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Simplified Soft-output Demapper Based on a Linear Transformation Technique for M-ary PSK

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Abstract: Combining channel coding with high-order modulation schemes, namely coded modulation (CM), is an efficient digital transmission technology. CM requires the demapper to provide a soft decision bit metric as a part of the inputs to the decoder. This paper proposes an efficient soft-output demapper for M-ary PSK. This novel demodulation algorithm continues to evolve the conventional MAX-Log-MAP algorithm and summarizes the final derivation into a form of matrix multiplication. The Computational complexity for getting one bit soft value of a signal by employing the proposed algorithm remains a constant with the increase of modulation order. Meanwhile, the theoretical analysis and simulation results prove that the novel simplified soft-output demapper can obtain the same performance as MAX-Log-MAP. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: Soft demapper, Phase-shift keying (PSK), Bit metric, Bit-interleaved coded modulation (BICM), Computation complexity.

1. Introduction

Along with the rapid development of communication service over the past couple of decades, the limited spectrum resource is lack increasingly. The signals are high-order modulated in order to achieve higher spectrum efficiency and data rates. For example, Digital video broadcasting via Satellite – Second Generation (DVB-S2) [1] supports four modulation modes scheme: quadrature phase-shift keying (QPSK), 8PSK, 16-ary amplitude phase-shift keying (16APSK) and 32APSK. QPSK and 8PSK are proposed as standard configurations for broadcasting business, while 16APSK and 32APSK are optional. When it comes to interactive serves, digital satellites news gathering and other professional business, all the four modulation modes are used as mandatory formats. 4-ary quadrature amplitude modulation (4QAM), 16QAM and 64QAM are employed by the third generation partnership project (3GPP) long-term evolution (LTE) [2], which takes into account some significant issues such as the reduction in network latency, a higher data rate and system capacity, etc. Furthermore, 128QAM is adopted by LTE-Advanced [3], which is the evolution version of LTE. In addition, Turbo code which is introduced by Berrou et al. [4] in 1993 has been considered as one of the physical layer forward error corrections (FEC) coding schemes according to the LTE-A standard protocol. It is well-known that Turbo code produces a near-capacity performance on additive white Gaussian noise (AWGN) channels due to the iterative decoding technique. Combining channel coding with high-order modulation schemes, namely coded modulation (CM) with a character without expanding frequency coding, is an efficient digital transmission technology. Bit-interleaved coded modulation (BICM) is introduced by Zehavi in 1992 [5]. The performance of BICM is improved over a Rayleigh fading channel by designing a bit-interleaver between a binary encoder and a non-binary modulator to make the correlated errors randomized.

BICM requires the demapper to provide a soft decision bit metric whose absolute value denotes the reliability of the decision as a part of the inputs to the decoder. Soft-output demapper is an essential qualification to get an excellent performance. The algorithm applied to a symbol-to-bit soft demapper is complicated. The classical maximum *a posteriori* probability based in the log-domain (Log-MAP) [6] calculates the log-likelihood-ratio (LLR) for every received symbol. Nevertheless, the complexity of computing LLR adopting Log-Map demapping algorithm is exponentially increasing, and it is hard to support a higher-order modulation mode. The proposed Maximum Log-MAP (MAX-Log-MAP) algorithm [7] overcomes the high-complexity exponential operations of Log-MAP with some unavoidable performance loss by omitting the correction function.

Albeit the computational complexity of MAX-Log-MAP is much lower than Log-MAP, researchers have not stopped digging into optimizing soft demappers. Numerous literatures have proposed their novel demodulation approaches from different perspectives for certain constellations. A recursive bit metric generation approach for PSK with Gray labeling is presented in [8]. All bit metrics are computed by a metric function of the most significant bit. In [9], a simplified algorithm for the soft-output demapper for 16QAM and 64QAM in the HIPERLAN/2 standard [10] is developed, which invokes a piecewise linear approximation to reduce the complexity. Literature [11] optimizes the coefficients according to piecewise linear LLR approximations, and the performance is very close to true LLRs. A novel low complexity two-stage soft demapper without performance loss for the rotated constellation in DVB-T2 [12] is designed by Kitaek Bae et al. [13]. The first stage is to calculate the LLRs by using the one dimensional LLR lookup table for a dominant channel, and the second stage calculates the other LLRs through selecting a new subset of candidates of $\sqrt{M}$ for M-QAM for the other channel.

