

## UD-DKF-based Parameters on-line Identification Method and AEKF-Based SOC Estimation Strategy of Lithium-ion Battery

Xuanju Dang, Bo Chen, Hui Jiang, Xiangwen Zhang, Xiru Wu, Yan Mo

School of Electronic Engineering and Automation, Guilin University of Electronic Technology,  
541004, China

Tel.: 13978338379, fax: 0773-2290806

E-mail: xjd69@163.com

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**Abstract:** State of charge (SOC) is a significant parameter for the Battery Management System (BMS). The accurate estimation of the SOC can not only guarantee the SOC remaining within a reasonable scope of work, but also prevent the battery from being over or deeply-charged to extend the lifespan of battery. In this paper, the third-order RC equivalent circuit model is adopted to describe cell characteristics and the dual Kalman filter (DKF) is used online to identify model parameters for battery. In order to avoid the impacts of rounding error calculation leading to the estimation error matrix loss of non-negative qualitative which result in the filtering divergence phenomenon, the UD decomposition method is applied for filtering time and state updates simultaneously to enhance the stability of the algorithm, reduce the computational complexity and improve the high recognition accuracy. Based on the obtained model parameters, Adaptive Extended Kalman Filter (AEKF) is introduced to online estimate the SOC of battery. The simulation and experimental results demonstrate that the established third-order RC equivalent circuit model is effective, and the SOC estimation has a higher precision. *Copyright © 2014 IFSA Publishing, S. L.*

**Keywords:** SOC, Third-order RC equivalent circuit, DKF, UD decomposition, AEKF.

### 1. Introduction

For the serious situation of the energy crisis and environmental pollution, it becomes more and more critical for research and development of new energy vehicles. The battery, a key component of the whole system, must be rationally used for ensuring the safety of the electric vehicles. In recent years, lithium-ion battery with advantages of high energy, long cycle life, no memory effect [1] and wide work temperature range, etc., has been gradually used in the electric car [2].

The SOC is a crucial parameter of battery management system (BMS), which can not be

measured by the sensor measurement directly, (and) only be indirectly estimated by modeling and the corresponding algorithm [3]. During the running time, the charge and discharge current size, temperature, self-discharge and life are effect the battery behavior. The complicated nonlinear for battery leads to hardly estimate for the SOC [4]. The accurately estimating the SOC is vital parameter for the BMS.

Currently, the SOC estimation methods mainly include the ampere-hour [5], open circuit voltage, the fuzzy logic [6], Kalman filtering, and neural network [7]. The Kalman filtering is a kind of real-time recursive optimal estimation algorithm to use

only for the linear system, therefore the Kalman filtering cannot be directly used to estimate the SOC for battery with the complex nonlinear process caused by the internal chemical characteristics. The Extended Kalman filter (EKF), which has strong correction action for the initial error of SOC, is suitable for the nonlinear system. However, the statistical characteristics fluctuates dramatically along with the actual working condition, which would cause estimation loss of accuracy and even divergences for the filtering.

In this paper, the dual Kalman filter (DKF) is proposed to online identify the model parameters. In the process of filtering, the UD decomposition method is applied for the time and state updates concurrently, namely UD – DKF, to enhance the stability of the algorithm, reduce the computational complexity and improve the high recognition accuracy. For the sake of quickly and accurately running the UD-DKF algorithm to ensure its convergence and effectiveness, the least square method is used to identify reasonable initial parameter values for the battery models collected by the intermittent discharge experiment. Meanwhile, based on the designed UD-DKF, the adaptive filter method is introduced, namely, adaptive extended Kalman filtering (AEKF), which is applied to estimate the state of charge for improving the SOC estimation accuracy. The experiments confirm its good performance of robustness and effectiveness.

## 2. Battery Model and Initial Parameter Identification

### 2.1. Battery Model

At present, the commonly used models include the neural network model, AC impedance model, the electrochemical model [8] and the equivalent circuit model [9], etc. However the RC network equivalent circuit model with its own superiority of simple structure and high precision has been widely applied in the BMS. In this paper, the third-order RC equivalent circuit model is adopted, as shown in Fig. 1 [10].

