Sensors & Transducers, Vol. 178, Issue 9, September 2014, pp. 214-218

Sensors & Transducers

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Research on Modified Root-MUSIC Algorithm of DOA Estimation Based on Covariance Matrix Reconstruction

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Received: 27 June 2014 /Accepted: 29 August 2014 /Published: 30 September 2014

Abstract: In the standard root multiple signal classification algorithm, the performance of direction of arrival estimation will reduce and even lose effect in circumstances that a low signal noise ratio and a small signals interval. By reconstructing and weighting the covariance matrix of received signal, the modified algorithm can provide more accurate estimation results. The computer simulation and performance analysis are given next, which show that under the condition of lower signal noise ratio and stronger correlation between signals, the proposed modified algorithm could provide preferable azimuth estimating performance than the standard method. *Copyright © 2014 IFSA Publishing, S. L.*

Keywords: DOA estimation, Root-MUSIC, Covariance matrix reconstruction.

1. Introduction

Root-Multiple signal classification (Root-MUSIC) is a classic method of the azimuth estimation algorithms. In this algorithm, the orthogonality of signal subspace and noise subspace is used to calculate the direction of arrival (DOA) [1, 2].

This approach results in good performance when the signals in space are independent of each other or they have a small correlation. Nevertheless, there exist some coherent signals generated by the multipath transmission and chromatic dispersion in a real environment. These coherent signals reduce the rank of the covariance matrix, leading to a situation that the algorithm cannot divide the signal subspace and noise subspace correctly. In this scenario, some steering vectors of coherent sources and noise subspace will be not orthogonal to each other, thus causes the omission of spatial spectrum estimation [3, 4].

Modified Root-MUSIC algorithm is proposed in this paper. By reconstructing and weighting the covariance matrix of received data, a new covariance matrix will be used for to calculate the DOA of signal. When the correlation exist in signals and in circumstances that a low signal noise ratio (SNR) and a small signals interval, the modified Root-MUSIC algorithm can estimate the signals more accurately than the standard algorithm.

2. Research Method

2.1. Signal Model

Consider a uniform linear array (ULA) which consists of M array elements, the spacing is d , λ is the carrier wavelength. Hypothesize there are N narrowband signals incident on each array element with an angle, as Fig. 1.

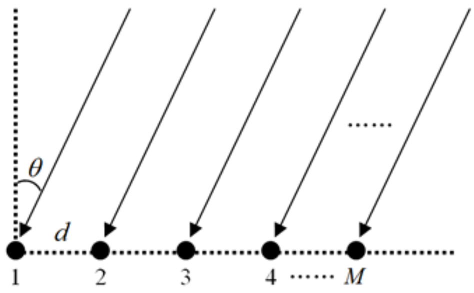

Fig. 1. Uniform linear array.

Assuming that there is a Gaussian white noise in the environment, with a zero mean and the variance σ_n^2, which has no correlation with signals. The data received by array elements can be expressed as the following model,

$$X(t) = AS(t) + N(t), \quad (1)$$

where A is defined as

$$A = [\alpha(\theta_1), \alpha(\theta_2), \cdots, \alpha(\theta_N)], \quad (2)$$

denotes the array manifold matrix, $\alpha(\theta_i)$ is defined as

$$\alpha(\theta_i) = [1, e^{j2\pi \sin(\theta_i)/\lambda}, \cdots, e^{j2\pi(M-1)\sin(\theta_i)/\lambda}], \quad (3)$$

denotes the steering vector. $S(t)$ is defined as

$$S(t) = [s_1(t), s_2(t), \cdots, s_N(t)]^T, \quad (4)$$

denotes the vector of source waveforms. $N(t)$ is defined as

$$N(t) = [n_1(t), n_2(t), \cdots, n_M(t)]^T, \quad (5)$$

denotes the vector of noise received by array elements [1].

2.2. Standard Root-MUSIC

Standard MUSIC algorithm handles with the covariance matrix of the received data R, divide the characteristic space into two subspaces by eigenvalue decomposition (EVD), signal subspace and noise subspace, using the orthogonality of the two subspaces to estimate the DOA of the signals. Define R as follows

$$R = \mathrm{E}\{x(t)x^H(t)\} = AR_S A^H + \sigma^2 I_M, \quad (6)$$

where A denotes the array manifold matrix, R_S is defined as

$$R_S = \mathrm{E}\{S(t)S^H(t)\}, \quad (7)$$

denotes the covariance matrix of signals, σ^2 denotes the noise power, I_M is the $M \times M$ unit matrix. Utilizing the EVD for R, two matrix can be get, S and U. S denotes the signal subspace, consisting of N large eigenvalues. U denotes the noise subspace, consisting of $M-N$ small eigenvalues. It can be proved that S and U are orthogonal [1], therefore, MUSIC spatial spectrum can be constructed as follows [5]

$$P_{MUSIC}(\theta) = \frac{1}{a^H(\theta)UU^H a(\theta)}, \quad (8)$$

Root-MUSIC algorithm [6-9] is a polynomial form of standard MUSIC, finding of the roots of polynomials instead of searching the space spectrum peak. Define the polynomial as