Against this background, a low-complexity soft-output demapper for M-ary PSK by using a linear transformation is presented. The proposed algorithm extends the derivations based on MAX-Log-MAP and makes a summing up of the results. We convert the final derivation into a linear transformation form of matrix multiplication and this demodulation matrix which is called “R-matrix” is easy to change and save. The contribution of this paper can be summarized as follows: using the saved R-matrix for demodulating can leave out the operations for computing Euclidean distances between the received symbol and every constellation point and the process of comparing them. In addition, this proposed demapper achieves the same performance as the MAX-Log-MAP demapper. This paper studies and describes the proposed approach based on Turbo-coded BICM system.

The rest of this paper is organized as follows. Section II describes Log-MAP and MAX-Log-MAP algorithm. In Section III, we introduce the proposed schemes by taking QPSK and 8PSK for examples. These theoretical results are supported in Section IV by means of simulation and conclusions are presented in Section V.

2. Theoretical Background

We consider the transmission of complex modulation symbols over an AWGN channel where the received signal vector $\boldsymbol{y_j}=(y_1, y_2, \ldots, y_K)$ is expressed as

$$y_j = x_j + n_j , \tag{1}$$

where $\boldsymbol{x_j}=(x_1, x_2, \ldots, x_K)$ presents the complex transmitted signal vector, and $\boldsymbol{n_j}=(n_1, n_2, \ldots, n_K)$ stands for the zero-mean complex additive white Gaussian noise with variance σ^2.

2.1. Log-MAP Demapper

At the receiver, demapper should compute the bit soft information which is important to soft decoding for every M-ary noisy signal *y*. It is known that the LLR can be calculated by

$$\Lambda_i = \log \frac{P(b_i = 1 \mid y)}{P(b_i = 0 \mid y)} , \tag{2}$$

$$= \log \frac{\sum_{x \in \chi_i^{(1)}} P(x \mid y)}{\sum_{x \in \chi_i^{(0)}} P(x \mid y)} , \tag{3}$$

$$= \log \frac{\sum_{x \in \chi_i^{(1)}} p(y \mid x)}{\sum_{x \in \chi_i^{(0)}} p(y \mid x)} , \tag{4}$$

where b_i is the i^{th} bit of the modulated signal in (2). $\chi_i^{(1)}$ and $\chi_i^{(0)}$ denote the subset of b_i=1 and b_i=0 respectively. The conditional probability density function (pdf) $p(y|x)$ is given by

$$p(y \mid x) = \left(\frac{1}{2\pi\sigma^2}\right)^{\frac{1}{2}} \exp\left(-\frac{|y - x|^2}{2\sigma^2}\right) , \tag{5}$$

2.2. MAX-Log-MAP Demapper

Log-MAP algorithm refers to the operation in the form of $log(exp(x_1)+ exp(x_2)+\ldots+ exp(x_n))$ which can be calculated with Jacobian algorithm as

$$\log(\exp(x)+\exp(y)) = \max(x,y)+\log(1+\exp(-|x-y|)) \\ = \max(x,y)+f_c(|x-y|) \quad (6)$$

where $f_c(.)$ denotes the correction function. Considering that $f_c(.)=0$ and (6) can be simplified as

$$\log(\exp(x)+\exp(y)) = \max(x,y)\,, \quad (7)$$

Thus, the LLR calculated by (4) is approximated as follow

$$\Lambda_i \approx \log\frac{\max_{x\in\chi_i^{(1)}} p(y\,|\,x)}{\max_{x\in\chi_i^{(0)}} p(y\,|\,x)}\,, \quad (8)$$

Then, we plug (5) into (8) and the MAX-Log-MAP algorithm is formulated as

$$\Lambda_i = -\frac{1}{2\sigma^2}\left(\min_{x\in\chi_i^{(1)}} |y-x|^2 - \min_{x\in\chi_i^{(0)}} |y-x|^2\right), \quad (9)$$

This near optimal algorithm with omitting correction function avoids the complex exponential and logarithmic operations. The performance of MAX-Log-MAP is almost on a par with Log-MAP in the high signal-to–noise ratio (SNR) region. As a consequence, MAX-Log-MAP is sufficiently widely available.

