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Capacitance: The Transducer that Converts Resistors into Electromagnetic Sensors

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Abstract: It is widely known since 1928 that each resistor at temperature T shows a voltage noise of spectral density 4kTR V^2/Hz, where k is the Boltzmann constant. However, the way in which this voltage arises from the exchanges of energy of this device with its thermal bath remains still unclear. Leaving aside strange features ascribed to its resistance R in order to justify how this random voltage is generated, we have considered for this purpose the electrical capacitance C of each resistor of volume V_Q sandwiched between its two equipotential terminals (plates). Using a new, Fluctuation-based noise model that excels the Dissipation-based model in use today, we have shown that Johnson noise is the measurable effect caused by truly impulsive charge noise that takes place in this reactive element with a mean square value of 4kT/R Coulombs^2/second. Hence, capacitance is the transducer that converts thermal fluctuations of electric field within V_Q into this random voltage called Johnson noise. The C^2/s units that emerge from this advanced noise model for the familiar value of the Nyquist density 4kT/R A^2/Hz and its partition into 2kT/R ultra wideband and 2kT/R band-limited noises, are two new results of this advanced model.

Keywords: Charge noise and fluctuations, Conduction current and dissipation, Displacement currents and shot noise, Noise out of thermal equilibrium, Two-terminal devices and complex immitances.

1. Introduction

Even though the voltage noise between terminals of any resistor (i. e. its Johnson noise) and its thermal origin are well-known since the pioneering works of J. B. Johnson [1] and H. Nyquist [2], the noise model developed from their results that is currently in use should be improved. We refer to the dissipation-based model (DBM) where capacitors and inductors do not generate noise. Concerning resistors this model has led to some myths like "Ohmic" resistances taken as noisy whereas dynamical ones are taken as noiseless without any proof. This strange situation in regard to the uniqueness of the magnitude resistance is well summarized by the voltage noise given by the circuit simulator PSPICE for a Schottky diode in thermal equilibrium (TE) at room temperature for example.

People wishing measurements to compare with the simulations could consider that the measurement of the voltage noise of a broad area Schottky diode (e. g. a power diode to have a low dynamical resistance r_d in the MΩ range for the diode in TE) is at the reach of today's instrumentation. This rather low $r_d \approx 1$ MΩ for the diode in TE at room T facilitates the measurement of its voltage noise by a typical Low Noise Amplifier (LNA) of much higher input resistance. If $R_{in} \geq 20$ MΩ the noise contribution of R_{in} becomes much lower than that of this diode under open circuit conditions. This is so because the noise density S_{Imeas}=(4kT/R$_{in}$+ 4kT/r$_d$) A^2/Hz at the input of the LNA is very close to that of the diode without R$_{in}$ (S_{Idiode}=4kT/r$_d$ see below).

For R$_{in}$=20 MΩ and in the low frequency region, the flat density of voltage noise we would measure is: S_{Vmeas}=4kT×(R$_{in}$×r$_d$)/(R$_{in}$+r$_d$) V^2/Hz that is 5 % lower than the S_{Vdiode} we are looking for. This departure that comes from the power divider created by R$_{in}$ and r$_d$ connected in parallel (R$_{in}$||r$_d$), is unavoidable but easy to consider. More important is the fact that the noise power measured would have one part coming from the resistor of R$_{in}$ ohms (device1) and twenty parts coming from the diode (device2) given the null correlation of these two noises that are generated in devices whose volumes are in different regions of space. Hence, the voltage noise we would measure is that of the diode "slightly contaminated by noise of R$_{in}$".

Concerning the voltage noise given by PSPICE for these two devices, we should simulate the voltage noise between terminals of our diode shunted by R$_{in}$ and shunted also by a dc current source I$_{dc}$. This source driving the diode allows the presence of the fluctuating voltage that we want to simulate between terminals. Using a low bias current like I$_{dc}$=10^{-25} A is a way to benefit from the above feature at the cost of adding S_{Ishot}=2qI$_{dc}$≈3×10^{-44} A^2/Hz to S_{Idiode}=4kT/r$_d$= 1,7×10^{-26} A^2/Hz, thus an irrelevant effect. Note that q is the electronic charge. Strictly speaking this I$_{dc}$≠0 puts the diode "slightly out of TE", but the voltage offset it produces between its terminals that is: ΔV$_{dc}$≈±(I$_{dc}$×r$_d$) volts, where the sign depends on the forward/reverse sense of I$_{dc}$, becomes irrelevant. Since this offset only is: ΔV$_{dc}$=±10^{-19} volts, we can assume that, for practical purposes, we are simulating the voltage noise of these two devices "both in TE" at room T.

Running PSPICE we could check all the above, but we would see that the density of voltage noise $S_{V\text{simul}}$ it would give would not be S_{Vmeas}, but twenty-one times lower: S_{Vmeas}=4kT×(R$_{in}$||r$_d$)/21. This low value given by PSPICE appears because PSPICE takes R$_{in}$ as "Ohmic" and ascribes a density 4kT/R$_{in}$ A^2/Hz to it, but concerning r$_d$ (that PSPICE calculates perfectly, see below) it is taken as noiseless. Due to this, the noise that PSPICE uses is: $S_{I\text{simul}}$=(4kT/R$_{in}$+0) =S_{Imeas}/21 and this explains why it is unable to give the voltage noise of the diode in TE. The right value of the r$_d$ that PSPICE obtains is observed in the cut-off frequency of the low-pass filtering action that it shows perfectly for the small noise S_{Vsimul}=S_{Vmeas}/21 V^2/Hz that it gives. This filtering is due to the junction capacitance C$_j$ of the diode that shunts the resistance (R$_{in}$||r$_d$).

Despite this filtering action suggesting that r$_d$ is "actual resistance" (i. e. *the ratio between sinusoidal voltage and current amplitudes found in phase in the circuit at each frequency*) PSPICE considers r$_d$ as if it was a "different" noiseless resistance. Due to this myth that flaws PSPICE, the noise it gives is not enough "to keep thermal equipartition in C$_j$", which is a notion of TE that should prevail (see below) over myths like "different resistances depending on the technology of the resistors where they come from". The magnitude "resistance", defined uniquely by the first sentence in italics of this paragraph, does not give room for two types of resistances. Hence, the striking "notch in TE" that S_{Vsimul} shows around ΔV$_{dc}$≈0 when the sign and the magnitude of I$_{dc}$ is varied to sweep between ±100 mV the dc voltage of the diode, is the "signature" of the aforementioned myth flawing the DBM undergoing this widely used circuit simulator.