$$f(z) = e_i^H B(z), \quad i = N+1, N+2, \ldots, M, \quad (9)$$

where e_i denotes $M-N$ feature vectors which corresponding to the small eigenvalues of the covariance matrix. $B(z)$ is defined as

$$B(z) = [1, z, z^2, \ldots, z^{M-1}]^T, \quad (10)$$

denotes the steering vector of array signals. On account of $B(z)$ and noise subspace is orthogonal, the polynomial can be changed as

$$f(z) = B^H UU^H B, \quad (11)$$

Under the condition that the covariance matrix and the signal number are known, the DOA of signals can be calculated by solving the roots of polynomial z as follow

$$\theta_i = \arcsin\left(\frac{\lambda}{2\pi d}\arg(z_i)\right) \quad i = 1, 2, \ldots, N, \quad (12)$$

2.3. Modified Root-MUSIC

The modified Root-MUSIC algorithm is put forward in this section.

As is well known that, for the ideal independent signal sources, the covariance matrix R has Toeplitz nature, whereas in reality, the Toeplitz performance is destroyed because of limited snapshot, system error and coherent sources. This kind of situation reduces the performance of DOA estimation which base on the covariance matrix decomposition. Therefore, the covariance matrix R can be changed into Toeplitz matrix [10] as follow

$$R_T = (R + IR^* I)/2\,, \tag{13}$$

where the symbol * denotes conjugate operation, I is a $M \times M$ reverse unit matrix, R^* denotes the conjugate matrix of R.

Do EVD with R_T, S_T and U_T will be get. In order to reduce the effects of imprecise source number estimation, a considerable method of weighting with U_T is as follow

$$\tilde{U}_T = [\lambda_1^\beta \mathrm{u}_1, \lambda_2^\beta \mathrm{u}_1, \ldots, \lambda_{M-N}^\beta \mathrm{u}_{M-N},], \tag{14}$$

where $\lambda_i (i = 1, 2, \ldots, M - N)$ is eigenvalues of the covariance matrix R_T, $\beta \in [0,1]$ denotes the weighting coefficient. By weighting the eigenvectors, different eigenvectors can make different functions in MUSIC spectrum that the algorithm could maintain a preferable estimation performance under the under-estimation of source number condition.

Replace U with the new $\tilde{U}_T$, the formula $f(z)$ is changed as follow

$$f(z) = B^H \tilde{U}_T \tilde{U}_T^H B\,, \tag{15}$$

Considering the easiness of root, the formula $f(z)$ could be rewrote in a further form

$$f(z) = z^{M-1} B^T(z^{-1}) \tilde{U}_T \tilde{U}_T^H B(z)\,, \tag{16}$$

Under the same condition above, the DOA of signals can still be calculated by formula,

$$\theta_i = \arcsin(\frac{\lambda}{2\pi d}\arg(z_i)) \quad i = 1, 2, \ldots, N\,, \tag{17}$$

3. Results and Discussion

Some simulation results will be presented in this section in order to prove the correctness and effectiveness of the modified algorithm which has be put forward earlier in the article.

Considering a uniform linear array (ULA) which consists of 10 array elements. The interelement spacing d is 0.5λ. The number of narrowband signals N is 2. Two narrow-band coherent sources arrive at the array in incidence angles θ_1, θ_2, the estimated values of which are $\hat{\theta}_1$ and $\hat{\theta}_2$. Define an estimation result as

$$\begin{cases} Success\,,\, if \; \left|\hat{\theta}_1 - \theta_1\right| + \left|\hat{\theta}_2 - \theta_2\right| < \left|\hat{\theta}_1 - \hat{\theta}_2\right| \\ fail\,,\, other \end{cases}, \tag{18}$$

Then the probability of success (POS) estimation η will be

$$\eta = \frac{N_{success}}{N_{all}} \times 100\%\,, \tag{19}$$

where $N_{success}$ denotes the number of success estimation, N_{all} denotes the number of estimation in total. The simulation results are shown as Fig. 2, Fig. 3 and Fig. 4.

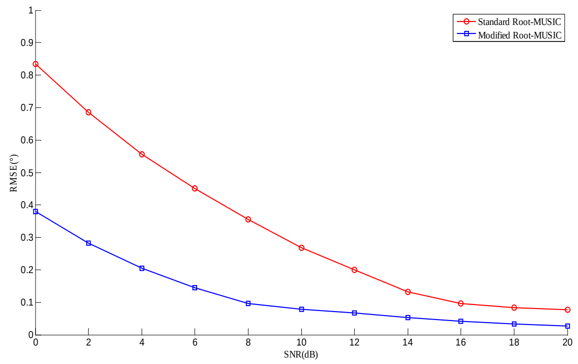

Fig. 2. DOA estimation RMSE against SNR.