3. Proposed Simplified Soft-output Demapper

Although MAX-Log-MAP has reduced a lot of complexity, it is still tedious to compute Euclidean distances between the received symbol and every constellation point and find the minimum among them. Therefore, the number of computation increases with the modulation order. This paper develops the MAX-Log-MAP algorithm and introduces the proposed schemes by taking QPSK and 8PSK for examples.

3.1. QPSK Demapper

Fig. 1 shows the QPSK constellation and illustrations of demapping for the first and second bit.

Assuming that the received signal y can be expressed as

$$y = y_I + jy_Q \\ = |y|e^{j\phi_y}\,, \quad (10)$$

where $\phi_y \in [-\pi,\pi]$. On the other hand, equation (9) can be further written as

$$\Lambda_i = -\frac{1}{2\sigma^2}\left(|y-s_i^{(1)}(r)|^2 - |y-s_i^{(0)}(r)|^2\right), \quad (11)$$

where $s_i^{(1)}(r)$ and $s_i^{(0)}(r)$ respectively denote the nearest constellation point included in $\chi_i^{(1)}$ and $\chi_i^{(0)}$ to y, and these two points are expressed by $e^{j\phi_1}$ and $e^{j\phi_0}$ as

$$e^{j\phi_1} = \cos\phi_1 + j\sin\phi_1 \\ e^{j\phi_0} = \cos\phi_0 + j\sin\phi_0\,, \quad (12)$$

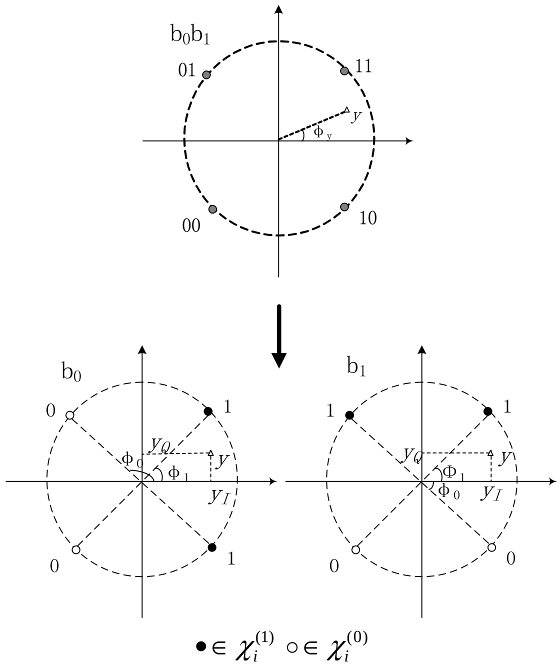

Fig. 1. QPSK constellation and the illustrations of demapping for b_0 and b_1.

Thus, we have

$$\begin{aligned}\Lambda_i &= -\frac{1}{2\sigma^2}\left(|y-e^{j\phi_1}|^2 - |y-e^{j\phi_0}|^2\right) \\ &= -\frac{1}{2\sigma^2}\left((y_I-\cos\phi_1)^2+(y_Q-\sin\phi_1)^2-(y_I-\cos\phi_0)^2-(y_Q-\sin\phi_0)^2\right) \\ &= -\frac{1}{\sigma^2}\left(-y_I\cos\phi_1 - y_Q\sin\phi_1 + y_I\cos\phi_0 + y_Q\sin\phi_0\right) \\ &= \frac{1}{\sigma^2}\left(y_I(\cos\phi_1-\cos\phi_0)+y_Q(\sin\phi_1-\sin\phi_0)\right) \\ &= [y_I \quad y_Q]\times\begin{bmatrix}\frac{1}{\sigma^2}(\cos\phi_1-\cos\phi_0) \\ \frac{1}{\sigma^2}(\sin\phi_1-\sin\phi_0)\end{bmatrix}\end{aligned} \quad (13)$$