For readers wishing to check PSPICE concerning voltage noise in diodes we shall give some numerical data after saying that PSPICE works splendidly for reverse voltages below four thermal voltages V$_T$. This means that diode voltages are: ΔV$_{dc}$=V≤-100 mV at room T (V$_T$≈25.9 mV at T=300 K). And moreover: as the reverse bias of the diode is higher, the accuracy increases. Though accordingly to the DBM, capacitors do not give noise, we shall consider C$_j$=100 pF as the depletion capacitance of this diode in TE at room T and whose I-V characteristic will be this one:

$$I = I_{sat} \times \left[\exp\left(\frac{V}{mV_T}\right) - 1\right], \qquad (1)$$

Taking a realistic ideality factor m=1.05 to show what its effect is and I$_{sat}$=2.7×10^{-8} A at room T to have r$_d$≈1 MΩ in TE, the dynamical resistance would be:

$$r_d(V) = \left[\frac{dI}{dV}\right]^{-1} = \left[\frac{I_{sat}}{mV_T}\exp\left(\frac{V}{mV_T}\right)\right]^{-1}, \qquad (2)$$

Eq. (1) with these values of m and I$_{sat}$ for the diode plus the capacitance C$_j$=100 pF in parallel, would form the model we need for simulations. Since the voltage of the diode in TE is V=0, we obtain:

$$r_d(0) = r_d = \frac{mV_T}{I_{sat}} = \frac{1.05\times0.0259}{2.7\times10^{-8}} = 1M\Omega, \qquad (3)$$

In this way we could simulate the voltage noise of our diode to compare it with measured values (e.g. in a commercial power Schottky diode to get a low r$_d$) or with values that we can justify theoretically from well-known basis. In any case, this disappointing "state of affairs" concerning the futility of the DBM for noise in junction diodes led us to develop a new noise model from a different basis. We mean a fluctuation-based model (FBM) that we developed to overcome the limitations of the DBM (that strictly speaking is valid only in TE) and to remove misconceptions and myths on this subject that still remain in regard to the voltage noise that we measure in two-terminal devices (2 TDs) like resistors, capacitors or L-C resonators.

Thus, the notion of a 2 TD is essential and in regard to measurements, the magnitude we can measure is a random voltage of spectral density in V^2/Hz, thus not random current that is never measured directly but inferred from a voltage caused by such current. The unawareness of this fact shows that voltage noise is not well-known today, despite the Fluctuation-Dissipation (F-D) framework that H. B. Callen and T. A. Welton proposed from a quantum physics approach for noisy processes in 1951 [3].

From a circuit point of view, the F-D framework means that conductance or resistance for dissipation of electrical energy, as well as reactance or susceptance to hold this energy that fluctuates, are both required to model a noisy device. When this need of a complex immitance to study electrical noise is accepted, the kT/C noise of capacitors and the Johnson noise of resistors become the same type of voltage noise that can be studied by a common fluctuation-based model (FBM) for voltage noise. It is worth noting that kT/C V^2 is the integral in frequency *f* of the Johnson noise of the resistor that any capacitor becomes as $f \rightarrow 0$ [4].

Resistors and capacitors are 2TDs that differ from inductors, whose two terminals vanish when they are shorted to hold magnetic energy liable to fluctuate within them. By contrast, the energy that fluctuates and is dissipated in a 2TD is electrical energy *U(t)* defined by its voltage *v(t)* at each instant of time and its capacitance C between terminals. Using *Q(t)* for the time-varying charge in one of the plates of C, this fluctuating energy is:

$$U(t) = \frac{1}{2}Cv^2(t) = \frac{Q^2(t)}{2C}, \tag{4}$$

Hence, Johnson noise *v(t)* reflects the energy *U(t)* that exists at any instant of time in the capacitance $C=(C_{str}+C_{mat})$ between terminals of a resistor. This is why C includes any stray value due to wiring (C_{str}) and the inherent term C_{mat} due to the non-null dielectric permittivity of its material between terminals [5]. The FBM for electrical noise that we refer to can be found in [6] and to show its starting point we shall consider the possible shot noise of our Schottky diode in TE.

2. Using Shot Noise to Remove a Myth

Eq. (1) states that our Schottky diode in TE is a 2TD where two currents, each of mean value I_{sat}, run with opposed senses. Hence, they cancel one another on average, but not at each instant of time because they come from different regions of the device. This means that they are uncorrelated and their associated effects like their shot noises will add in power. Let us do this sum manually because to calculate shot noise in a 2TD PSPICE uses its net current, thus being useless for our diode whose net current in TE is null: $I_{dc}=(I_{sat}-I_{sat})=0$.

Then, the total shot noise density of our diode in TE will be: $S_{Ishot}=2qI_{sat}+2qI_{sat}=4qI_{sat}$ A^2/Hz. Since the magnitude we measure is not a current noise density like this but its effect as voltage between terminals, we must obtain the spectral density S_{Vdiode} of voltage noise (in V^2/Hz) due to S_{Ishot}. Multiplying S_{Ishot} by $(r_d)^2$ we obtain this spectral density of voltage noise:

$$S_{Vdiode} = 4qI_{sat} \times (r_d)^2 = m4kTr_d \tag{5}$$

Since m=1.05 we have: $S_{Vdiode} \approx 4kTr_d$ V^2/Hz, that is the noise density that PSPICE is unable to give due to the myth about electrical noise giving rise to two types of resistances. Hence, removing from PSPICE (i. e. from the DBM) this harmful myth, our knowledge about electrical noise will improve. Added to the above, we observe that working with shot noise linked with the displacement currents of single electrons in a capacitive device can be a good way to obtain cogent results in regard to electrical noise.

Considering that shot noise comes from electrons that are displaced between the terminals of a 2TD, we envisaged years ago that displacement currents (like those I_{sat} due to electrons crossing the depleted region of our Schottky diode) could give rise to the voltage noise of 2TDs like resistors and capacitors. It is worth noting that our Schottky diode in TE is both a resistor and a capacitor. From the cut-off frequency of its r_d-C_j parallel circuit: $f_c=1/(2\pi r_d C_j) \approx 1.6$ kHz, we can say that for $f<160$ Hz, our diode is a quite good resistor of 1 MΩ and for $f>16$ kHz it is a quite good capacitor of 100 pF.

The above paragraph shows the way we envisaged the FBM published in [6] that we had used previously to explain electronic noises out of TE like the flicker noise of vacuum devices [4] and the 1/*f* resistance noise of solid-state devices [5]. Concerning the density $S_{Vdiode}=4kTr_d$ V^2/Hz we have given as the true voltage noise of the diode, we will say that this value is its flat amplitude below f_c. This is so because the truly flat density of shot noise $S_{Ishot}=4qI_{sat}$ A^2/Hz is low-pass filtered by the r_d-C_j parallel circuit of the diode to give this Lorentzian spectrum of voltage noise:

$$S_{Vdiode}(f) = \frac{4kTr_d}{1+\left(\frac{f}{f_c}\right)^2} \tag{6}$$

Since Eq. (6) means that "we are keeping thermal equipartition in C_j" because its integral from $f \rightarrow 0$ to $f \rightarrow \infty$ gives the mean square voltage noise kT/C_j V^2 [6], we have said confidently that the S_{Vsimul} of PSPICE for our Schottky diode in TE was untrue.

The reason why PSPICE works splendidly when our diode has reverse voltages below V = -100 mV is because in this case, the net current of the diode tends to be: $I_{dc} \approx -I_{sat}$. From Eq. (1), the condition $I_{dc}=-0{,}99I_{sat}$ is found for V=-125 mV. This means that PSPICE will take $0.99I_{sat}$ as the net current it uses to obtain the shot noise density of the diode as $S_{Ishot}=2q \times 0{,}99I_{sat}$ A^2/Hz. Since $I_{dc}=-0.99\ I_{sat}=I_{sat}\times(0.01-1)$, the two displacement currents giving shot noise in this case are: the reverse saturation current -Isat and a fraction of the forward current I_{sat} that has been weakened down to $0{,}01I_{sat}$ by the reverse voltage (V=-125 mV) of the diode biased by $I_{dc}=-0{,}99I_{sat}$. The relative error for the amount of shot noise is only: (1.01-0.99)/1.01≈2 %.

