Sensors & Transducers, Vol. 181, Issue 10, October 2014, pp. 71-76

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LMI-Based H_∞ Anti-Rollover Control Algorithm of Vehicle Active Suspension

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Received: 3 July 2014 /Accepted: 30 September 2014 /Published: 31 October 2014

Abstract: In order to improve the anti-rollover ability for vehicles, a 4 DOF vehicle rollover dynamics model is established, base on which we have designed an active suspension anti-rollover controller and proposed the H_∞ control strategy. Simulations were carried out using Matlab/Simulink, and results show that the proposed control scheme can not only reduce the roll angle and roll angular velocity, but also improve the rollover stability of the vehicle and reduce the probability of vehicle rollover accidents. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Active suspension, Dynamics model l, Anti-rollover control, H_∞ control, Active safety.

1. Introduction

In recent years, China has seen a rapid growth of its automobile and transportation industries. While meeting public transportation needs, vehicles have also become a major concern in serious traffic accidents resulted from vehicle rollover. In the United States, the hazard of vehicle rollover accidents ranks the second place among all traffic accidents, immediately after crashing accidents [1].

In 2004, rollover accidents and deaths accounted for 4 % and 8.6 % respectively of the total number of road accidents and deaths of the year in China. There are four typical types of traffic accidents, namely head-on collisions, side collisions, rear-end collisions and rollover collisions, among which the rollover collision may cause the most serious damage because the occupant survival space is squeezed to various degrees after the accident. When a rollover occurs, there will be drastic "secondary collisions" during the rolling process which causes severe or even fatal injuries. This is why the consequence of a rollover is likely to be disastrous. Therefore, researches on vehicle roll stability and rollover control systems are of great significance. At present there are mainly three automobile anti-rollover control technologies: 1) Active steering control; 2) Differential braking control; 3) Semi-active suspension and active suspension control. The active suspension anti-rollover control demonstrates better results than the differential braking control and the semi-active suspension control, and it does not change the driver's intention compared with the active steering control.

Therefore, it is a good option to improve the automobile roll stability by utilizing the active suspension. Many scholars have conducted various studies on the vehicle active suspension anti-rollover control. References [2-4] study and analyze the rollover stability of vehicle active suspension system through simulation, but don't provide applicable control algorithm. Reference [5] employs LQ algorithm, analyzing and studying the improvement of vehicle rollover stability with active suspension. Reference [6] presents a fuzzy control algorithm for active suspension of sport utility vehicles, using fishhook and step steer maneuvers, and has been proven to reduce the tendency to rollover. In this paper, an Active Suspension Axle model is defined as the controlled object, and H_∞ norm as the robust performance evaluation index. The linear output feedback controller is obtained by employing LMI (linear matrix inequality)-based H_∞ controller design method, after tuning the system roll performance index with appropriately selected weighting matrix array. In MATLAB/SIMULINK simulation, results show that with properly selected weighting function matrix, the active suspension system can obtain considerably better rollover performance than the passive suspension, effectively improving vehicle's roll stability and anti-rollover performance.

2. Dynamic Model of Vehicle Rollover

2.1. Vehicle Rollover Factors

We define the factor in vehicle rollover, the lateral load transfer ratio, as the evaluation index of vehicle rollover stability. The lateral load transfer ratio is expressed as [7]:

$$LTR = \frac{F_{ZR} - F_{ZL}}{F_{ZR} + F_{ZL}},$$

where F_{ZR} denotes the contact force between the right wheel and the ground, and F_{ZL} denotes that between the left wheel and the ground. When the vehicle moves straight forward steadily, if the mass is distributed symmetrically left and right, then $F_{ZL} = F_{ZR}$ and $LTR = 0$; If the left or the right side wheel is raised off the ground, then $F_{ZL} = 0$ or $F_{ZR} = 0$, and $LTR = 1$ or -1; With no wheel raised from the ground up $|LTR| < 1$, and the vehicle is in a rollover-stable state; the smaller $|LTR|$ is, the better rollover stability the vehicle has. According to theoretical rollover study, the dynamic lateral load transfer ratio can be expressed as:

$$LTR = \frac{2(C\dot{\phi} + K_s\phi)}{mgB}, \tag{1}$$

where C is the suspension damping coefficient, K_s is the suspension roll rigidity.