We describe the final equation in the form of matrix multiplication. In the same way, the other bit soft information is also expressed as (13). As a consequence, using one equation to calculate two LLRs of a symbol simultaneously is given by

$$[\Lambda_{b0} \quad \Lambda_{b1}] = [y_I \quad y_Q] \times \begin{bmatrix} \frac{1}{\sigma^2}(\cos\phi_1^{'} - \cos\phi_0^{'}) & \frac{1}{\sigma^2}(\cos\phi_1 - \cos\phi_0) \\ \frac{1}{\sigma^2}(\sin\phi_1^{'} - \sin\phi_0^{'}) & \frac{1}{\sigma^2}(\sin\phi_1 - \sin\phi_0) \end{bmatrix}, \quad (14)$$

where $\phi_1^{'}$ and $\phi_0^{'}$ stand for the phase angles in the higher bit (b_0) constellation; ϕ_1 and ϕ_0 are for the lower bit (b_1) constellation. We call the 2×2 matrix the demodulation matrix namely "R-matrix" in (14). As can be seen from the result, it is only required to know three parameters for computing LLRs: the real and imaginary value of a signal, phase angles and SNR estimation.

The QPSK constellation can be divided into four regions by ϕ_y :

$$\begin{aligned} &\text{i: } \phi_y \in [-\pi, -\frac{\pi}{2}) \\ &\text{ii: } \phi_y \in [-\frac{\pi}{2}, 0) \\ &\text{iii: } \phi_y \in [0, \frac{\pi}{2}) \\ &\text{iv: } \phi_y \in [\frac{\pi}{2}, \pi), \end{aligned} \quad (15)$$

When the noisy signal is received by demodulator, it is not to calculate and compare the Euclidean distances between a symbol and every constellation points for the proposed demapper, but to judge the region where the symbol is located according to (15) and then plug the corresponding values of ϕ_1 and ϕ_0 into (14). Table 1 lists the phase angles ($\phi_1^{'}$ and $\phi_0^{'}$) of the two nearest constellation points ($s_i^{(1)}(y)$ and $s_i^{(0)}(y)$, which is respectively included in $\chi_i^{(1)}$ and $\chi_i^{(0)}$) by the signal y as well as the values for calculating R-matrix.

Table 1. The process to compute the first column of R-matrix for QPSK constellation.

	$\phi_1^{'}$	$\phi_0^{'}$	$\cos\phi_1^{'} - \cos\phi_0^{'}$	$\sin\phi_1^{'} - \sin\phi_0^{'}$
i	$-\frac{\pi}{4}$	$-\frac{3\pi}{4}$	1.4142	0
ii	$-\frac{\pi}{4}$	$-\frac{3\pi}{4}$	1.4142	0
iii	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	1.4142	0
iv	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	1.4142	0

Similarly, the second column of R-matrix can be calculated. Thus, we obtain a constant (for the same SNR estimation) R-matrix:

$$\begin{bmatrix} \frac{1.4142}{\sigma^2} & 0 \\ 0 & \frac{1.4142}{\sigma^2} \end{bmatrix}, \quad (16)$$

It is no need to judge the region (i/ii/iii/iv) for QPSK constellation because the R-matrix is constant.

In conclusion, demapper converts the real and imaginary values of each symbol to a 1×2 matrix, and multiplies it by the R-matrix which has been saved in the case of QPSK.

3.2. 8PSK Demapper

Fig. 2 shows the 8PSK Gray mapping constellation and illustrations of demapping for b_0, b_1 and b_2.