More striking, however, is the exponential change of the resistance of the diode r_d(V) with the voltage V set by its bias current I_{dc}. We mean Eq. (2) that works perfectly in PSPICE showing that the $r_d(V=0) \approx 1$ MΩ of the diode in TE, now is: r_d(V=-125 mV)=4.6 MΩ. Since C_j=100 pF does not vary, the cut off frequency of the diode with I_{dc}=-0.99I_{sat} (thus with V=-125 mV) is 4.6 times lower f_c≈350 Hz. All in all, PSPICE would be showing us that the mean square voltage noise of our diode out of TE is half its value in TE. This is so because this noise comes from a Lorentzian spectrum with 4.6 lower bandwidth and only 2.3 times higher amplitude than these two values of the Lorentzian noise spectrum of the diode in TE (V=0 V).

Previous paragraph not only shows the basis of the noise "tunability" we used in [5] to show the synthesis of the 1/*f* spectrum of resistance noise in solid-state devices, but also the suitability of shot noise to handle the voltage noise of 2TDs that are out of TE. Besides, it reinforces the idea of shot noise as a good starting point for a new noise like our FBM. The key role of the capacitance C_j that we have shown in this Section and 4kT/R C^2/s of mean square charge noise given in the Abstract, should pave the way for the new noise model that we discuss in the next Section.

3. A Fluctuation-Dissipation Noise Model

Fig. 1-a sketches a resistor made from a rod of conductive material with two tinned iron caps where its two access wires are soldered. Fig. 1-b is a first order circuit between its two terminals. For the notions to come, stray inductance for its wires is not needed. Taking the impedance *Z(jω)* to study the noise voltage of this 2TD leads to the circuit of Fig. 1-c, where two circuit elements give rise to two orthogonal voltages from the single current they share. The sum of these voltages done by their series connection gives *v(t)*, the voltage of Johnson noise at each frequency *f*=*ω*/(2π), but the annoying frequency-dependent resistance R(*ω*) and capacitance C*(*ω*) of this circuit, are not easy to link with physical phenomena. Hence, a clever choice for its noise *v(t)* is the admittance *Y(jω)* =1/*Z(jω)*.

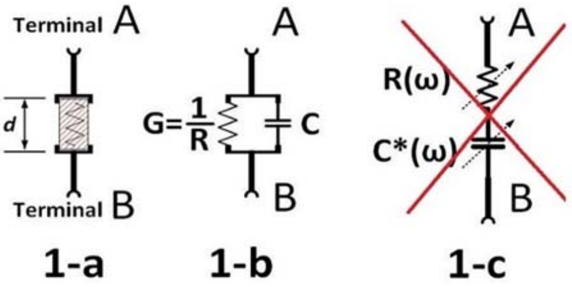

Fig. 1. a) Longitudinal cross section of a cylindrical resistor; b) A simple circuit to study its voltage noise; c) The series circuit of its complex impedance *Z(jω)*.

Admittance is a parallel notion where two mutually orthogonal currents add to give rise to a voltage shared by its conductance *G* and its susceptance *B*=*ωC*. The shared voltage *v(t)* is the Johnson noise and these two currents (the displacement and conduction ones linked to *C* and *G*) are mutually orthogonal due to the time derivative $\partial v(t)/\partial t$ of currents in *C*. From these ideas, we proposed that random passages of single electrons between terminals of a resistor, would give rise to its Johnson noise [6]. The random series of tiny voltage steps of amplitude ΔV=±*q*/*C* caused by such passages mean fluctuations of energy accumulated in C that are subsequently dissipated by R as *v(t)* (the voltage in C) decays with time-constant τ_{relax}=R×C.

This Fluctuation-Dissipation dynamics for voltage noise agrees with the F-D framework of [3]. The full agreement found if each electron passed with a null transit time (τ_t=0), led us to look for a cogent proof about such instantaneous translocations. This proof is the key result of [7] that has completed our FBM of [6]. To prove that this null τ_t is possible due to the capacitance *C* of any 2TD, we will study in the next Section how we measure electrical current. This result converts our advanced FBM into a powerful tool to study electrical and electronic noises.

4. Displacement Currents and Shot Noise

The direct measurement of small voltages is a well-known task in noise studies. Those people believing that small currents are measured directly and not from voltages they produce, could find useful this Section with regard to the design of *i-v* converters to measure small currents in noise experiments [8]. The reason of this usefulness is that the main task of this Section is to review the measurement of low-level currents in 2TDs like resistors and capacitors.

"Thermal agitation of electric charge" entitling [2] suggests what causes Johnson noise. Hence, finding how electric charge agitates in the volume V_Q of a 2 TD will likely show us how Johnson noise is generated in resistors. Since electrical measurements are the tool to monitor this agitation, we will consider them briefly. Whereas a voltmeter gives directly a voltage value, the value of any current is always inferred from a voltage one. Using a sensing resistor, a Hall probe or a current clamp (transformer) current is converted into voltage that we measure to infer the value of the current that has caused this voltage. Hence, electrical current is known after measuring the effect it produces: a voltage between two terminals. This leads to consider the way we measure very low currents (e. g. femtoAmps).

Let us think of a current involving a few electrons per second on average. "Average" comes from the idea of current as a flow of discrete, corpuscular electrons. Focusing in our task, let us consider again the resistor made from a rod of conductive material of dielectric relaxation time τ=ε/σ, cladded by its two metal caps at a distance *d* (terminals) that is sketched in Fig. 1-a. To better follow the notions to come, let us use Fig. 2 for a resistor made from a rectangular rod of material instead of the cylindrical one of Fig. 1-a.

Used to charge-matter interactions that take place by discrete packets of charge of *q*=-1,6×10^{-19} C each that we call electrons, let us show how electrons that "jump" between terminals of the 2 TD of Fig. 2 can agree with measurements. By the verb "to jump" we mean that if an electron travels from one terminal to the other, it will travel as a whole because to the best of our knowledge, pieces of electron are not found in low energy experiments like noise measurements. This leads to consider the possible behavior of the quantum of charge in a 2 TD and the fluctuations of energy that it can produce in it. Bluntly speaking, let us use quantum electronics for our task.

Each time we say "the current $i(t)$", we are actually considering two simultaneous currents at each instant of time. One enters the 2TD at position x in space by terminal A, while the other leaves the 2TD at position $(x+d)$ by terminal B. Simultaneous measurements at two points separated by a distance d in space requires using Especial Relativity (just basic skills). Consider two observers at plates A and B of Fig. 2, whose clocks were synchronized by an electromagnetic (EM) signal going from the observer at point x (plate A) to the observer at point $(x+d)$ (plate B). Needless to say, an EM signal going from observer in plate B to that in plate A would synchronize equally well their clocks following Einstein's lections.