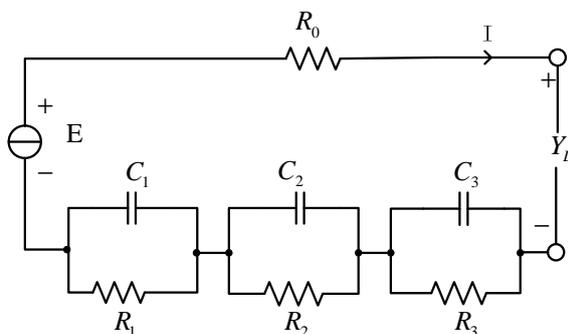


Fig. 1. Third-order RC equivalent circuit model of battery.

Its mathematical expressions are described as:

$$\frac{dU^1}{dt} = -\frac{U^1}{R_1 C_1} + \frac{I}{C_1}, \quad (1)$$

$$\frac{dU^2}{dt} = -\frac{U^2}{R_2 C_2} + \frac{I}{C_2}, \quad (2)$$

$$\frac{dU^3}{dt} = -\frac{U^3}{R_3 C_3} + \frac{I}{C_3}, \quad (3)$$

$$Y_L = E - IR_0 - U^1 - U^2 - U^3, \quad (4)$$

$$SOC = SOC_{initial} + \frac{1}{C_n} \int \eta I dt, \quad (5)$$

where the model parameters are defined by references [10-12].

### 2.2. Initial Model Parameter Identification

In this paper, the UD-DKF algorithm is employed for on-line identification model parameter, while the algorithm has good robustness for the initial values of parameters, but all the parameters in the model are unknown, the initial values of model parameters must be given within a reasonable range, which enable to run the UD-DKF algorithm for on-line identification model parameter quickly and accurately for guaranteeing its convergence and effectiveness. For obtaining reasonable initial values of model parameters, this paper selects a power lithium-ion battery, 20 Ah/24 V, pack made by a certain manufacturer as the research object. For the intermittent discharge experiment at the room temperature, the experimental results are exhibited in Fig. 2. The test equipment is Electric Vehicle Test System (EVTS) made by American ARBIN Instruments.

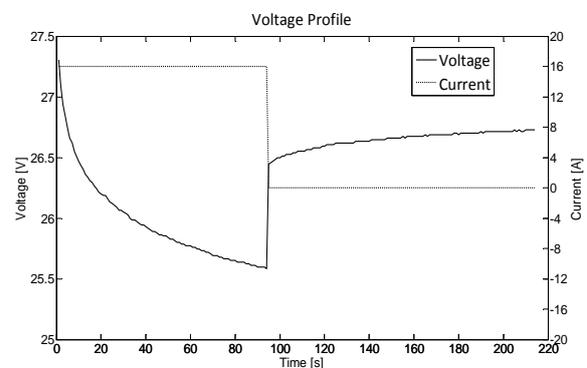


Fig. 2. Intermittent discharge experiment results.

The initial model parameter identification method mainly adopts the least-square method [13, 14], by

which the initial values of the ohm internal resistance  $R_0$ , polarization capacitance  $C_1, C_2, C_3$ , polarization resistance  $R_1, R_2, R_3$ , can be obtained to provide a foundation for the parameters on-line identification.

### 3. UD DKF-based Parameter on-line Identification

#### 3.1. UD Decomposition Principle

For the high dimension of state parameter of the system, the classical Kalman filter algorithm has a larger computing rounding errors due to computer words long restricted, thus, resulting in the prediction estimation error variance matrix  $P_{k,k-1}$  and estimation error covariance matrix  $P_{k,k-1}$  lose non-negative qualitative leading to filter divergence. In order to guarantee the filtering stability and reduce the amount of calculation time, the UD decomposition method [15, 16] is utilized in the process of filtering, by which the prediction estimation error variance matrix  $P_{k,k-1}$  and estimation error variance matrix  $P_{k|k}$  are decomposed into triangle U matrix and diagonal matrix D unit, which is expressed by