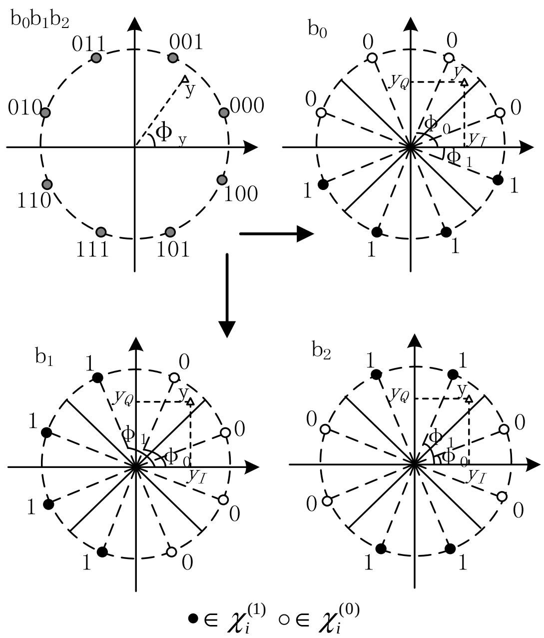

Fig. 2. 8PSK with Gray mapping constellation and the illustrations of demapping for b_0, b_1 and b_2.

We can divide the constellation into 2^m (m donates the modulation order, and here m=3) regions by ϕ_y in the same way with QPSK demapper constellation:

$$\begin{aligned} &\text{i: } \phi_y \in [-\pi, -\frac{3\pi}{4})\text{; ii: } \phi_y \in [-\frac{3\pi}{4}, -\frac{\pi}{2})\text{;} \\ &\text{iii: } \phi_y \in [-\frac{\pi}{2}, -\frac{\pi}{4})\text{; iv: } \phi_y \in [-\frac{\pi}{4}, 0)\text{;} \\ &\text{v: } \phi_y \in [0, \frac{\pi}{4})\text{; vi: } \phi_y \in [\frac{\pi}{4}, \frac{\pi}{2})\text{;} \\ &\text{vii: } \phi_y \in [\frac{\pi}{2}, \frac{3\pi}{4})\text{; viii: } \phi_y \in [\frac{3\pi}{4}, \pi), \end{aligned} \quad (17)$$

In the following, using the highest bit (b_0) as an example to introduce the process of calculating a LLR. Assuming that the signal *y* is mapped to the location where the triangle icon is presented in Fig. 2. It can be judged that *y* belongs to region vi by (17). We list the semblable Table 2 according to Table 1.

Table 2. The process to compute the first column of R-matrix for 8PSK Gray mapping constellation.

	$\phi_1^{''}$	$\phi_0^{''}$	$\cos\phi_1^{''}-\cos\phi_0^{''}$	$\sin\phi_1^{''}-\sin\phi_0^{''}$
i	$-\frac{7\pi}{8}$	$\frac{7\pi}{8}$	0	-0.7654
ii	$-\frac{5\pi}{8}$	$\frac{7\pi}{8}$	0.5412	-1.3066
iii	$-\frac{3\pi}{8}$	$\frac{\pi}{8}$	-0.5412	-1.3066
iv	$-\frac{\pi}{8}$	$\frac{\pi}{8}$	0	-0.7654
v	$-\frac{\pi}{8}$	$\frac{\pi}{8}$	0	-0.7654
vi	$-\frac{\pi}{8}$	$\frac{3\pi}{8}$	***0.5412***	***-1.3066***
vii	$-\frac{7\pi}{8}$	$\frac{5\pi}{8}$	-0.5412	-1.3066
viii	$-\frac{7\pi}{8}$	$\frac{7\pi}{8}$	0	-0.7654

