

## A Novel Modified Algorithm with Reduced Complexity LDPC Code Decoder

\* Song Yang, Bao Nanhai, Cai Chaoshi

School of Information Engineering, Communication University of China, Beijing, 100024, China

\* Tel.: 18500077220

\* E-mail: [icysongyang@163.com](mailto:icysongyang@163.com)

*Received: 31 May 2014 / Accepted: 29 August 2014 / Published: 31 October 2014*

---

**Abstract:** A novel efficient decoding algorithm reduced the sum-product algorithm (SPA) Complexity with LPDC code is proposed. Base on the hyperbolic tangent rule, modified the Check node update with two horizontal process, which have similar calculation, Motivated by the finding that sun-min (MS) algorithm reduce the complexity reducing the approximation error in the horizontal process, simplify the information weight small part. Compared with the exiting approximations, the proposed method is less computational complexity than SPA algorithm. Simulation results show that the author algorithm can achieve performance very close SPA. *Copyright © 2014 IFSA Publishing, S. L.*

**Keywords:** LDPC, Sum-product algorithm (SPA), Sun-min (MS) algorithm, Reduced complexity.

---

### 1. Introduction

Low-density parity-check (LDPC) codes were proposed by Gallager in the 1960s [1], but the technology were forgotten until the late 1990s when MacKay [2], than the LDPC code has attracted academia and industry by the excellent performance. Iterative decoding of binary Low-density parity-check (LDPC) codes has recently been shown to approach the capacity Shannon limits within 0.0045 dB [3]. The attractive feature of the LDPC code is decoding algorithm, Sum-Product Algorithm (SPA) [4] is the most efficient iterative decoding algorithm of the LDPC code, but the SPA has many communication complexity limits practice. Many SPA-based simplified algorithm are propose, such as reducing the computational complexity [5]. The min-sum algorithm (MS) [6] is one of suboptimal approximation about the SPA, which simplifies the horizontal process of the SPA. Similarly, in order to simplify the MS decoding algorithm, the normalized MS algorithm (Normalized-MS) [7], modified MS

algorithm (offset-MS) is proposed [8]. Recently, many low-complexity decoding algorithms have been proposed about MS algorithm. In [7], through Jacobian approach was simplify the check node update computation based on two random values, than a dual min-sum algorithm was proposed while only the minimum value was considered[9].

In [10], achieves essentially optimal performance by applying scaling in the decoder's extrinsic information which significantly simplifies the check node update computation.

In this paper, motivated by [11], a horizontal process correction term is used to improve the decoding performance of the Min-Sum algorithms, the author modified the Check node update with two horizontal process and simplify the information weight small part which use MS algorithm is proposed. It can effectively reduce the computational complexity and can ensure the decoding efficiency.

The remainder of this paper is organized as follows. The Sum-Product Algorithm (SPA) and min-sum algorithm (MS) are introduced in Section 2,

presents our proposed algorithms in section 3, simulation results and discussions are given in Section 4. Finally, we conclude this paper in Section 5.

## 2. SPA and MS

The SPA belongs to the family of message passing decoding algorithms which are based on the bipartite graph representation of the code, the decoding consists of iterative message passing between bit-nodes and check nodes in the graph. Next, we describe the SPA with log-likelihood ratio (LLR) messages. We denote by  $L_{mn}$ ,  $Z_n$  and  $Z_{mn}$ , the bit-to-check message from bit  $m$  to check  $n$ , the check-to-bit message from check  $n$  to bit  $m$ , and the decoder output for bit  $m$ . The SPA is summarized in three steps.

### 2.1. Sum-product Algorithm (SPA)

Step 1: Initialization.

$$Z_{nm}(x_n) = L(x_n | y_n) = \frac{2y_n}{\sigma^2}, \quad (1)$$

Step 2: Iteration:

a) Horizontal step (check node update)

For each  $m, n$

$$L_{mn} = 2 \tanh^{-1} \left( \prod_{n' \in N(m) \setminus n} \tanh \left( \frac{Z_{mn'}}{2} \right) \right), \quad (2)$$

b) Vertical step (bit node update)

For each  $n$

$$Z_n = r_n + \sum_{m \in M(n)} L_{mn}, \quad (3)$$

For each  $m, n$

$$Z_{mn} = Z_n - L_{mn}, \quad (4)$$

Step 3: Decision

If  $Z_{mn} \geq 0$ , then  $x_n = 0$  and if  $Z_{mn} \geq 0$ , then  $x_n = 1$ . Otherwise, go to step 1. If a certain number of decoding iterations is reached and the algorithm does not halt, then a decoding failure is reported. SPA has better performance but with high complexity. The horizontal process calculates the hyperbolic tangent function and the hyperbolic arctangent function.