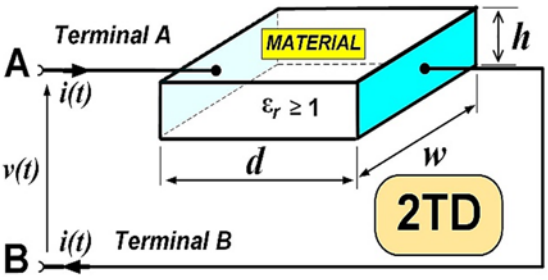

Fig. 2. Physical structure of a two-terminal device of volume $V_Q = w \times h \times d$ called resistor or capacitor, depending on frequency. Note its instantaneous "voltage" between its terminals and its two equal currents at each instant of time.

Although the notion of a corpuscle-like electron that leaves one plate at instant t_0 and arrives in the other at instant $(t_0+\tau_t)$, after a transit time $\tau_t>0$ seems possible, we will discard it for two reasons. The first one is because it makes impossible a cogent measurement of $i(t)$. With this corpuscular approach our two observers would never find the two simultaneous currents $i(t)$ of equal magnitude at positions x and $x+d$ that we need. If a current $i(t)$ starts to bring some charge $+\partial Q$ in plate A, another current of equal magnitude $i(t)$ also will start to extract charge $+\partial Q$ out of plate B.

Corpuscular electrons passing with transit time $\tau_t \neq 0$ would puzzle our observers. If the observer at plate A said: "a quantum of charge is entering the 2TD at $t=t_0$", the observer at plate B should say: "Really? Nothing occurs here at instant $t=t_0$." Then, if an electron passes between terminals, its electrical transit time will be null. This is better understood by taking Fig. 2 as a current sensing resistor of resistance R_s whose voltage $v(t)$ will allow us to infer its current by Ohm's Law $i(t)=v(t)/R_s$. The voltmeter giving the difference of electrical potential in plates A and B at each instant of time that is: $v(t)$, is what synchronizes as described our two imaginary observers. The right operation of myriads of ammeters that work in this way suggests that if the observer at plate A said: "a quantum of charge is entering this 2TD at $t=t_0$", the observer at plate B should have reasons to say: "a quantum of charge is exiting this 2TD at $t=t_0$".

The above situation is possible if an electron arrival in plate A created a simultaneous disturbance in plate B of Fig. 2 to warn its observer and vice versa. For an observer within V_Q and next to plate A, an electron arriving there through terminal A would create an electric field ΔE pointing inwards plate A that would reach plate B as an electric field pointing outwards plate B. The inward pointing field in plate A would tell observer A that an electron has just arrived there whereas this same field pointing outwards in plate B, would tell observer B that an electron has just left the 2TD through terminal B. Since this field that is born in a plate (no matter if inwards or outwards) reaches the other plate at the speed of the EM wave in V_Q, the arrival of an electron in one plate and its departure from the other become two simultaneous events. The quantum of charge translocated in this way is what equates the two currents of the 2TD.

Reviewing the measurement of weak currents in a 2TD we find that its capacitance could help electrons to jump suddenly between its terminals. To show the easiness of an electron to pass between terminals of resistors as a displacement current of null dwell-time ($\tau_t=0$), we will say that the amount of energy needed for this passage is truly small ($\Delta U \ll kT$). Making $Q=q$ in Eq. (4) we get $\Delta U=1.6\times10^{-7}$ eV for $C_{stray}=0.5$ pF. For $kT\approx26$ meV at room T we obtain $\Delta U=0.000006kT$, or less if we add the unavoidable $C_{mat}>0$ [5] to C_{stray}. Hence, the mean rate λ of electrons jumping between terminals due to thermal activity should be a huge number. Note that after an electron translocated in this way the voltage of C will shift by $\Delta V=\pm q/C$ volts depending on the sense of $\Delta E=(q/C)/d$, the step of electric field that is equivalent to, or that accounts for this monoelectronic translocation.

Since each of these jumps is the type of fluctuations of energy that we proposed as the origin of Johnson noise in resistors [6], it is not difficult to show the way this voltage noise is generated by these fluctuations occurring randomly in time and sign, with average rate λ. Thinking of a very cold resistor, Fig. 3 shows how its Johnson noise is built in time by these fluctuations that have been represented in the horizontal axis as Dirac's delta functions.

To give a number for λ let us use Eq. (4) for a single electron. This gives $\Delta U=q^2/(2C)$ Joules or $q/(2C)$ eV that we will take as the "mean energy per fluctuation that enters the resistor". The energy remaining in C after each fluctuation will continue its dissipation driven by the noise voltage $v(t)$ resulting after the $\pm q/C$ volts increment due to such fluctuation and $v(t)$ will continue decaying with time-constant

τ=RC. The null mean of the Johnson noise means that its causes (i. e. thermal fluctuations of electric field ΔE) will be 50 % positive (ΔE along x) and 50 % negative (ΔE along $-x$) on average. This balanced distribution agrees with the randomness of a thermal bath driving these sudden translocations of electrons between terminals.

Dividing by R the mean square voltage noise of C in TE that is kT/C V^2 due to thermal equipartition, we obtain P_d=kT/(RC) watts as the mean power dissipated in the resistor in TE. Equating P_d to λ times the mean energy per fluctuation $\Delta U=q^2/(2C)$ that enters the 2TD (that would be the mean power entering the resistor) the mean rate of fluctuations that results is [6]:

$$\lambda = \frac{2kT}{q^2 R} \tag{7}$$

In a resistor of R=50 Ω at room T we have $\lambda \approx 6\times10^{15}$ fluctuations/second. Hence, the Johnson noise of 50 Ω resistors or that of transmission lines of Z_0=50 Ω looks like a continuous voltage. But thinking of a very cold resistor at T→0, its Johnson noise would be something like the spiky voltage shown in Fig. 3.

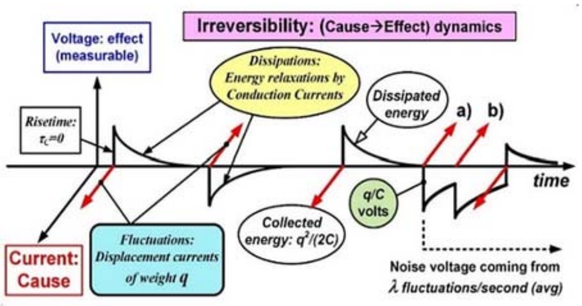

Fig. 3. 2-D model for Johnson noise generated from a random series of fluctuations in the volume V_Q of a resistor that are converted into voltage noise by its capacitance C.

Let us use the FBM for a 50 Ω resistor put in the test fixture of C_{stray}=0.5 pF of a noise analyzer to measure its Johnson noise at room T. Taking the material used for the resistor as a conductive one with a dielectric relaxation time of $\tau=\varepsilon/\sigma$=1 ps, the capacitance due to this is: $C_{mat}=\tau/R$=0.02 pF. Taking $C=C_{stray}+C_{mat}\approx$0.5 pF for simplicity, the mean square voltage noise between terminals of this C at room T will be: kT/C=8.3×10^{-9} V^2 due to thermal equipartition. This voltage noise driving R=50 Ω means P_d=166 pW as the mean power dissipated in this resistor at room T. Following [3, 6] if you observe voltage noise there must be dissipation in the noisy device.