The first colunm $\begin{bmatrix} 0.5412 \\ -1.3066 \end{bmatrix}$ of R-matrix are checked from Table 2. With the same method, all R-matrixes corresponding to the 8 regions can be enumerated as follows (leave out the coefficient $\frac{1}{\sigma^2}$):

$$\begin{aligned}
&\text{i: } \begin{bmatrix} 0 & -1.3066 & 0.5412 \\ -0.7654 & 0.5412 & -0.5412 \end{bmatrix}; \\
&\text{ii: } \begin{bmatrix} 0.5412 & -0.7654 & 0.5412 \\ -1.3066 & 0 & -0.5412 \end{bmatrix}; \\
&\text{iii: } \begin{bmatrix} 0.5412 & -0.7654 & -0.5412 \\ -1.3066 & 0 & -0.5412 \end{bmatrix}; \\
&\text{iv: } \begin{bmatrix} 0 & -1.3066 & -0.5412 \\ -0.7654 & -0.5412 & -0.5412 \end{bmatrix} \\
&\text{v: } \begin{bmatrix} 0 & -1.3066 & -0.5412 \\ -0.7654 & 0.5412 & 0.5412 \end{bmatrix}; \\
&\text{vi: } \begin{bmatrix} 0.5412 & -0.7654 & -0.5412 \\ -1.3066 & 0 & 0.5412 \end{bmatrix} \\
&\text{vii: } \begin{bmatrix} 0.5412 & -0.7654 & 0.5412 \\ -1.3066 & 0 & 0.5412 \end{bmatrix}; \\
&\text{viii: } \begin{bmatrix} 0 & -1.3066 & 0.5412 \\ -0.7654 & -0.5412 & 0.5412 \end{bmatrix},
\end{aligned} \tag{18}$$

Those R-matrixes shown in (18) should be stored. Demapper checks and fetches the right one against the stored lookup table when the phase angle region of a signal is determined, and then the three LLRs of an 8PSK symbol are obtained by

$$\begin{aligned}
&[\Lambda_{b0} \quad \Lambda_{b1} \quad \Lambda_{b2}] \\
&= [y_I \quad y_Q] \times \begin{bmatrix} \frac{1}{\sigma^2}(\cos\phi_1^{''} - \cos\phi_0^{''}) & \frac{1}{\sigma^2}(\cos\phi_1^{'} - \cos\phi_0^{'}) & \frac{1}{\sigma^2}(\cos\phi_1 - \cos\phi_0) \\ \frac{1}{\sigma^2}(\sin\phi_1^{''} - \sin\phi_0^{''}) & \frac{1}{\sigma^2}(\sin\phi_1^{'} - \sin\phi_0^{'}) & \frac{1}{\sigma^2}(\sin\phi_1 - \sin\phi_0) \end{bmatrix}
\end{aligned} \tag{19}$$

There are several calculating comparisons about the optimal Log-MAP, suboptimal MAX-Log-MAP and the proposed R-matrix algorithm in Table 3.

Table 3. The calculating comparisons of computing the soft information on the i^{th} bit by different algorithms. M= 2^m .

	Log-MAP	MAX-Log-MAP	R-matrix
Exponential operation	M+1	0	0
Addition	4M-2	3M+1	1
Multiplication	2M+1	2M	2
Comparison operation	0	M-2	0
Table look-up	0	0	≤ M

From the table above we can see that the computational complexity for getting one bit-soft value of using Log-MAP and MAX-Log-MAP increases with rising *m*, while the proposed method only requires two times of multiplication and addition once. Although there are *M* times at most for table searching (except for QPSK), R-matrix demapper still owns a low-complexity in the whole demodulation process compared with the two former algorithms.

4. Simulation Result

A frame of 1024 bits is considered and at least 70 error frames are encountered. The encoder is Turbo code with generator polynomials (feedback, redundancy) $(13, 15)_{oct}$, and the code rate is 1/3. The modulation and demodulation adopt QPSK, 8PSK and 16PSK with Gray mapping. Simulations are executed under the condition of AWGN channel.

Fig. 3 compares the LLRs (the soft information if b_0 and b_1) which are obtained by changing the value of the imaginary axis with fixed value of real axis of Log-MAP, MAX-Log-MAP and R-matrix for QPSK signals. It's observed that the LLRs calculated by the three demappers are basically the same. Because there are altogether four constellation points, namely the each subset $\chi_i^{(1)}$ or $\chi_i^{(0)}$ includes two points and they are far apart from each other, the correction function has a small effect on LLR. In consequence, an R-matrix demapper is the most reasonable one of the three demappers for QPSK to significantly reduce large computational complexity without compromising the performance.