### 2.2. Min-sum Algorithm (MS)

The check node update computation of the min-sum algorithm is [9]. MS algorithm simplifies the horizontal process as follows.

$$L_{mn} = \prod_{n' \in N(m) \setminus n} \text{sgn}(L(R_{mk})) \left( \min_{n' \in N(m) \setminus n} |L(R_{mk})| \right), \quad (5)$$

This simplification uses only minimum operations. Thus the complexity of the Min-Sum algorithm is significantly less than that of the SPA. However, this simplified structure brings some loss of performance. The performance loss can be alleviated by some modifications, which can be close to that of SPA. In this paper, the Author simplify the horizontal process, reduce the computational complexity, and better performance than MS algorithm.

## 3. Performance Algorithm Analysis

The SPA Horizontal step form(2), we can obtain

$$y = 2 \tanh^{-1} \left( \tanh \left( \frac{x_1}{2} \right) \tanh \left( \frac{x_2}{2} \right) \cdots \tanh \left( \frac{x_n}{2} \right) \right), \quad (6)$$

The horizontal process can be simplified to a process with only two inputs. We define the horizontal process result for the two part inputs

$$y_1 = 2 \tanh^{-1} \left( \tanh \left( \frac{x_1}{2} \right) \tanh \left( \frac{x_2}{2} \right) \cdots \tanh \left( \frac{x_{n/2}}{2} \right) \right), \quad (7)$$

$$y_2 = 2 \tanh^{-1} \left( \tanh \left( \frac{x_{n/2+1}}{2} \right) \tanh \left( \frac{x_{n/2+2}}{2} \right) \cdots \tanh \left( \frac{x_n}{2} \right) \right), \quad (8)$$

Submitting (7), (8) into (6)

$$y = 2 \tanh^{-1} \left( \tanh \left( \frac{y_1}{2} \right) \tanh \left( \frac{y_2}{2} \right) \right), \quad (9)$$

The check node update process can be conceded two horizontal process. We just consider reduce the complex of horizontal

Then, from the Jacobi an approach [5]:

$$\begin{aligned} L(U \oplus V) &= \log \left( \frac{1 + \exp\{L(U) + L(V)\}}{\exp\{L(U)\} + \exp\{L(V)\}} \right) \\ &= \text{sgn}\{L(U)\} \cdot \text{sgn}\{L(V)\} \min\{|L(U)|, |L(V)|\} \\ &\quad + \log(1 + \exp\{-|L(U) + L(V)|\}) \\ &\quad - \log(1 + \exp\{-|L(U) - L(V)|\}), \end{aligned} \quad (10)$$

Use Jacobian approach into (9)

$$\begin{aligned} y &= \text{sgn}\{y_1\} \cdot \text{sgn}\{y_2\} \min\{y_1, y_2\} \\ &\quad + \log(1 + \exp\{-|y_1 + y_2|\}) \\ &\quad - \log(1 + \exp\{-|y_1 - y_2|\}), \end{aligned} \quad (11)$$

If  $y_1 < y_2$

$$y = |y_1| + \log(1 + \exp\{-|y_1 + y_2|\}) - \log(1 + \exp\{-|y_1 - y_2|\}) \quad (12)$$

From [10], we know the second term is smaller than the third term,

$$y \approx |y_1| - \log(1 + \exp\{-|y_1 - y_2|\}), \quad (13)$$

Then, we calculate the  $y_2$  through the min-sum algorithm,

$$y_2 = \text{sgn}\{x_k\} \cdot \min\{x_k\}, k \in [1 \sim \frac{n}{2}], \quad (14)$$

We obtain the following approximation:

$$y = |y_1| - \log(1 + \exp\{-|y_1 - x_k|\}), \quad (15)$$

## 4. Simulation Result

In this section, we report simulations that have been recorded two different code length about LDPC, which will be selected ( $N=576$  and  $N=2304$ ) and the code rate is  $R=1/2, 2/3, 3/4, 5/6$ . Via SP algorithm, MS algorithm and our improved algorithm, both Fig. 1 and Fig. 2 depict BER with 100 iterations.

