

Effect of a Rauch-Tung-Striebel Algorithm on Different Global Navigation Satellite System Time Transfer

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Abstract: Global Navigation Satellite System (GNSS) Precise point positioning (GNSS PPP) is widely applied in high precision international time transfer. With the development of GNSS precision orbit and clock products, GNSS time transfer technology has also attracted the attention of researchers. Traditional PPP has a long convergence time. In this paper, the improved algorithm based on the RTS method is used to calculate the time transfer results of different navigation systems to evaluate the improvement of this method on the time transfer results of different systems. The results show that the time transfer link based on Global Positioning System (GPS) and BeiDou Navigation Satellite System (BDS) can both achieve sub-nanosecond precision. The frequency stability of the time transfer results can reach $2E-14$ with the average time of $1E6$ seconds. The improved method can effectively reduce the influence of the convergence of the forward filter on the time transfer results.

Keywords: GNSS PPP, Kalman filter, Time transfer, Frequency and time, Time stability.

1. Introduction

With the development of GNSS, satellite navigation technology has been widely used in many fields of our life. In recent decades, many high-precision time transfer technologies have also been achieved through satellites. GNSS PPP time comparison technology is one of important technology for BIPM to conduct international time transfer [1]. GNSS PPP time transfer technology has been widely applied for low cost, high precision, and easy maintenance [2]. Though the traditional static PPP has high accuracy, its convergence time is long [3, 4]. It takes approximately 20 min required for 95 % of solutions to reach a horizontal accuracy of 20 cm or better for static PPP [5, 6]. PPP time transfer has the

same result. For different satellite navigation systems, the number of satellites, satellite elevation angles, and signal quality observed by the user at the same time are different. These factors can also cause PPP to have different convergence times.

Traditional static PPP only uses forward filter. The first series of positioning and time transfer data will have poor accuracy. For post-processing calculation, the fixed interval smoother can improve the accuracy. The fixed interval smoother combines forward and backward filter result by the noise matrix. This method can reduce the influence of the one-way filter convergence problem on the calculation of the initial part of the data. But introducing the inverse filtering result directly introduces additional noise, especially in the last part of the result. In order to solve the above

problems, based on previous work a post-processing RTS method is investigated in this paper [7].

2. Method

The traditional static PPP model has multiple model corrections, and it can be written as:

$$\begin{aligned} P_{r,j}^S &= \rho_r^S + c \cdot (dt_r - dt_{s,j}) + T_{r,j}^S + I_{r,j}^S + \\ &\quad + (d_{r,j} - d_j^S) + \zeta_j + \varepsilon_{r,j}^S(P_{r,j}^S) \\ L_{r,j}^S &= \rho_r^S + c \cdot (dt_r - dt_{s,j}) + T_{r,j}^S - I_{r,j}^S + \\ &\quad + \lambda_j^S (w_{r,j}^S + N_{r,j}^S) + \zeta_j + \xi_{r,j}^S(L_{r,j}^S) \end{aligned} \quad (1)$$

where $P_{r,j}^S$ and $L_{r,j}^S$ are the code and carrier phase observations, ρ_r^S is the geometric distance, c is the speed of light in vacuum, dt_r and $dt_{s,j}$ are the receiver and satellite clock offset, $T_{r,j}^S$ is the tropospheric delay, $I_{r,j}^S$ is the ionospheric delay, $(d_{r,j} - d_j^S)$ is the hardware latency, λ_j^S is the wavelength of carry phase, $(w_{r,j}^S + N_{r,j}^S)$ is the ambiguity parameter, ζ_j is the correction not listed, $\varepsilon_{r,j}^S(P_{r,j}^S)$ and $\xi_{r,j}^S(L_{r,j}^S)$ are the observation noise.

GNSS PPP uses a Kalman filter for parameter estimation. The estimation parameters include antenna coordinates, receiver clock, tropospheric parameters and ambiguity. The Kalman filter model is shown in Equation 2:

$$\begin{aligned} K &= Q * H * (H * Q * H + R)^{-1} \\ Xp &= X + K * l \\ Qp &= (I - K * H) * Q \end{aligned} \quad (2)$$

where K is the Kalman gain, X and Xp are the state vector, Q and Qp are the covariance matrix of states, l is the innovation (measurement - model), R is the covariance matrix of measurement error, H is the transpose of the design matrix, I is an identity matrix.