In order to analysis the difference by employing the three demappers, we show the LLR of one bit for Gray-labeled 16PSK in Fig. 4. It can be seen that MAX-Log-MAP and R-matrix have nearly the same computed result, while the optimal algorithm Log-MAP owns the larger values than the two former algorithms.

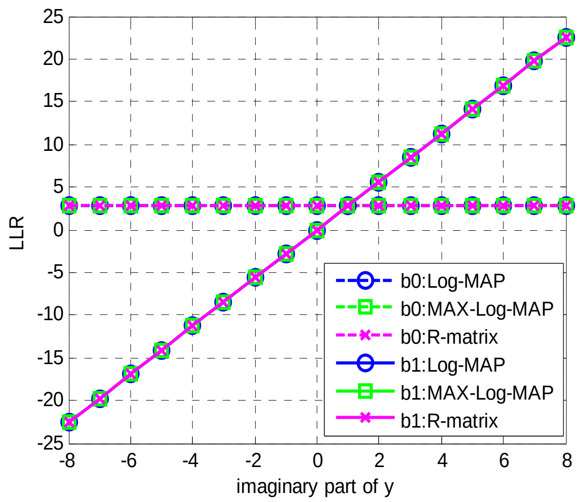

Fig. 3. The comparison of LLRs calculated by Log-MAP,MAX-Log-MAP and R-matrix for QPSK.

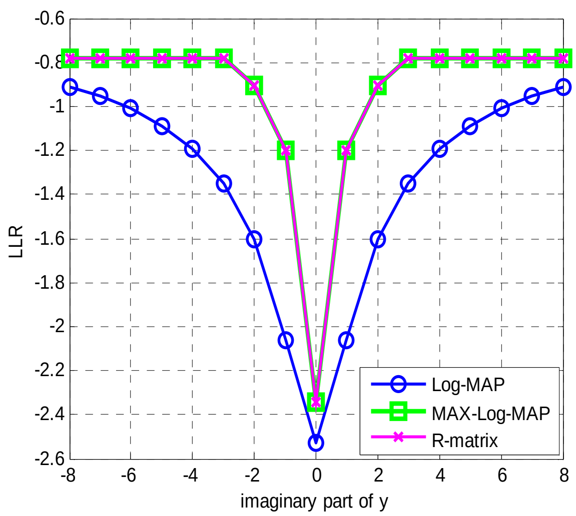

Fig. 4. The comparison of one bit soft value calculated by Log-MAP, MAX-Log-MAP and R-matrix for 16PSK.

The BER curves for QPSK, 8PSK and 16PSK are shown in Fig. 5. It's evident that the performance of the three demappers has no significantly difference as the theoretical analysis for Fig. 3. As expected, Log-MAP demapper presents the optimal performance for 8PSK and 16PSK. At the same time, the overlapped BER curves obtained by MAX-Log-MAP and R-matrix demappers confirm that the proposed soft demapper achieves the same demodulating performance as MAX-Log-MAP demapper.

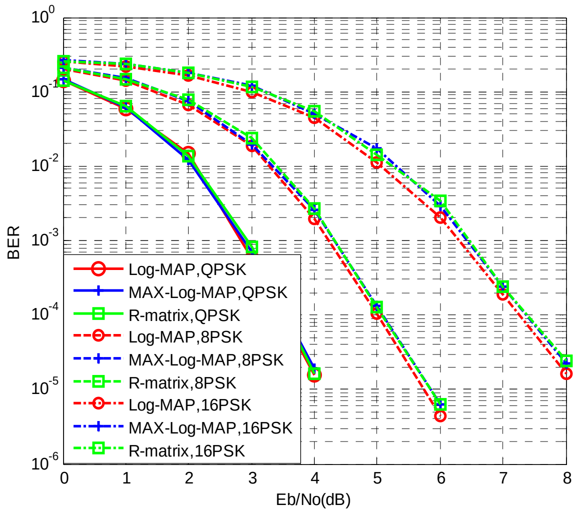

Fig. 5. The BER curves for QPSK, 8PSK and 16PSK by employing the three demappers.