2.2. Single Axle Vehicle Rollover Model

In a passive suspension system, the damping system of the shock absorber is fixed, and the suspension only passively adapts to the changes of driving environment and conditions. An active suspension has a power mechanism consisting of a hydraulic source, hydraulic cylinders, and electromagnetic valves which adjusts the hydraulic pressure in the cylinder so as to control the state of the vehicle in real time. By changing the hydraulic cylinder pressure on the left and right sides of the active suspension, a rolling moment opposite to the vehicle rolling direction is generated to reduce the rollover tendency resulted from a sharp steering, so the roll stability of the vehicle is improved. Since the front and rear axles of the vehicle are symmetrical, we conduct our study on one of these axles. A half vehicle model with four degrees of freedom is set up as shown in Fig. 1 [8].

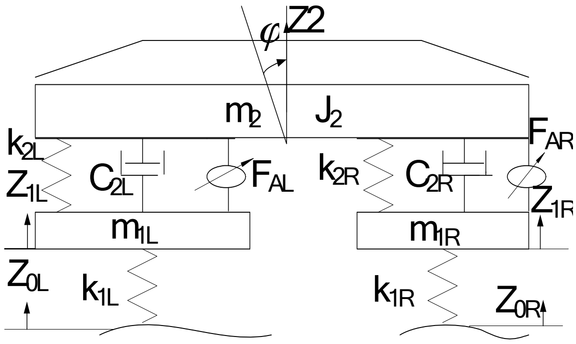

Fig. 1. Simplified single axle model of active suspension system.

According to Newton's second law, the dynamics equations are established as follows.

$$m_2\ddot{Z}_2 = -k_{2L}(Z_2+\varphi B/2-Z_{1L}) - C_{2L}(\dot{Z}_2+\dot{\varphi}B/2-\dot{Z}_{1L}) + F_{AL} - k_{2R}(Z_2-\varphi B/2-Z_{1L}) - C_{2R}(\dot{Z}_2-\dot{\varphi}B/2-\dot{Z}_{1L}) + F_{AR}, \tag{2}$$

$$m_{1L}\ddot{Z}_{1L} = k_{2L}(Z_2+\varphi B/2-Z_{1L}) + C_{2L}(\dot{Z}_2+\dot{\varphi}B/2-\dot{Z}_{1L}) - F_{AL} - k_{1L}(Z_{1L}-Z_{0L}), \tag{3}$$

$$m_{1R}\ddot{Z}_{1R} = k_{2R}(Z_2-\varphi B/2-Z_{1R}) + C_{2R}(\dot{Z}_2-\dot{\varphi}B/2-\dot{Z}_{1R}) - F_{AR} - k_{1R}(Z_{1R}-Z_{0R}), \tag{4}$$

$$J_2\ddot{\varphi} = -k_{2L}B(Z_2+\varphi B/2-Z_{1L})/2 - C_{2L}B(\dot{Z}_2+\dot{\varphi}B/2-Z_{1L})/2 + F_{AL}B/2 + k_{2R}B(Z_2-\varphi B/2-Z_{1R})/2 + C_{2R}B(\dot{Z}_2-\dot{\varphi}B/2-\dot{Z}_{1R})/2 - F_{AR}B/2 + m_2v^2h_g/R_s \tag{5}$$

where m_2 is the vehicle body mass; m_{1L}, m_{1R} are the left and right unsprung mass; J_2 is the moment of inertia of vehicle sprung mass around the vertical axle; B is the wheel track; k_{1L}, k_{1R} are the left and right wheel rigidity; k_{2L}, k_{2R} are the left and right suspension rigidity; C_{2L}, C_{2R} are the left and right damping coefficients of the shock absorber, Z_2 is the vertical displacement of vehicle body, φ is the roll angle; Z_{1L}, Z_{1R} are the vertical displacement of the left and right wheels; Z_{0L}, Z_{0R} are the roughness of contact point between the left wheel and the ground, and that between the right wheel and the ground; F_{AL}, F_{AR} are the left and right active control force; v is the velocity of vehicle; R_s is the vehicle cornering radius; h_g is the mass centroid height.