$$\begin{cases} D_{nn} = P_{nn} \\ U_{in} = \begin{cases} 1 & i = n \\ \frac{P_{in}}{D_{nn}} & i = n-1, n-2, \dots, 1 \end{cases} \end{cases} \quad (6)$$

$$\begin{cases} D_{jj} = P_{jj} - \sum_{k=j+1}^n D_{kk} U_{jk}^2 \\ U_{ij} = \begin{cases} 0 & i > j \\ 1 & i = j \\ \frac{P_{ij} - \sum_{k=j+1}^n D_{kk} U_{ik} U_{jk}}{D_{jj}} & i = j-1, j-2, \dots, 1 \end{cases} \end{cases} \quad j = n-1, n-2, \dots, 1 \quad (7)$$

#### 3.2. On-line Identification for UD-DKF Parameter

The identification of model parameters is the prerequisite for the SOC estimation. The model parameters for battery are online identified to accurately describe the internal dynamic characteristics of the battery influenced by many factors.

At present, many methods for on-line identification of the model parameters by the dual Kalman filtering method (DKF) are proposed. The identification for the single ohmic resistance is proposed in the references [17, 18]; The on-line

identification for the ohm internal resistance and capacity in the battery is given in [19, 20]. However, not all the parameters of the battery model are developed in above references. Therefore, the internal dynamic characteristics of the battery cannot be fully described. In this paper, the dual Kalman filtering algorithm is employed to online estimate all parameters of the model, in which the two separate Kalman filter are used for estimating the state and parameters for system, respectively. In each sampling cycle, the measurement data are applied to update the system parameters and state interchangeably and recursively, thus estimating for system state and parameters [21]. The experimental results prove that the proposed method is effective, and the estimation precision for the model is higher.

The dual Kalman filtering is combined with the UD decomposition approach to online estimate the model parameters, namely the UD-DKF, thus enhancing the filtering stability and reducing the computational complexity [22, 23]. In the references [24, 25], UD decomposition method is applied on KF (EKF) or AEKF, but just be utilized in the time update, not in the observation update. In this paper, DKF is employed to online identify model parameters and the UD decomposition method is applied on time update and state update simultaneously in the process of filtering in order to improve accuracy of the identification. The algorithm flow chart is presented in Fig. 3:

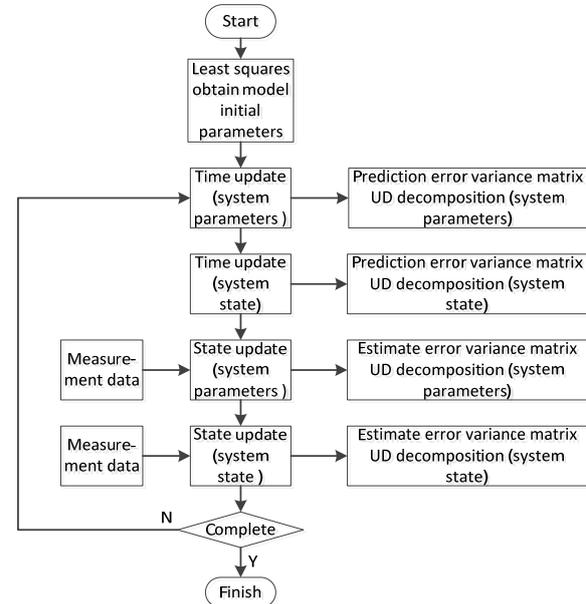


Fig. 3. The algorithm flow chart.