5. Conclusions

In this paper, a low-complexity soft-output "R-matrix" demapper for M-ary PSK based on using a linear transformation is presented. We develop the final derivation of calculating the LLR based on MAX-Log-MAP and the result is summarized into a form of matrix multiplication. In terms of computational complexity, the proposed demapper only requires two times of multiplication and addition once computing one bit soft value, meanwhile, it needs M times at most for table lookup in the process above. Nonetheless, the complexity for computing LLR of one bit remains unchanged with rising modulation order. The theoretical analysis and simulation results prove that the proposed demapper is equivalent to the MAX-Log-MAP demapper in the terms of performance. Moreover, the R-matrix demapper can bring the low demodulation complexity and gain a satisfactory performance as well especially for QPSK.

References

[1]. Digital Video Broadcasting (DVB); Second generation framing structure, channel coding and modulation systems for Broadcasting, Interactive Services, News Gathering and other broadband satellite applications, *European Telecommunications Standards Institute*, EN 302 307, V1.1.2, 2006, 74 p.

[2]. S. Sesia, I. Toufik, and M. Baker, LTE – the UMTS long term evolution, *Wiley*, 2009.

[3]. 3GPP Technical Report 36.814, Evolved universal terrestrial radio access (e-utra); further advancements

for e-utra physical layer aspects (release 9), (http://www.3gpp.org).
[4]. C. Berrou, A. Glavieux, and P. Thitimajshima, Near Shannon limit error-correcting coding: turbo codes, in *Proceedings of the IEEE International Conference on Communication*, Geneva, Switzerland, May 1993, pp. 1064-1070.
[5]. E. Zehavi, 8-PSK trellis codes for a Rayleigh channel, *IEEE Transactions on Communication*, Vol. 40, Issue 5, May 1992, pp. 873–884.
[6]. J. Erfanian, S. Pasupathy, and G. Gulak, Reduced complexity symbol detectors with parallel structures for ISI channels, *IEEE Transactions on Communication*, Vol. 42, Issues 2-4, 1994, pp. 1661–1671.
[7]. P. Robertson, E. Villebrun, and P. Hoeher, A comparison of optimal and sub-optimal MAP decoding algorithms operating in the log domain, *in Proceedings of the IEEE International Conference on Communications*, Seattle, WA, USA, June 18–22, 1995, Vol. 2, pp. 1009–1013.
[8]. L. Wang, D. Xu, and X. Zhang, Recursive bit metric generation for PSK signals with Gray labeling, *IEEE Communication Letters*, Vol. 16, Issue 2, February 2012, pp. 180–182.
[9]. F. Tosato and P. Bisaglia, Simplified soft-output demapper for binary interleaved COFDM with application to HIPERLAN/2, in *Proceedings of the IEEE International Conference on Communications*, New York, NY, USA, April 28 – May 2, 2002, Vol. 2, pp. 664–668.
[10]. ETSI TS 101 475, Broadband radio access networks (BRAN); HIPERLAN type 2; physical (PHY) layer, v1.2.2, *European Telecommunications Standards Institute*, 2001.
[11]. R. Yazdani and M. Ardakani, Efficient LLR calculation for non-binary modulations over fading channels, *IEEE Transaction on Communications*, Vol. 59, Issue 5, May 2011, pp. 1236–1241.
[12]. Digital Video Broadcasting (DVB); Frame Structure Channel Coding and Modulation for a Second Generation Digital Terrestrial Television Broadcasting System (DVB-T2), ETSI EN Std. 302 755 V1.3.1, *European Telecommunications Standards Institute*, Apr. 2012.
[13]. K. Bae, K. Kim, and H. Yang, Low complexity two-stage soft demapper for rotated constellation in DVB-T2, in *Proceedings of the IEEE International Conference on Consumer Electronics (ICCE'12)*, January 2012, pp. 618–619.