For forward filtering, the state parameters and the covariance matrix of the current epoch are calculated from the state parameters and the covariance matrix of the previous epoch. For post processed calculations, the fixed interval smoothed filter is used to combine the result of forward and backward filtering. For a calculation process with a total of N epoch, for epoch k , the state parameters and covariance matrix of epoch k can be calculated from epoch 1 to epoch k by forward filtering, and from epoch n to epoch k by backward filtering. The fixed interval smoothed filter combines the forward and backward results in a smoother, then get the final result. In this way, the results of the unconverged part of the forward filter are improved. However, this calculation introduces additional noise at the end of the data. Thus, a new

method based on Rauch-Tung-Striebel (RTS) algorithm is applied in this paper.

First, for a total of N epoch, a series of data from the epoch 1 is calculated by backward filtering. For post processed calculation, sufficient data are used to calculate the covariance matrix and state parameters for epoch 1 with high precision. The station geodetic coordinates, clock offsets, troposphere, ambiguity and corresponding covariance and noise matrices are used as initial information for forward filtering [8]. The result of this forward filtering from epoch 1 to epoch N is the final result. In this way, the influence of convergence at the beginning and the end of the calculation results can be reduced.

3. Calculation and Analysis

The PPP time comparison data UTC(TP)-UTC(ORB) of ORB and TP stations from MJD 59580 to 59587 are shown in follow section. The precise orbit and clock files are from GeoForschungsZentrum (GFZ). The calculation results of GPS and BDS are shown in following part. The time comparison results between the two stations published by the International Bureau of Weights and Measures (BIPM) are used as reference.

3.1. Time Transfer Results

Time transfer results using the above seven-day data processing are presented. The three methods mentioned above are all used for the calculation, and the time transfer results between the two stations through different two satellite navigation systems are different.

As can be seen from the Fig. 1, benefit from the high accuracy of the calculation results of the traditional static PPP, the results calculated by the above three methods are very close. Only the results of the forward filtering have an obviously difference from the reference value at the beginning of each day.

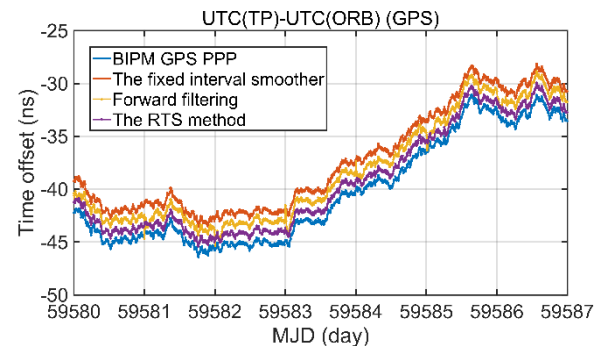


Fig. 1. The GPS PPP time transfer result of UTC(TP)-UTC(ORB) calculated by the above three methods and the results published by the BIPM. The observation data are from TP and ORB laboratories during MJD59580 to MJD59587.

From Fig. 2, it can be seen that the calculation results of Beidou are similar to GPS. The results calculated by the RTS method and the fixed interval smoother are very close to the reference values, and the results calculated by the forward filtering are slightly fluctuating.

In order to perform a detailed comparison of the impact of various methods on the results of different systems, the calculation results are shown as the difference between the clock offset between the two stations calculated by the above three methods and the reference.

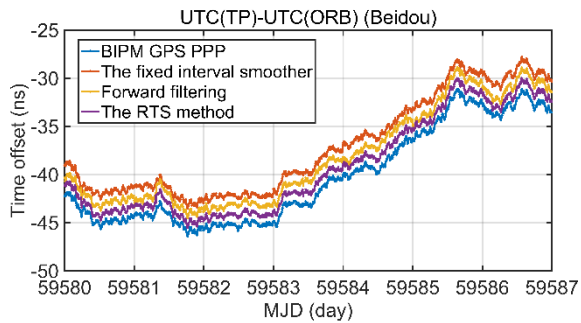


Fig. 2. The BDS PPP time transfer result of UTC(TP)-UTC(ORB) calculated by the above three methods and the results published by the BIPM. The observation data are from TP and ORB laboratories during MJD59580 to MJD59587.