According to the established three order RC battery equivalent circuit model, the terminal voltage across capacitor  $C_1, C_2$  and  $C_3$  are selected as a state vector, i.e.  $X_k = [U_k^1 \ U_k^2 \ U_k^3]^T$ , the discrete state equation and measurement equation are written as:

$$\begin{cases} X_k = \begin{pmatrix} e^{\frac{\Delta t}{\tau_1}} & 0 & 0 \\ 0 & e^{\frac{\Delta t}{\tau_2}} & 0 \\ 0 & 0 & e^{\frac{\Delta t}{\tau_3}} \end{pmatrix} X_{k-1} + \begin{pmatrix} R_1(1-e^{\frac{\Delta t}{\tau_1}}) \\ R_2(1-e^{\frac{\Delta t}{\tau_2}}) \\ R_3(1-e^{\frac{\Delta t}{\tau_3}}) \end{pmatrix} i_{k-1} + w_{k-1} \\ Y_k = E_k - i_k R_0 - U_k^1 - U_k^2 - U_k^3 + v_k \end{cases} \quad (8)$$

where  $Y_k$  is the battery output voltage at time index  $k$ ,  $E_k$  is the open circuit voltage of battery at time index  $k$ ;  $\Delta t$  is the sampling time,  $i_k$  is the circuit current at time index  $k$ , namely the system control input,  $\tau_1 = R_1 C_1$ ,  $\tau_2 = R_2 C_2$ ,  $\tau_3 = R_3 C_3$ ,  $w_k$  and  $v_k$  are the system zero mean random process noise and measurement noise, respectively, mainly caused by the sensor error and model error, and the corresponding covariance are  $Q_k$  and  $R_k$ .

The eight parameters for the battery including the ohm internal resistance  $R_0$ , open circuit voltage  $E$ , capacitance  $C_1$ ,  $C_2$ ,  $C_3$  and resistance  $R_1$ ,  $R_2$ ,  $R_3$  regard as a parameter vector  $\theta = [R_0 \ C_1 \ R_1 \ C_2 \ R_2 \ C_3 \ R_3 \ E]^T$ , which can be considered be as slowly changing, and the state equation and measurement equations are represented as:

$$\begin{cases} \theta_k = \theta_{k-1} + r_{k-1} \\ Y_k = g(x_k, u_k, \theta_k) + e_k \\ = C_{\theta,k} \theta_k - U_k^1 - U_k^2 - U_k^3 + e_k \end{cases}, \quad (9)$$

where  $C_{\theta,k} = [-i_k \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1]$ , the disturbance error  $r_k$  and  $e_k$  are used to describe the slow time-varying characteristics of the parameters, and the covariance are  $Q_k^\theta$  and  $R_k^\theta$ , respectively.

UD-DKF algorithm is summarized as followed:

1) Parameter prediction

$$\hat{\theta}_{k|k-1} = \hat{\theta}_{k-1|k-1} \quad (10)$$

2) State prediction

$$\hat{X}_{k|k-1} = A_{k|k-1} \hat{X}_{k-1|k-1} + B_{k|k-1} i_{k-1} \quad (11)$$

$$\begin{cases} A_{11} = e^{\frac{-\Delta t}{\tau_{1,k}}} & A_{22} = e^{\frac{-\Delta t}{\tau_{2,k}}} \\ A_{33} = e^{\frac{-\Delta t}{\tau_{3,k}}} & B_{11} = \hat{R}_{1,k} (1 - A_{11}) \\ B_{22} = \hat{R}_{2,k} (1 - A_{22}) & B_{33} = \hat{R}_{3,k} (1 - A_{33}) \\ A_{k|k-1} = \begin{pmatrix} A_{11} & 0 & 0 \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \end{pmatrix} & B_{k|k-1} = \begin{pmatrix} B_{11} \\ B_{22} \\ B_{33} \end{pmatrix} \end{cases} \quad (12)$$

where  $\hat{\tau}_{1,k}$ ,  $\hat{\tau}_{2,k}$ ,  $\hat{\tau}_{3,k}$ ,  $\hat{R}_{1,k}$ ,  $\hat{R}_{2,k}$ ,  $\hat{R}_{3,k}$  are the data of parameter vector  $\hat{\theta}_{k|k-1}$  on the corresponding line.