With regard the mean energy per fluctuation that enters this resistor $\Delta U=q^2/(2C)$=0.16μeV, it is truly low, only is six parts per million the thermal energy kT≈26 meV at room T. The frequency of a photon of this energy only is: $\Delta U/h$=39 MHz. From Eq. (7), the capacitance shunting this resistor would sense its 50 Ω resistance as a "dense rain" of $\lambda=6.5\times10^{15}$ fluctuations per second. If this was a 50 MΩ resistor its shunting capacitance only would undergo 6.5×10^{9} fluctuations per second, thus 10^6 times lower as it would be the mean power dissipated in this case that would drop from 166 pW to $P_d=166\times10^{-18}$ W. This picture about how a capacitance "is sensing" its shunting resistance is very useful to envisage the effect of its voltage noise in the output signal of oscillators that use as resonator an L-C tank. We will show this later after a closer look at the way thermal energy pervading the volume V_Q of a 2TD is collected by its capacitance acting as an antenna for this purpose.

5. Charge Noise in C and Thermal Photons

From Maxwell equations, an electrical field is equivalent to positive and negative charge separated in space. The thermal bath that pervades the volume V_Q of a 2TD is electromagnetic energy (EME) traveling under the form of EM fields oscillating with different frequencies. When the EME travels in vacuum or through solid matter, it behaves as the continuous EM wave that was predicted by J. C. Maxwell. But when this EME interacts with charges existing in matter, it is absorbed or emitted in the form of packets of energy proportional to its frequency that are called "photons". The notion of EME as a set of individual photons that travel in space or through matter would be doubtful because its discrete nature is observed when it stops its travel and disappears as soon as it interacts with matter (to create an electron-hole pair for example).

Previous paragraph warns the reader about the doubtful meaning of sentences like "If a photon of energy U arrives in, or is absorbed by an electron…". Using the Plank constant h, it would be better to write: "If EME of frequency U/h arrives in, or is absorbed by an electron…". Once this warning has been given, we shall continue by saying that those oscillating fields of the thermal bath in the volume V_Q of a resistor can interact with some electrons of its inner material. This occurs with those conducting electrons of the material, often called "electrons in its conduction band (CB) or just free electrons", to mean that they can take part in conduction processes, a feature that electrons trapped in donor atoms embedded in this material, would not have. Neutral impurity atoms in n-doped silicon that are ionized by thermal energy produce free electrons in the CB of this material. This is the method we use to have free electrons occupying energy levels of its CB near its bottom level.

Free electrons that are already in the CB, in low energy states close to its bottom, can absorb thermal energy in order to occupy new states of higher energy within the CB and they continue being free electrons. In dual form, electrons in high energy states of the CB can emit EME to occupy states of lower energy in the CB. Concerning the inner material of our resistor with free electrons in energy levels near the bottom of its CB, the presence of its capacitance C sets a new degree of freedom for these free electrons that are able to gain or to lose energy to jump between energy levels within the CB. Given the tiny amount of energy $\Delta U=q^2/(2C)$ shown in Section 4, the presence of the capacitance C means that there are new energy levels very close to the bottom of the CB, available to free electrons able to exchange EME with the thermal bath in the volume V_Q. This leads to consider the energy requirements of these exchanges and their effect.

If EME of frequency $\Delta U/h$ and with the proper polarization arrived in the V_Q of a resistor when its voltage in C is null, an electron of the CB could absorb it "to set a voltage of q/C V in C". If we prefer: "to occupy one of the new energy states that the existence of C creates near the bottom of the CB". In this way the energy ΔU of the photon thus absorbed would enter this resistor and would appear as a sudden step of electric field in its C with amplitude $\Delta E=(q/C)/d$ V/m and null rise time. Hence, the absorption of this photon of the thermal bath gives rise to a voltage step between terminals of $\Delta V=q/C$ volts amplitude and null rise time that would set $v(t)=+q/C$ volts in this resistor.

By contrast, if $v(t)$ already was $v_i(t)=+q/C$ V when this EME arrived in V_Q it could not be absorbed to rise $v_i(t)$ up to $v_f(t)=v_i(t)+q/C$ volts because the energy (and thus the frequency of the EME) required for it now is: $(2q)^2/(2C)-q^2/(2C)=3\Delta U$. However, the capacitance with a voltage $v_i(t)=+q/C$ V would have in its cathode an electron with an energy ΔU that could emit on its own by a spontaneous photon, or by a photon that was stimulated by EME of frequency $\Delta U/h$ that could exist in the thermal bath. In these two cases, an amount ΔU of EME would pass from the resistor to its thermal bath. And if $v(t)$ already was $v_i(t)=+q/C$ volts and an EME of frequency $3\Delta U/h$ arrived in V_Q with the proper polarization, a free electron with energy close to the bottom of the CB could absorb a photon of this EME to set in C a voltage $v(t)=+2q/C$ volts. In this case, an energy of 3ΔU coming from the absorbed photon would enter this resistor. It would appear as a sudden step of electric field of amplitude $\Delta E=(q/C)/d$ V/m in its C.

Given the broad energy band for the photons of a thermal bath at room T and their statistical arrival in the 2TD, the noise voltage in C could depart markedly from $v(t)=0$ from time to time. We refer to much larger departures than the $\Delta V=q/C\approx0.3$ μV that results in this case where C≈0.5pF. Given the huge value of λ, a fast burst of fluctuations of the same sign that could vary $v(t)$ by 30 μV, for example, does not seem unlike. All the above and Fig. 3 show the way the Johnson noise of a resistor appears in time as this 2TD exchanges energy with its thermal bath due to its capacitance C between terminals. The sense of the flow of energy (inwards/outwards) at instant t_0 where a fluctuation takes place, will depend on the relative signs of the previous voltage $v(t)$ existing in C at t_0^- and that of the step $\Delta V=\pm q/C$ Volts generated by the fluctuation.

The energy that enters a resistor by its fluctuations and its unceasing dissipation due to the presence of $v(t)$ driving its resistance R, sustain in time a charge noise in its capacitance C whose measurable effect is the Johnson noise $v(t)$ at temperature T (see Fig. 3). It is worth noting that the charge noise due to fluctuations comes from the individual behavior of each electron, whereas the incessant dissipation of energy coming from the current $i_c(t)=v(t)/R$ results from the collective behavior of electrons as they vary in the plates of the capacitance C. This last behavior is transferred to the charge noise of the displacement current $i_d(t)=-i_c(t)$ that also exists in V_Q as it is shown in Fig. 4.

In the dielectric relaxation taking place in the 2TD of Fig. 2 when its voltage $v(t)$ decays as $\exp(-t/RC)$, the conduction current $I_c(t)=v(t)/R$ is counterbalanced at each instant of time by the displacement current due to the electric field that decays in C. This must be so to keep null the current $i(t)$ that enters or leaves the 2TD through its terminals A or B because in TE, it is under open circuit conditions. Hence, the current associated to R (i. e. $i_c(t)=v(t)/R$) and the displacement current in C (i. e. $i_d(t)=C\times(\partial v(t)/\partial t)$ are both non-null but they cancel one another at each instant of time. From such exact cancellation it could be said that the voltage noise generated by these two currents giving rise to zero current at each instant of time should be null, at first sight. "At first sight" means that as we will show below, the mean square charge noise due solely to fluctuations is: 2kT/R coulombs/second, thus half the shot noise in the resistor.