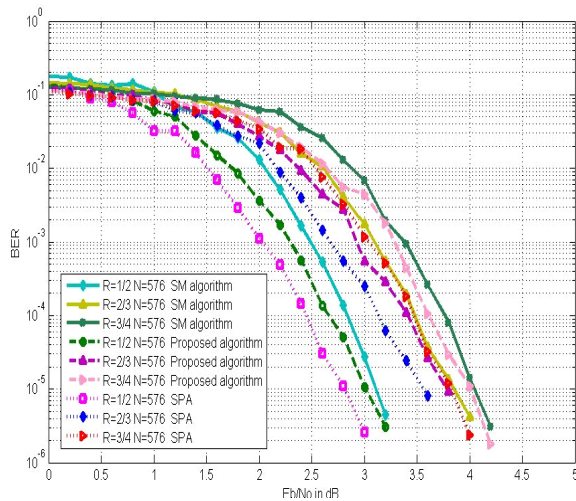


Fig. 1. LDPC code with  $N=576$  at most 100 iterations.

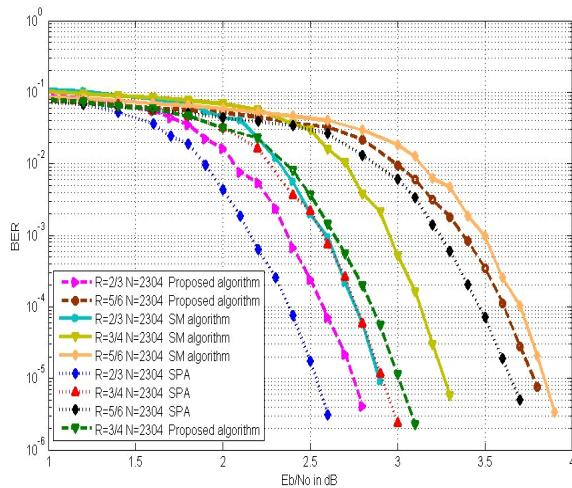


Fig. 2. LDPC code with  $N=2304$  at most 100 iterations.

The encoded bits are binary phase-shift keying (BPSK) modulated and transmitted with AWGN channel. Both Fig. 1 and Fig. 2 show that our algorithm has better performance than the min-sum algorithm and has almost the same performance as the SPA. Our method has 0.2-0.3 dB gain with min-sum algorithm. Mathematical transformations reduce computational complexity. It comes to the following conclusion: our improved algorithm is feasible and effective.

## 5. Conclusions

In this article, a novel effective decoding algorithm for LDPC codes has proposed. Based on the hyperbolic tangent rule, the Check node update with two horizontal processes is modified. Two horizontal processes with different methods are calculated. Compared with the existing approximations for the SPA, the proposed method obtains less computational complexity and better performance than min-sum algorithm with small increasing computational complexity.