Due to symmetry of left and right suspensions, the relationship between structure parameters in Equations (1), (2), (3) and (4) is expressed as follows:

$$m_{1L} = m_{1R} = m_1, k_{2L} = k_{2R} = k_2, C_{2L} = C_{2R} = C_2,$$
$$k_{1L} = k_{1R} = k_1.$$

By combining Equations (1), (2), (3) and (4), and setting system state variables as:

$$X = \begin{bmatrix} \dot{Z}_2 & Z_2 & \dot{Z}_{1L} & Z_{1L} & \dot{Z}_{1R} & Z_{1R} & \dot{\varphi} & \varphi \end{bmatrix}^T,$$

the system state equation can be written as:

$$\dot{X}(t) = AX(t) + B_1\omega(t) + B_2U(t), \tag{6}$$

where $U(t)$ is the control input matrix, $\omega(t)$ is the known input matrix, and

$$U(t) = [F_{AL} \quad F_{AR}]^T, \omega(t) = [Z_{0L} \ Z_{0R} \ v^2 / R_S]^T.$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{2C_2}{m_2} & -\frac{2k_2}{m_2} & \frac{C_2}{m_2} & \frac{k_2}{m_2} & \frac{C_2}{m_2} & \frac{k_2}{m_2} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{C_2}{m_1} & \frac{k_2}{m_1} & -\frac{C_2}{m_1} & -\frac{k_1+k_2}{m_1} & 0 & 0 & \frac{C_2B}{2m_1} & \frac{k_2B}{2m_1} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{C_2}{m_1} & \frac{k_2}{m_1} & 0 & 0 & -\frac{C_2}{m_1} & -\frac{k_1+k_2}{m_1} & -\frac{C_2B}{2m_1} & -\frac{k_2B}{2m_1} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{C_2B}{2J_2} & \frac{k_2B}{2J_2} & -\frac{C_2B}{2J_2} & -\frac{k_2B}{2J_2} & -\frac{C_2B^2}{2J_2} & -\frac{k_2B^2}{2J_2} \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 & \frac{k_1}{m_1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{k_1}{m_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{m_2h_g}{J_2} \end{bmatrix}^T,$$

$$B_2 = \begin{bmatrix} 0 & \frac{1}{m_2} & 0 & -\frac{1}{m_1} & 0 & 0 & 0 & \frac{B}{2J_2} \\ 0 & \frac{1}{m_2} & 0 & 0 & 0 & -\frac{1}{m_1} & 0 & \frac{B}{2J_2} \end{bmatrix}^T$$

3. The H∞ Control Strategy of Active Suspension

3.1. Description of the LMI-based H∞ Control System

According to the established vehicle rollover dynamic model, the state space of the controlled object is described as [9, 10]:

$$\begin{cases} \dot{x} = Ax + B_1w + B_2u \\ z = C_1x + D_{11}w + D_{12}u \\ y = C_2x + D_{21}w + D_{22}u \end{cases}, \tag{7}$$

where x is the state, z is the controlled output, and y is the measured output.

The control system is shown in Fig. 2. $W(t)$ is the external disturbance signal, i.e. the disturbance arising from the road excitation and vehicle cornering at a high speed. $u=K(s)*y$ denotes the control input, where $K(s)$ is the dynamic feedback controller. $G(s)$ is the generalized controlled object which includes the actual controlled object, the weighting function and the evaluation function, and it is expressed as follows [11]:

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix}, \tag{8}$$

The closed-loop transfer function from w to z is:

$$T_{zw}(s) = G_{11}(s) + G_{12}(s)K(s)[I - G_{22}(s)K(s)]^{-1}G_{21}(s), \tag{9}$$

The design requirements of the control system are:

1) The closed-loop system is internally stable, that is to say all eigenvalues of the state matrix of the closed-loop system are on the left half of the open complex plane.

2) For an arbitrary disturbance w, the closed-loop system generates suppression against the disturbance, which means the formula [12]:

$$\|T_{zw}(s)\| < \gamma, \tag{10}$$

is satisfied. The $K(s)$ meeting Formula (10) is called the H∞ suboptimal controller, while the $K(s)$ minimizing $\|Tzw(s)\|$ is called the H∞ optimal controller.

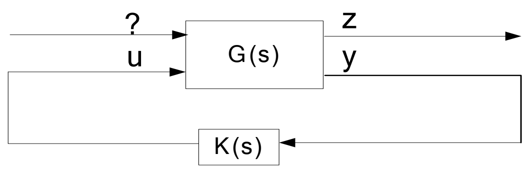

Fig. 2. diagram of control system.