This exact cancellation that discards any magnetic effect after each fluctuation, is considered in Fig. 4. It was found looking for magnetic effects that could produce the type of fluctuations we had proposed in our FBM for electrical noise [8]. Added to this, and from our attempts in the past century to measure the voltage noise of the shot noise assigned to a dc current $I_{dc}=1$ μA in a 1 MΩ resistor, we knew that "conduction current in resistors is noiseless" (see below). Hence, we realized that the null current of previous paragraph was 50 % noisy -$i_d(t)$- and 50% noiseless: -$i_c(t)$- (see Fig. 4). And since the mean square charge noise due solely to $i_d(t)$ is 2kT/R coulombs/second, we conclude that the total mean charge noise of the resistor was: 4kT/R C^2/s (or A^2/Hz) as it is written in the title of [7], where this 50 %-50 % partition of the Nyquist noise was proposed for the first time (to the best of our knowledge).

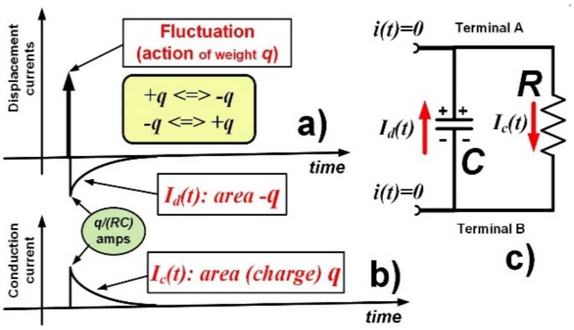

Fig. 4. Displacement and conduction currents triggered by a fluctuation causing a sudden voltage step of $\Delta V=q/C$ V in the 2TD of Fig. 2.

Considering that $i_c(t)$ and $i_d(t)$ are currents that are distributed in V_Q and go in opposed senses between plates of Fig. 2, we may use Fig. 4 to understand them better in a resistor with $v(t)=0$ between terminals up to the instant t_0, where the sudden jump of an electron from plate A to plate B takes place. This event would set $v(t_0^+)=+q/C$ volts in C. As soon as $v(t_0^+)$ was set, a conduction current of $[q/C]/R$ amps would start in R. It would be: $i_c(t)=+[q/(RC)]\times\exp[-(t-t_0)/RC]$ (see Fig. 4-b). Integrating $i_c(t)$ from $t=0$ to $t\rightarrow\infty$ the charge that we obtain suggests that the charge passing from plate A to plate B "through R" is $+q$ coulombs. This leads to the idea of a thermal action that displaces a charge $+q$ C from plate B to plate A and the subsequent device reaction by the conduction current $i_c(t)$ to restore the 2TD to its initial state just before the fluctuation, thus to its state at the instant $t=t_0^-$. Note how the current $i_d(t)$ goes unnoticed in this action/reaction notion.

From the currents shown in Fig. 4 however, we see that the charge displaced in one sense by a fluctuation and that displaced back by its subsequent displacement current $i_d(t)$ would cancel one another in time, though at different instants. Given this cancellation, no other displacement of charge would be needed if the internal dipole due to the fluctuation in this 2TD disappears in this way. Then this intriguing question arises: what is the actual charge displaced by the current $i_c(t)$ that we use to consider as "the passage of electrical charges through R"? Since the presence of $i_d(t)$ is undeniable, we would say that the conduction current $i_c(t)$ "should not displace charge at all". This sounds heretic because $i_c(t)$ suggested that the charge "passing from plate A to plate B through R" is $+q$ coulombs.

On the other hand, we knew long time ago that the voltage noise of a 1 MΩ resistor in TE and its voltage noise measured while a dc current $I_{dc}=1$ µA was set in it, were equal within experimental error. Years ago, we used to do this experiment as a calibrating routine of a home-made noise measuring setup. The 1 µW power dissipated by I_{dc} is low enough so as to not heat noticeably the big resistor of 1 MΩ/1 W that we used as a source of flat noise density $S_{VJohn}=1.7\times10^{-14}$ V^2/Hz. Viewing that the voltage noise density we measured for $I_{dc}=1$µA and for $I_{dc}=0$, was always S_{VJohn}, we have been considering that displacement currents are noisy, but conduction currents are noiseless for an unknown reason. Hence, currents like the two I_{sat} of a diode that occur in a space charge region (i. e. in a capacitive 2TD) should produce shot noise giving rise to voltage noise in the 2TD under study.

Regarding to this experiment, we shall say that if the current $I_{dc}=1$ µA was the flow of discrete electrons passing from the negative terminal to the positive one of this resistor, the number of electrons crossing it would be: 6.2×10^{12} electrons per second, whose shot noise converted into voltage noise by R=1 MΩ should give a density of voltage noise $S_{Vcond}=2qI_{dc}R^2$ V^2/Hz. This $S_{Vcond}=3.2\times10^{-13}$ V^2/Hz is much higher than S_{VJohn}. If I_{dc} was generating this noise, the experiment would reveal it unambiguously. Hence, conduction currents do not generate voltage noise.

From the null charge that should displace $i_c(t)$ when we consider that this task is done by $i_c(t)$ (see Fig. 4) and from the role of resistance "representing a series of dissipations of capacitive energy that are triggered by fluctuations" we could give a possible reason why conduction current is noiseless. If it would not involve the passage of discrete charges between two terminals, it would not produce voltage noise. Mimicking $i_c(t)$, conduction current should involve the occurrence of a series of dissipations of capacitive energy in the 2TD. If this was so, "dissipation current" would be a better name than conduction current for this type of current I_{dc} different from displacement ones. A requirement that I_{dc} should accomplish is to put the resistor out of TE, but "leaving unaltered the displacement currents that generate shot noise in the resistor in TE".

Searching for a model for conduction current from the above ideas we tried to keep the role of resistance in TE: "representing a series of discrete dissipations of capacitive energy that are triggered by fluctuations" but using an amount of energy for each dissipation that was proportional to $(V_{dc})^2$, the square of the dc voltage between terminals $V_{dc}=R\times I_{dc}$. With this starting point and inspired by switched capacitor circuits emulating resistors, we envisaged a model for conduction current in resistors that would keep unaltered the displacement currents (fluctuations) that give rise to their Johnson noise [8]. However, this is a subject still under study that falls out of the scope of this work.

The action-reaction dynamics of Figures. 3 and 4 is an old notion we used concerning charge noise in a 2TD before reading [3]. From the noise viewpoint, there are λ sudden actions per unit time and λ slower reactions in opposed sense associated to dissipations as shown in Fig. 4 (a). Fig. 4 (c) is the lumped circuit that results if we separate the interpenetrating currents $i_c(t)$ and $i_d(t)$ that are distributed in the volume V_Q. Regarding the sense of $i_c(t)$ going from plate A to plate B, it is clear because the positive charge is in plate A. This makes the voltages $V_A>V_B$ and applying Ohm's Law, $i_c(t)$ will go downwards "through R", as shown in Fig. 4.

Concerning the sense of $i_d(t)$, we must consider the electric field E set in C by the fluctuation and its time derivative. Since E points downwards (from plate A to plate B), but decreases as the voltage $v(t)$ decays in time, its time derivative is negative. Hence, $i_d(t)$ will go upwards, as shown. Since the external currents $i(t)$ of the 2TD of Fig. 2 are both null while $v(t)$ relaxes, we also obtain $i_d(t)=-i_c(t)$. This is the reason for the negative displacement current in Fig. 4 (a) starting at $t=t_0^+$, just when the fluctuation is already gone.