## References

- [1]. R. G. Gallager, Low-density parity-check codes, *IRE Transactions on Information Theory*, Vol. IT-8, January 1968, pp. 21–28.
- [2]. J. C. MacKay, Good error-correcting codes based on very sparse matrices, *IEEE Transactions on Information Theory*, Vol. 45, Issue 2, 1999, pp. 399–431.
- [3]. S.-Y. Chung, G. D. Forney, Jr., T. J. Richardson, and R. Urbanke, On the design of low-density parity-check codes within 0.0045 dB of the Shannon limit, *IEEE Communication Letters*, Vol. 5, Issue 2, February 2001, pp. 58–60.
- [4]. T. Richardson and R. Urbanke, The capacity of low-density parity-check codes under message-passing decoding, *IEEE Transactions on Information Theory*, Vol. 47, Issue 2, 2001, pp. 599–618.
- [5]. X. Hu, E. Eleftheriou, D. Arnold, and A. Dholakia, Efficient implementations of the sum-product algorithm for decoding LDPC codes, in *Proceedings of the IEEE Global Telecommunications Conference*, 2001, Vol. 2, pp. 1036–1036E.
- [6]. N. Wiberg, H. A. Loeliger, and R. Koetter, Codes and iterative decoding on general graphs, *European Transactions on Telecommunication*, Vol. 6, Issue 5, 1995, pp. 513–526.
- [7]. J. Chen, A. Dholakia, E. Eleftheriou, M. P. C. Fossorier, and X. Hu, Reduced-complexity decoding algorithm for low-density parity-check codes, *IEEE Transactions on Communication*, Vol. 53, Issue 8, 2005, pp. 1288–1299.
- [8]. N. Pandya, and B. Honary, Low-complexity decoding of LDPC codes, *IET Transactions on Electronics Letters*, Vol. 43, Issue 18, 2007, pp. 990–991.
- [9]. J. Li, and X. D. Zhang, Reduced-complexity belief propagation decoding for low-density parity-check codes, *Electronic Letters*, Vol. 44, Issue 3, 2008, pp. 220–222.

- [10]. S. Papaharalabos and P. T. Mathiopoulos, Simplified sum-product algorithm for decoding LDPC codes with optimal performance, *Electronic Letters*, Vol. 45, Issue 2, 2009, pp. 116–117.
- [11]. Qian Chen, Lei Weilong, Wang Zhaocheng, Low complexity LDPC decoder with modified sum-product algorithm, *Tsinghua Science and Technology*, Vol. 18, No. 1, 2013, pp. 57-61.

2014 Copyright ©, International Frequency Sensor Association (IFSA) Publishing, S. L. All rights reserved.  
(<http://www.sensorsportal.com>)



## CALL FOR PAPERS

IEEE SENSORS 2014 is intended to provide a forum for research scientists, engineers, and practitioners throughout the world to present their latest research findings, ideas, and applications in the area of sensors and sensing technology. IEEE SENSORS 2014 will include keynote addresses and invited presentations by eminent scientists. The Conference solicits original state-of-the-art contributions as well as review papers.

### Topics of interest include but are not limited to:

Phenomena, Modeling, and Evaluation  
Chemical and Gas Sensors  
Biosensors  
Optical Sensors  
Mechanical and Physical Sensors  
Sensor/Actuator Systems  
Sensor Networks  
Other Sensor Topics  
- Materials, processes, circuits, signals and interfaces, etc.

### Publication Plans

Presented papers will be included in the Proceedings of IEEE SENSORS 2014 and in IEEE Xplore. Authors may submit extended versions of their paper to the IEEE Sensors Journal.

### Exhibition & Partnership Opportunities

The Conference exhibit area will provide your company or organization with the opportunity to inform and display your latest products, services, equipment, books, journals, and publications to attendees from around the world.

### For further information, contact:

Lauren Pasquarelli  
Conference Catalysts, LLC  
Phone: +1 352 872 5544  
[lauren@conferencecatalysts.com](mailto:lauren@conferencecatalysts.com)

### Conference Officials

General Co-Chairs  
Cándid Reig - University of Valencia, Spain  
Lina Sarro - TUDelft, The Netherlands  
Technical Program Chair  
Ignacio R. Matias - Public University of Navarra, Spain  
Tutorial Chair  
Arnaldo D'Amico - URome, Italy  
Special Sessions Chair  
Alexander Fish - Bar-Ilan University, Israel  
Publicity Chair  
Edward Grant - North Carolina State University, USA  
Liasion with Industry and Administration Chair  
Javier Calpe - Analog Devices, Spain  
Local Chair  
Jaime Lloret - Polytechnic University of Valencia, Spain  
Treasurer  
Mike McShane - Texas A&M University, USA

### Important Dates

Special Session Proposal deadline: March 28, 2014  
Tutorial Proposal Submission Deadline: March 28, 2014  
Abstract Submission Deadline: April 11, 2014  
Author Notification: June 20, 2014  
Final Full Paper Submission (4 Pages): July 25, 2014  
Presenting Author and Early-Bird Registration Deadline: July 25, 2014  
All submissions shall be checked by means of [CrossCheck](#) to prevent plagiarism.

Conference web site: [ieeesensors2014.org](http://ieeesensors2014.org)