3.2. The LMI-based Controller Design

The design of the active suspension anti-rollover control system of the vehicle follows below principles: the controlled object is an internally-stable closed-loop system; given an arbitrary T>0 and the disturbance signal $\omega \in [0,+\infty]$, the formula below is satisfied [13]:

$$\int_0^T \{x^T(t)Qx(t)+u^T(t)Ru(t)\}dt < \varepsilon\int_0^T w^T(t)w(t)dt < \infty, \qquad (11)$$

For a given constant $\varepsilon>0$, Q and R which are symmetric positive semidefinite matrix and symmetric positive definite matrix respectively, are the weighting matrix.

$$Q=\begin{bmatrix} q_1 & & & \cdots & & & \\ & q_2 & & & & & \\ & & \ddots & & & & \\ \vdots & & & q_m & & & \\ & & & & 0 & & \\ & & & & & \ddots & \\ & & & & & & 0\end{bmatrix},$$

$$R=\begin{bmatrix} r_1 & & & 0 \\ & \ddots & & \\ & & \ddots & \\ 0 & & & r_n\end{bmatrix}.$$

The smaller ε is, the better disturbance suppression performance of system is. With dynamic properties of the disturbance suppression regulated by and the control input signal by R, the performance evaluation signal can be expressed as follows:

$$z = C_1 x + D_{12} u, \qquad (12)$$

where

$$C_1=\begin{bmatrix} Q^{\frac{1}{2}} \\ 0 \end{bmatrix}, D_{12}=\begin{bmatrix} 0 \\ R^{\frac{1}{2}} \end{bmatrix},$$

$$G(s)=\begin{bmatrix} A & \frac{1}{\sqrt{\varepsilon}}B_1 & B_2 \\ C_1 & 0 & D_{12} \\ C_2 & \frac{1}{\sqrt{\varepsilon}}D_{21} & 0 \end{bmatrix}.$$

According to the given generalized controlled object and the given conditions, the solution of *K(s)* meets the design requirements of the control system.

4. Simulation and Results Analysis

The electrohydraulic active suspension anti-rollover half vehicle model used in simulation and experimental testing is shown in Fig. 1. The settings of parameters are:
m_1=110 kg, m_2=405 kg, k_1=138000 N/m, k_2=15200 N/m, d_2=1760 N.m/s, B=1.3 m, J_2=300 kg.m^2, h_g=0.5 m, C=1760 N.m.s/rad, K_s=6680 N.m/rad, g=9.81m/s^2.

Assuming a vehicle velocity of 54 km/h, R_s=20 m, a stable steering wheel angle of 45 degrees, and without considering the influence of the road surface roughness, that is to say: $Z_{0L}=Z_{0R}=0$, we simulated the vehicle moving at a certain stable speed on a curve road. Using Matlab to calculate [14], the simulation was carried out with and without LMI-based control under Matlab/Simulik [15, 16], and we compared results of the two scenarios. The selection of the weighting matrixes Q and R has an important influence on all aspects of system performance. By repetitive tuning and testing, it is discovered that when $q_1= q_2= q_3= q_4= q_5= q_6=0.001$, $q_7= q_8=10$, $\gamma=0.11$ and $\gamma_1=\gamma_2=0.5$, the active suspension anti-rollover system demonstrates considerable effects. The simulation results are shown in Fig. 3, Fig. 4, Fig. 5 and Fig. 6.

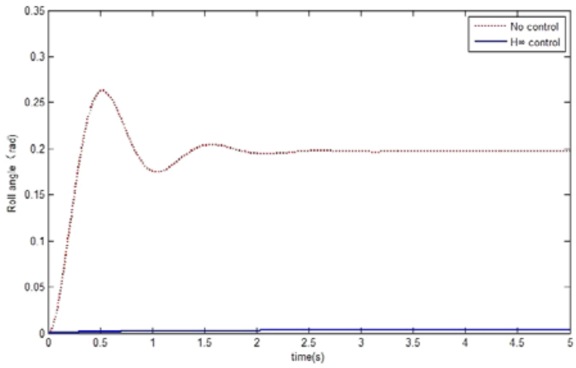

Fig. 3. Curves of roll angle vs. time.

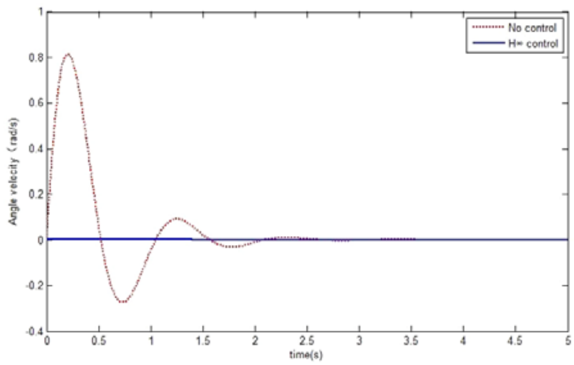

Fig. 4. Curves of roll angle velocity vs. time.