3.2. Analysis of Time Transfer Results

The differences of the results of the different methods are shown below. The difference between the forward filtering method and the RTS method are shown in Fig. 3 and Fig. 4.

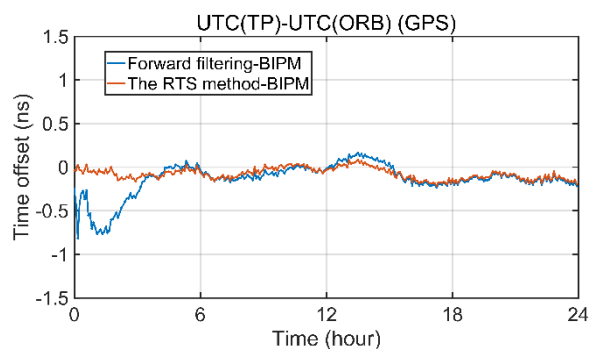


Fig. 3. Difference between time comparison results calculated by RTS algorithm and Forward filter GPS PPP. The observation data are from ORB and TP station, during MJD 59580.

As can be seen from Fig. 3, the difference between the GPS PPP results calculated by the two methods and the reference value is on the order of nanoseconds. When compared with the RTS method, the result with

forward filter has an obvious convergence process. In the first 6 hours of each day's data, the result of forward filter has large fluctuations. The root mean square (RMS) of the calculated results for the seven days data are used to quantitatively compare the two methods. The RMS of the results using the forward filtering for seven consecutive days is 319 ps, and the RMS of the first 6 hours of each day is 427 ps. The RMS of the results using the RTS method for seven consecutive days is 115 ps, and the RMS of the first 6 hours of each day is 88 ps, much better than the forward filter.

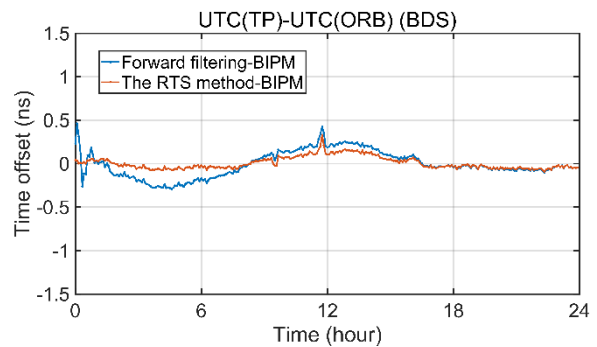


Fig. 4. Difference between time comparison results calculated by RTS algorithm and Forward filter BDS PPP. The observation data are from ORB and TP station, during MJD 59582.

The BDS data shows the same characteristics to GPS data. In the first 12 hours, there is a difference of several hundred picoseconds between the forward filter and the RTS method. But in the last 12 hours the two are very close. The RMS of the results using the forward filtering for seven consecutive days is 351 ps, and the RMS of the first 6 hours of each day is 471 ps. The RMS of the results using the RTS method for seven consecutive days is 125 ps, and the RMS of the first 6 hours of each day is 98 ps.

It can be seen that the RTS method improves the calculation results of the forward filtering of BDS and GPS, and it is more significant in the initial part of the calculation, which reduces the offset between the calculation results and the standard value. At the same time, it is similar to the result of forward filtering after convergence, and the characteristics of forward filtering are maintained. In contrast, GPS converges faster than BDS. After the forward filter has converged, and when using the RTS method, the time transfer accuracy of the two is comparable.

The difference between the fixed interval smoother method and the RTS method are shown in Fig. 5 and Fig. 6.

Fig. 5 shows that the results calculated with these two methods by GPS PPP are similar, the difference from reference value is within 0.5 ns. The results calculated by the two methods are obviously different in the last half series data. The calculation result of the RTS method is closer to the result calculated by the

forward filtering. The RMS of the results with the fixed interval smoother for seven consecutive days is 134 ps, and the RMS of the last 6 hours of each day is 151 ps. The RMS of the results using the RTS method for of the last 6 hours of each day is 138 ps. The result with RTS method has higher accuracy in the last 6 hours of daily data.