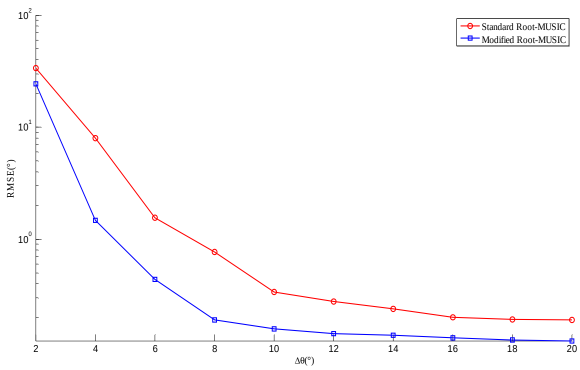

Fig. 3. DOA estimation RMSE against $\Delta\theta$.

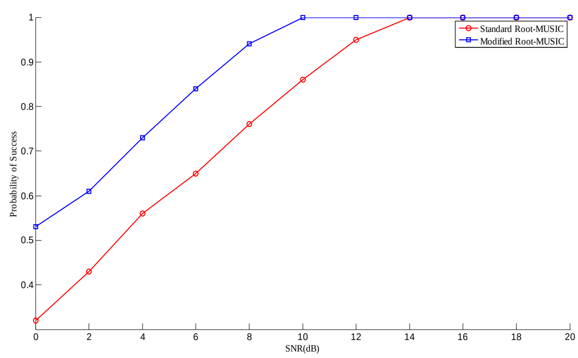

Fig. 4. POS against SNR.

Fig. 2 shows the DOA estimation root mean square errors (RMSE) of two methods, the standard Root-MUSIC, the modified Root-MUSIC respectively, against the SNR of signals, under the condition of $\theta_1 = \pi/6$, $\theta_2 = \pi/3$ .Using 100 Monte Carlo simulations to calculate the RMSE, it could be defined as

$$RMSE = \sqrt{\frac{1}{100}\left(\sum_{j=1}^{100}\left(\hat{\theta}_{1,j} - \theta_1\right)^2 + \sum_{j=1}^{100}\left(\hat{\theta}_{2,j} - \theta_2\right)^2\right)}, \quad (20)$$

As we can see from Fig. 2, under the condition of higher SNR, 20 dB, e.g., the two methods have good performance of DOA estimation, with the [RMSE (standard), RMSE (modified)]=[0.08°, 0.02°]. With the decrease of the SNR, the RMSE of standard Root-MUSIC algorithm increases gradually, nevertheless, modified Root-MUSIC algorithm still has a receivable performance, 0dB, e.g., with the [RMSE(standard), RMSE(modified)]=[0.83°, 0.38°]. The curves in Fig. 2 indicate that the modified algorithm has a lower RMSE than the standard which means more precise azimuth estimation under the same SNR.

Fig. 3 shows the DOA estimation RMSE of two methods, the standard Root-MUSIC, the modified Root-MUSIC respectively, against the angle difference between two signals $\Delta\theta = |\theta_1 - \theta_2|$, under the condition of $SNR = 10\,dB$.

As shown in Fig. 3, the x axis is the angle difference $\Delta\theta$, and the y axis is the DOA estimation RMSE, using a logarithmic coordinates axes because of the large value. When $\Delta\theta$ is relatively small, 2° e.g., the correlation between the two signals is strong, also the mutual interference between them should not be neglected, with the [RMSE (standard), RMSE (modified)]=[33.7°, 24.2°], unable to estimate the DOA of signals correctly both. When $\Delta\theta$ is an appropriate value, 8°, e.g., with the [RMSE(standard), RMSE(modified)]=[1.55°, 0.43°].

The curves in Fig. 3 indicate that the modified algorithm has a lower RMSE than the standard. It is known that the correlation and mutual interference between signals will be stronger with the decrease of the Angle difference. By reconstructing and weighting the covariance matrix of received data, the modified algorithm reduces the coherence of signal, which provides more precise azimuth estimation.

Fig. 4 shows the probability of success (POS) estimation of two methods against the SNR of signals, under the condition of $\theta_1 = \pi/6$, $\theta_2 = \pi/3$. As shown in this figure, the x axis is the SNR, and the y axis is the POS estimation, range [0,1]. With the increase of SNR, the POS of the two methods are both gradually increased. In the same SNR, the modified Root-MUSIC has a higher success probability than the standard one. 0 dB, e.g., with [POS(standard), POS(modified)]=[0.32, 0.53]. Another advantage is that to reach the condition of POS=1, the modified method need a smaller SNR than the standard method, with [SNR(standard), SNR (modified)]=[14 dB, 10 dB]. The curves in Fig. 4 indicate that the modified algorithm has a higher POS than the standard which means more efficient DOA estimation under the same SNR.

4. Conclusion

Simulation results show that, under the condition of lower SNR and stronger correlation between signals, the modified Root-MUSIC algorithm could provide preferable DOA estimating performance than the standard method by reconstructing and weighting the covariance matrix of received signal without increasing computational complexity, which has good reference value.

Acknowledgements

The author acknowledges the funding of following science foundation: National Natural Science Foundation of China (Grant Nos. 60908028, 60971068, 10979065, and 61275201).

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