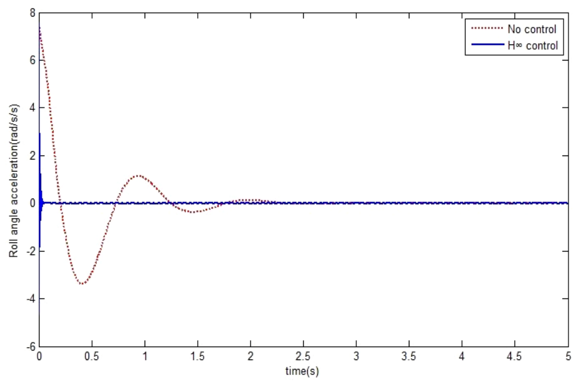

Fig. 5. Curves of roll angle acceleration vs. time.

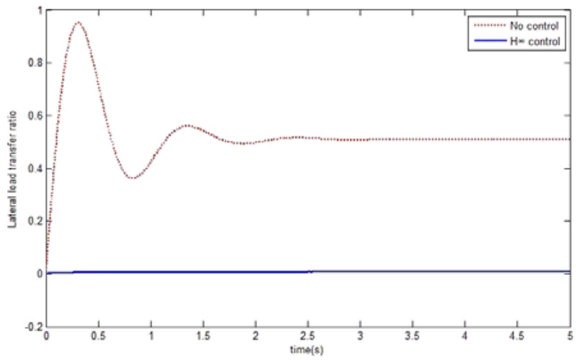

Fig. 6. Curves of rollover factor vs. time.

The simulation results and the data in Table 1 show that with LMI-based control, the average roll angle was reduced by 98.73 % compared to uncontrolled scenario, the maximum roll angle reduced by 98.93 %, and the root mean square value of the roll angle reduced by 98.71 %.

The average roll angle velocity was reduced by 98.50 % the maximum roll angle velocity reduced by 98.79 %, and the root mean square value of roll angle velocity reduced by 99.37 %.

The average roll angle acceleration was reduced by 78.66 %, the maximum roll angle acceleration reduced by 0 %, and the root mean square value of roll angle acceleration reduced by 68.48 %.

Table 1. Comparison of results.

	Mean value		**Maximum**		**Root mean square value**	
	No control	**H∞ control**	**No control**	**H∞ control**	**No control**	**H∞ control**
Roll angle (rad)	0.1902	0.0024	0.2630	0.0028	0.1949	0.0025
Roll angle velocity (rad/s)	0.0401	0.0006	0.8158	0.0098	0.1932	0.0012
Roll angle acceleration (rad/s2)	0.0942	0.0201	7.5938	7.5938	1.4707	0.4635
lateral load transfer ratio	0.5194	0.0066	0.9515	0.0073	0.5338	0.0068

The average lateral load transfer ratio was reduced by 98.72 %, the maximum lateral load transfer ratio reduced by 99.23 %, and the root mean square value of lateral load transfer ratio reduced by 98.72 %.

Compared to the uncontrolled scenario, the maximum roll angle acceleration under H_∞ control does not change; this is mainly because of, according to curves of Fig. 4 and Fig. 5, the impact of disturbance from the rapidly-changing external roll angel velocity. All other rollover performance indicators have demonstrated significant improvement which proves a good control performance.

5. Discussion and Conclusion

In this paper, an explorative study is made on the LMI-based H_∞ anti-rollover control of vehicle active suspension. According to simulation results we can conclude:

1) The designed H_∞ controller has good control performance, improving vehicle rollover stability and vehicle roll state, and reducing the probability of vehicle rollover accidents. The simulation results show that the LMI-based H_∞ control method can achieve high control precision.

2) According to different performance requirements, by changing the parameter matrixes Q and R we can obtain different optimal control force feedback gain matrix K, based on which the active force controller can be designed to achieve the desired performance.

3) Compared with the semi-active suspension, the cost of installation and use of the active suspension is higher and so is the energy consumption. But on the other hand, the LMI-based active suspension anti-rollover system utilizes the electrohydraulic active suspension structure which produces greater ant-rollover torque than the semi-active suspension, so it provides better anti-rollover control performance.

Acknowledgements

It is a project supported by the National Natural Science Foundation (51305372).

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