The notion of a mean square voltage is familiar for voltage signals. For charge signals, however, we must consider the charge in one plate of a capacitor and its mean square, since the net charge entering a 2TD like this one is null, as it should be known. From a charge noise point of view, the two opposed displacement currents of Fig. 4 (a) have a null overlap in time due to their cause-effect link [6, 9]. This means that they are uncorrelated and their effects will add quadratically or "in power". Once stated this, we shall go to Fig. 4 (a) to consider the charge noise associated with each fluctuation. To do this, we must integrate current in time.

Since the charge brought to C (we mean: to its plate A for example) by the fluctuation of null dwell time (Dirac's delta) is $+q$ C, its square will be: q^2 C^2. With regard to the charge brought to the same plate A of C by the slow current $i_d(t)$ coming after, it is $-q$ C and its square will be: q^2 C^2. In this way, the mean square charge noise caused by each fluctuation in a 2TD like a resistor, a capacitor or an L-C tank circuit will be: $(q^2+q^2)=2q^2$ C^2. Mimicking quadratic voltages taken as "power over a resistance of 1Ω" in signal theory, let us call "charge noise power" P_Q to quadratic values of charge noise in a capacitor. Using this notion, we can say that given the huge rates λ for fluctuations giving Johnson noise and its random sign, they should give rise to a quite constant charge noise power in C.

Working with charge noise power we shall consider first the power due solely to fluctuations. Because of their null dwell-time, these impulsive currents of mean square charge q^2 C^2 each, do not overlap in time and their effects will add in power. The mean charge noise power of fluctuations taking place at the rate of Eq. (7) will be λ times q^2 C^2/s (i. e. coulombs^2/second). Multiplying Eq. (7) by q^2, we obtain P_{Qf}=2kT/R C^2/s, a numerical result that is only half the Nyquist density S_I=4kT/R A^2/Hz.

Since P_{Qf}=2kT/R C^2/s or S_I=2kT/R A^2/Hz are both magnitudes having to do with displacement currents causing shot noise, the lacking 50 % of S_I should be the shot noise due to displacement currents different from the fast fluctuations of null dwell-time. This leads to consider the noise of those decaying displacement currents $i_d(t)$ of opposed sign to their preceding fluctuation that were explained in regard to Fig. 4-a. This slow charge noise power is also P_{Qs}=2kT/R C^2/s. Thus, the total charge noise power P_Q that produces voltage noise in the resistor in TE at temperature T is:

$$P_Q = P_{Qf} + P_{Qs} = \frac{4kT}{R} \tag{8}$$

To read properly Eq. (8), we must say that despite the equal charge noise power 2kT/R C^2/s of P_{Qf} and P_{Qs}, they have very different spectra. Whereas P_{Qf} comes from the individual behavior of the quantum of charge, P_{Qs} reflects the collective behavior of electric charges in the 2TD. The null dwell time of the fluctuations converts their displacement currents into an impulsive current noise whose spectral density is flat in frequency from the electrical viewpoint. This means that any electrical measurement in the 2TD should find voltage steps with null risetime reflecting the impulsive nature of these displacement currents producing shot noise of flat density S_I=2kT/R A^2/Hz. By contrast, the slower displacement currents tracking exponential decays of $v(t)$ will produce shot noise with Lorentzian spectrum of amplitude S_I=2kT/R A^2/Hz and cut-off frequency f_c=1/(2πRC) due to the time constant τ=RC of those exponential decays.

All in all, the Johnson noise of resistors will have two components. Its faster component a) will be a Lorentzian term of amplitude 2kTR V^2/Hz and cut-off frequency f_c=1/(2πRC) coming from the flat density of P_{Qf} (S_I=2kT/R A^2/Hz) being filtered by the circuit of Fig. 4 ©. Its second term, component b), will come from the Lorentzian shot noise of amplitude S_I=2kT/R A^2/Hz and f_c=1/(2πRC) being filtered by the circuit of Fig. 4 (c) whose time constant is τ=RC. This means that the -3 dB cut-off frequency f_{cs} of this slower term will be somewhat lower than f_c, and that its roll-off at high frequencies will be -40 dB/dec, two times faster than the -20 dB/dec of the component a). The sum in power of these two spectra of uncorrelated noises will show the well-known density S_V=4kTR V^2/Hz of Johnson noise at low frequencies, but its -3 dB bandwidth will be somewhat lower than f_c.

Concerning a value for frequencies close to f_c where the otherwise flat S_V will be decaying, we will consider again the noise measuring setup of C_{stray}= 0.5 pF for the Johnson noise of a resistor of R=50 Ω whose small but non null C_{mat} is included in C_{stray}. The time constant τ=RC=25 ps leads to f_c≈6.4 GHz. Hence, the Johnson noise of this resistor up to 2 GHz and even in the S band (2-4 GHz) would show the density S_V=4kTR V^2/Hz that everybody knows.

By contrast, this S_V density in the K band (8 GHz to 26 GHz) would be a neatly decreasing spectrum that would acquire a final roll-off approaching -20 dB/dec. Since measurements at these frequencies are not easy and most people use to work at "low f" (e. g. tens of MHz to avoid the filtering due to τ=RC) the limit f_c is barely reached and those measuring a flat S_V=4kTR V^2/Hz at f<<f_c, consider that the density S_I=4kT/R A^2/Hz that they infer from S_V is flat up to the limit given in [2] from quantum reasons.

6. Two Charge Noises: Empirical Evidence

The two different components of the phase noise of electronic oscillators (their carrier line of non-null width and a broad pedestal of much lower amplitude) is a striking feature that we are going to consider here. In frequency domain, the phase noise of an oscillator means that the spectral content of its output signal is not the line of null width $\delta(f\text{-}f_0)$ of a pure sinusoid of frequency f_0 (the expected "carrier" by design). Phase noise leads to find spectral content different from the above Dirac's delta.

The first noise term we refer to is "phase noise close to the carrier" because it is observed as a broadening of the otherwise monochromatic line at $f=f_0$. To keep this Section short, we will say that this part of the output spectrum is a Lorentzian line centered at f_0 that recalls the output spectrum of optical oscillators like lasers. In lasers, the broadening of their output line is already known as an effect of spontaneous emissions of photons that disturb the coherent optical energy that is stored in their resonant cavity, usually a Fabry-Perot resonator.

In dual form, the L-C tank of an electronic oscillator (i. e. its resonator) should sustain in time a coherent or pure sinusoidal voltage signal, and the electronic loop of the oscillator is designed to do this task in the best way possible. Since the L-C tank has a capacitance C that is connected in parallel with an inductance L, it is evident that any sinusoidal voltage sustained in time by the feedback electronics of the oscillator will be disturbed by the spontaneous translocations of single electrons between the plates of C (i. e. fluctuations) that are the origin of voltage noise in 2TD.