It can be seen from Fig. 6 that the difference between the time transfer results of the two stations through BDS PPP and the reference value is maintained within 0.5 nanoseconds. The calculation result of the RTS method is closer to the result calculated by the forward filtering. The RMS of the results with the fixed interval smoother for seven consecutive days is 122 ps, and the RMS of the last 6 hours of each day is 130 ps. The RMS of the results using the RTS method for of the last 6 hours of each day is 127 ps. The result with RTS method has higher accuracy in the last 6 hours of daily data.

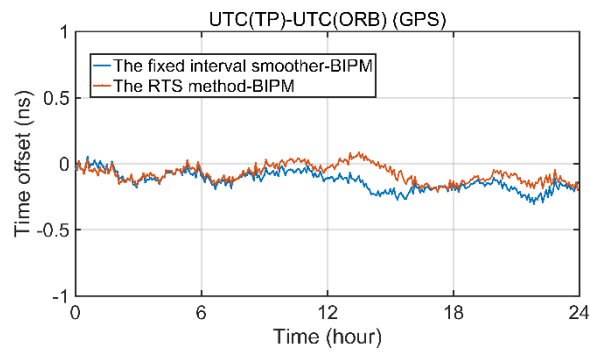


Fig. 5. Difference between time comparison results calculated by RTS algorithm and Fixed interval smoother GPS PPP. The observation data are from ORB and TP station, during MJD 59580.

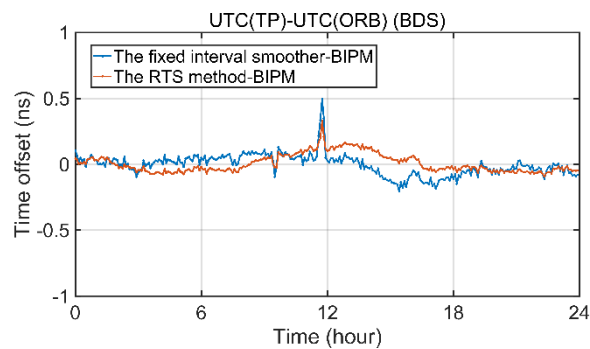


Fig. 6. Difference between time comparison results calculated by RTS algorithm and Fixed interval smoother BDS PPP. The observation data are from ORB and TP station, during MJD 59582.

The time transfer results calculated by the two navigation systems using these two methods are considerable precise, and the offset between the result and the reference value is within 0.5 nanoseconds. During the last 6 hours of data, the time-transfer results calculated using the RTS algorithm are slightly closer to the reference values.

3.3. Frequency Stability Analysis

The stability analysis is shown in following parts. Overlapping Allan deviation is used to evaluate the frequency stability of the alignment results [9].

Fig. 7 and Fig. 8 show the overlapping Allan variance of UTC(TP)-UTC(ORB) calculated by the three methods via GPS and BDS PPP and published by BIPM. It can be seen, for both GPS and BDS, the results of the RTS method and the fixed interval smoother have little difference compared to the value published by BIPM. However, the frequency stability of the results of the forward filtering is an order of magnitude lower than the others at an average time of 1000 seconds. As the average time increases to 100,000 seconds, the frequency stability of the results of the forward filtering raised to the same level as the others. It can be seen that both methods improve the short-term stability and reduce the impact of non-convergence compared with forward filtering for different navigation systems.

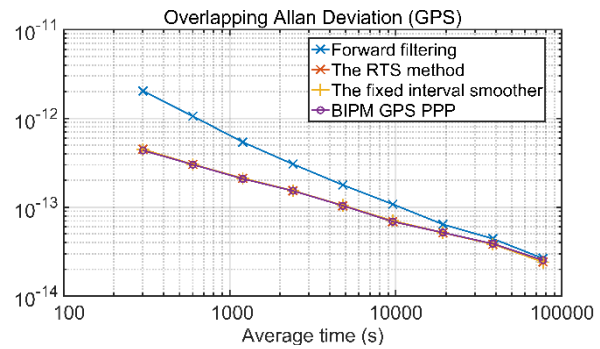


Fig. 7. Overlapping Allan deviation of UTC(TP)-UTC(ORB) calculated by different methods from MJD 58590 to MJD 59587.

