and settling time, tuning parameter can be adjusted such that required specifications are met with.

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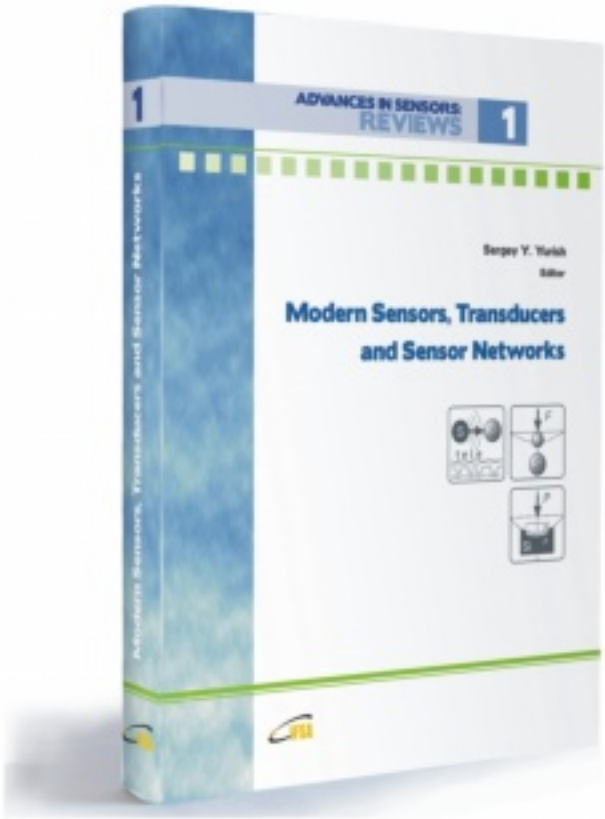

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