Due to the null risetime of each voltage step of $\Delta V = \pm q/C$ volts that is added randomly (both in time and sign) to the otherwise sinusoidal oscillation of voltage in the capacitance of an L-C tank (see Fig. 5), the electronics of the oscillator loop (its Automatic Amplitude Control AAC), can do nothing to avoid the appearance of these steps. We mean nothing to remove the impulsive current generating them in C, hence in the output signal. To react against one of these voltage departures of the output signal from a perfect sinusoid, the AAC must wait for the voltage ΔV in C (effect) that indicates the occurrence of a fluctuation "that is already gone". From this picture about how Johnson noise or kT/C noise enters into the resonator, we can envisage the presence of a kind of "roughness" in the output signal (phase noise) that is unavoidable.

From Eq. (7) for λ and $\Delta V = \pm q/C$, it is quite easy to obtain the phase noise due solely to these voltage steps of null risetime. Fig. 5 taken from [10] shows the way each fluctuation disturbs the sinusoidal voltage (output signal) that would exist in the capacitance of the L-C resonator used by these oscillators.

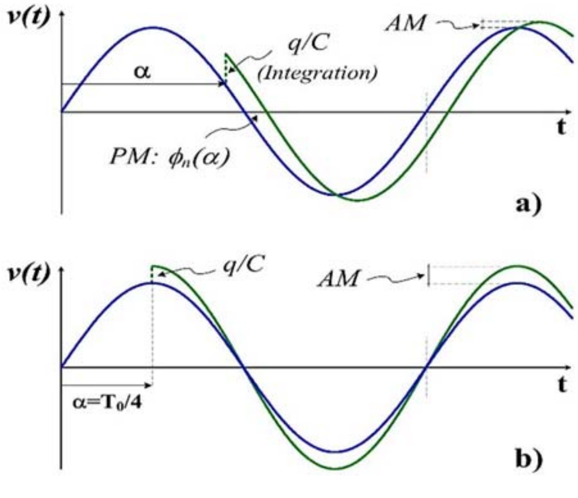

Fig. 5. Effects of a fluctuation taking place in the capacitance of an L-C oscillator when the damping of this change of energy is small during one period (see [10]).

Evaluating the phase noise due to disturbances like those shown in Fig. 5, taking place randomly both in time and sign with the average rate λ of Eq. (7), this phase noise spectrum is found [10]:

$$\mathcal{L}(\omega - \omega_0) = \frac{2D}{(\omega - \omega_0)^2 + D^2} \qquad (9)$$

Eq. (9) is the Lorentzian line or "phase noise close to the carrier" that we commented at the beginning of this Section. This line agrees perfectly with a classical paper in the field published by D. B Leeson [11]. Thus, the Lorentzian line of the output spectra of electronic oscillators comes from the truly impulsive current that gives rise to P_{Qf}. This would explain the deep analogy between spontaneous emissions of photons in lasers and spontaneous translocations of single electrons in L-C resonators due to thermal activity and why only half the Nyquist noise ($P_{Qf}=2FkT/R$) gives phase noise close to the carrier. To include the total losses of the L-C resonator "loaded by its surrounding electronics" we have used its unloaded resistance R divided by the noise figure F of the electronics around the L-C tank (see below). The single-sided spectrum of Fig. 6 taken from [10] shows the narrow line around f_0 (the carrier frequency) coming from $P_{Qf}=2FkT/R$ C^2/s and the flat pedestal of $P_{Qs}=2FkTR$ V^2/Hz divided by $(V_O)^2/2$, the mean square of the sinusoidal voltage of amplitude V_O (thus $P_O=(V_O)^2/2R$) sustained in C by the loop.

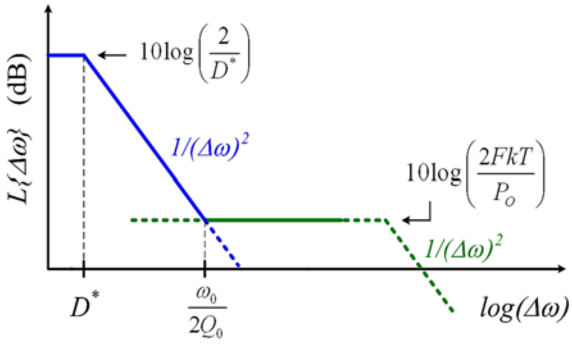

Fig. 6. Lorentzian Line of Phase Noise (e. g. "carrier" line) due to fluctuations in C, together with the Phase Noise Pedestal generated by the AAC (see [10]).

Concerning the slow 50 % of Nyquist noise we have called P_{Qs} we may say that gives rise to the pedestal of phase noise far from the carrier. This band-limited noise, being slower than P_{Qf}, is liable to be affected by the electronics of the loop and when this occurs, the band around f_0 of the aforementioned pedestal (i. e. its width) increases. The amplitude of this pedestal suggesting that only half the Nyquist noise $P_{Qs}=2kT/R$ C^2/s is involved in its generation, is another clue about the partition of the familiar density $S_I=4kT/R$ A^2/Hz of Nyquist noise into the fast and the slow shot noises that we have shown in [7]. Including the noise of the electronics by its noise figure F, the pedestal of phase noise results proportional to $P_{Qs}=2FkT/R$ C^2/s and the same occurs with the magnitude D (phase diffusivity) of Eq. (9) that becomes proportional not to P_{Qf}=2kT/R C^2/s, but to P_{Qf}=2FkT/R C^2/s, as it must be from the meaning of noise figure F in the FBM [6, 10].

It is worth noting that the best fitting to the phase noise of electronic oscillators has been achieved by using instantaneous translocations of charge between nodes of the oscillator circuit [12]. Nevertheless, the packets of charge that these authors have used involve thousands of electrons (e. g. packets of fC, 10^{-15} C or more) and the physical meaning of those sets of many electrons jumping synchronously between two nodes of the circuit is unclear. Nevertheless, the merit of this work using “sudden voltage changes” is undeniable and really useful if the user of the program is not very concerned for the physical meaning of this proposal that “works well in simulations”.

7. Conclusions

The model currently used for electrical noise in resistors is a dissipation-based model that needs to be improved to explain the noise of other two-terminal devices (2TD). With this notion in mind, we proposed a fluctuation-based model for electrical noise in 2TDs that fits in the Fluctuation-Dissipation framework that was proposed by Callen and Welton. It is a fluctuation-based model where the electrical capacitance between terminals of a 2TD is the reactive element that allows its thermal interaction. The voltage that appears in this capacitor whilst energy is being exchanged with its thermal bath, is Johnson noise in resistors, kT/C noise in capacitors and the voltage noise of 2TDs in general. Hence, the capacitance of each resistor between its terminals is the transducer that generates its density of Johnson noise S_V=4kTR V^2/Hz that we can measure.

Concerning the density S_I=4kT/R A^2/Hz that we can infer from S_V, this fluctuation-based model shows that S_I is formed by two different noises. The first term is impulsive shot noise of flat density S_{If}=2kT/R A^2/Hz until the quantum limit f_Q≈kT/h (f_Q≈6 THz at room T). The instantaneous translocations of electrons between the terminals of the device (i. e. fluctuations) give this ultrawideband noise. By contrast, the second term is a Lorentzian shot noise of amplitude S_{Is}=2kT/R A^2/Hz whose cut-off frequency is equal to that of the 2TD (where the voltage noise is being measured). The phase noise of oscillators that use L-C resonators agrees with the impulsive origin of S_{If} and with this 50 %-50 % partition: S_I= S_{If}+ S_{Is}=4kT/R C^2/s or A^2/Hz.

Acknowledgements

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