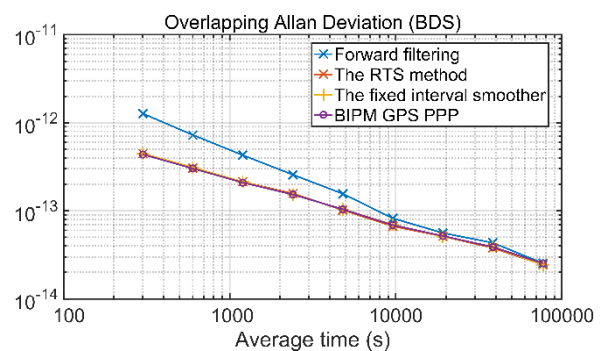


Fig. 8. Overlapping Allan deviation of UTC(TP)-UTC(ORB) calculated by different methods from MJD 58590 to MJD 59587.

Fig. 9 and Fig. 10 depicts the overlapping Allan variance calculated from 0:00 to 12:00 on MJD 59580 using the three methods by GPS and BDS PPP. The frequency stability of the results calculated with forward filtering is obviously worse than the other two

methods. It can be seen that in the previous part of the calculated data, both methods effectively improve the end effect with forward filtering.

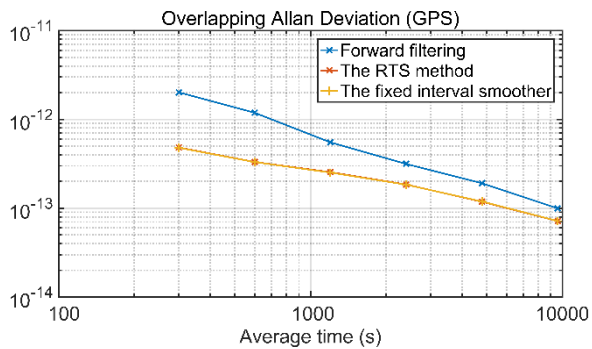


Fig. 9. Overlapping Allan deviation of UTC(TP)-UTC(ORB) calculated by different methods from 0:00 to 12:00 on MJD 59580.

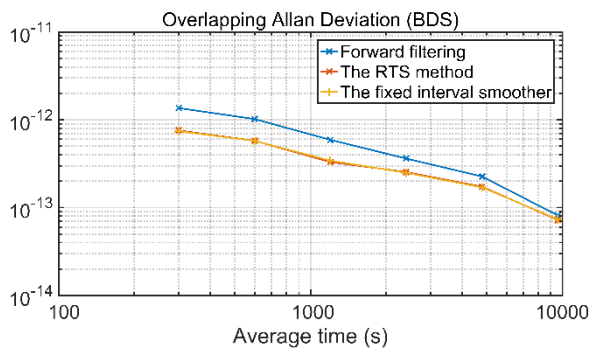


Fig. 10. Overlapping Allan deviation of UTC(TP)-UTC(ORB) calculated by different methods from 0:00 to 12:00 on MJD 59580.

The overlapping Allan deviation from 12:00 to 24:00 on MJD 59580 calculated by the three methods differs little from each other. Therefore, this part of the results is presented in Table 1.

Table 1 depicts the overlapping Allan variance calculated from 12:00 to 24:00 on MJD 59580 using the three methods. In the last half part of the calculated data, the results of the three methods have the same order of magnitude of frequency stability. The result calculated by the fixed interval smoother shows slightly worse frequency stability when the average time reach 10000 seconds. This is caused by the direct introduction of backward filtering results.

4. Conclusion

The results show that the PPP time transfer results based on GPS and BDS have high precision. The RMS of the offset between the reference values and PPP time transfer results using forward filtering, the fixed interval smoother and RTS methods are within 0.5 ns. Long-term frequency stability of time transfer results can reach $2E-14$ with the average time of $1E6$ seconds.

Table 1. Overlapping Allan deviation of UTC(TP)-UTC(ORB) calculated by different methods from 12:00 to 24:00 on MJD 59580.

Average time (s)	Overlapping Allan deviation		
	Forward filtering	The RTS method	The fixed interval smoother
GPS PPP			
3.00E+02	4.01E-13	4.03E-13	4.04E-13
6.00E+02	2.97E-13	2.98E-13	2.99E-13
1.20E+03	2.03E-13	2.02E-13	2.00E-13
2.40E+03	1.48E-13	1.47E-13	1.46E-13
4.80E+03	1.03E-13	9.87E-14	9.76E-14
9.59E+03	3.07E-14	2.99E-14	3.26E-14
BDS PPP			
3.00E+02	4.19E-13	4.17E-13	4.35E-13
6.00E+02	2.70E-13	2.70E-13	2.73E-13
1.20E+03	1.46E-13	1.47E-13	1.46E-13
2.40E+03	9.68E-14	9.82E-14	1.00E-13
4.80E+03	7.28E-14	7.35E-14	7.34E-14
9.59E+03	3.73E-14	3.17E-14	3.28E-14

For different navigation systems, the calculation results of the convergence time are different. Both the fixed interval smoother and the RTS methods can considerably improve the influence of the forward filter non-convergence on the accuracy of time transfer results. In the first few hours of the calculation, the error is reduced by a factor of several, and the frequency stability is improved by about an order of magnitude. But in the last hours of the results, the frequency stability of results calculated by the RTS method and the forward filter method are both slightly better than the fixed interval smoother.

In conclusion, the improved RTS method can effectively reduce the influence of the convergence of the forward filter on the time transfer results for GNSS PPP, and do not introduce large noise. This method can achieve sub-nanosecond PPP time transfer.

References

- [1]. Kouba, J., Héroux, P., Precise Point Positioning Using IGS Orbit and Clock Products, *GPS Solutions*, 5, 2001, pp. 12–28.
- [2]. Zumberge J. F., Heftin M. B., Jefferson D. C., et al., Precise point positioning for the efficient and robust analysis of GPS data from large networks, *Journal of Geophysical Research*, 102, B3, 1997, pp. 5005-5017.
- [3]. Choy, S., Bisnath, S. & Rizos, C. Uncovering common misconceptions in GNSS Precise Point Positioning and its future prospect, *GPS Solutions*, 21, 2017, pp. 13–22.
- [4]. Seepersad G., Bisnath S., Challenges in assessing PPP performance, *J Appl Geodesy*, 8, 3, 2014, pp. 205–222.
- [5]. Bisnath S., Gao Y., Current state of precise point positioning and future prospects and limitations.

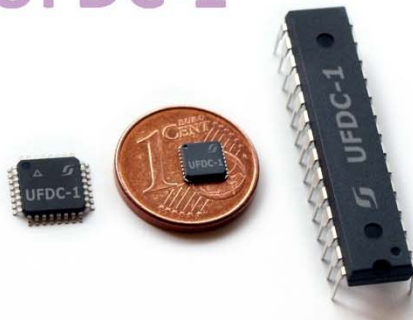
- Observing Our Changing Earth, *International Association of Geodesy Symposia*, Springer, Berlin, Heidelberg, 133, 2009, pp. 615–623.
- [6]. Hèroux P. et al., Products and applications for precise point positioning—moving towards real-time, in *Proceedings of ION GNSS 2004 Meeting*, Institute of Navigation, Long Beach, California, USA, 21–24 September 2004, pp 1832–1843.
- [7]. Mengshi C et al, An Rauch-Tung-Striebel Algorithm to Improve the End Effect of Static GNSS PPP, in *Proceedings of the 4th IFSA Frequency & Time Conference*, Corfu, Greece, 21-23 September 2022, pp. 10-13.
- [8]. Springer Handbook of Global Navigation Satellite Systems. Springer Handbooks, *Springer*, Cham.
- [9]. Allan D. W., Barnes J. A., A Modified "Allan Variance" with Increased Oscillator Characterization Ability, in *Proceedings of the Thirty 5th IEEE Annual Frequency Control Symposium.*, 1981, pp. 470-475.



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