3) Parameter prediction error covariance matrix and its UD decomposition

$$P_{k|k-1}^\theta = P_{k-1|k-1}^\theta + Q_{k-1}^\theta = U_{\theta,k|k-1} D_{\theta,k|k-1} U_{\theta,k|k-1}^T, \quad (13)$$

$$F_{\theta,k} = D_{\theta,k|k-1} U_{\theta,k|k-1}^T H_{\theta,k}^T, \quad (14)$$

$$G_{\theta,k} = U_{\theta,k|k-1} F_{\theta,k}, \quad (15)$$

$$S_{\theta,k} = C_{\theta,k} G_{\theta,k} + R_k^\theta, \quad (16)$$

$$\begin{aligned} H_{\theta,k} &= \left. \frac{dg(\hat{X}_{k|k-1}, u_k, \theta_k)}{d\theta_k} \right|_{\theta_k = \hat{\theta}_{k|k-1}} \\ &= \frac{\partial g(\hat{X}_{k|k-1}, u_k, \theta_k)}{\partial \theta_k} + \frac{\partial g(\hat{X}_{k|k-1}, u_k, \theta_k)}{\partial \hat{X}_{k|k-1}} \frac{d\hat{X}_{k|k-1}}{d\theta_k}, \quad (17) \\ &= [-i_k - \Delta C_1 - \Delta R_1 - \Delta C_2 - \Delta R_2 - \Delta C_3 - \Delta R_3] \end{aligned}$$

$$\Delta C_1 = \frac{e^{\frac{(-\Delta t)}{\hat{\tau}_{1,k}}} (\hat{U}_{k-1|k-1}^1 - i_{k-1} \hat{R}_{1,k}) \Delta t}{\hat{\tau}_{1,k} \hat{C}_{1,k}}, \quad (18)$$

$$\Delta C_2 = \frac{e^{\frac{(-\Delta t)}{\hat{\tau}_{2,k}}} (\hat{U}_{k-1|k-1}^2 - i_{k-1} \hat{R}_{2,k}) \Delta t}{\hat{\tau}_{2,k} \hat{C}_{2,k}}, \quad (19)$$

$$\Delta C_3 = \frac{e^{\frac{(-\Delta t)}{\hat{\tau}_{3,k}}} (\hat{U}_{k-1|k-1}^3 - i_{k-1} \hat{R}_{3,k}) \Delta t}{\hat{\tau}_{3,k} \hat{C}_{3,k}}, \quad (20)$$

$$\Delta R_1 = \frac{e^{\frac{(-\Delta t)}{\hat{\tau}_{1,k}}} \hat{U}_{k-1|k-1}^1 \Delta t}{\hat{\tau}_{1,k} \hat{R}_{1,k}} - \frac{i_{k-1} e^{\frac{(-\Delta t)}{\hat{\tau}_{1,k}}} \Delta t}{\hat{\tau}_{1,k}} - i_{k-1} (e^{\frac{(-\Delta t)}{\hat{\tau}_{1,k}}} - 1), \quad (21)$$

$$\Delta R_2 = \frac{e^{\frac{(-\Delta t)}{\hat{\tau}_{2,k}}} \hat{U}_{k-1|k-1}^2 \Delta t}{\hat{\tau}_{2,k} \hat{R}_{2,k}} - \frac{i_{k-1} e^{\frac{(-\Delta t)}{\hat{\tau}_{2,k}}} \Delta t}{\hat{\tau}_{2,k}} - i_{k-1} (e^{\frac{(-\Delta t)}{\hat{\tau}_{2,k}}} - 1), \quad (22)$$

$$\Delta R_3 = \frac{e^{\frac{(-\Delta t)}{\hat{\tau}_{3,k}}} \hat{U}_{k-1|k-1}^3 \Delta t}{\hat{\tau}_{3,k} \hat{R}_{3,k}} - \frac{i_{k-1} e^{\frac{(-\Delta t)}{\hat{\tau}_{3,k}}} \Delta t}{\hat{\tau}_{3,k}} - i_{k-1} (e^{\frac{(-\Delta t)}{\hat{\tau}_{3,k}}} - 1), \quad (23)$$

where  $\hat{U}_{k-1|k-1}^1$ ,  $\hat{U}_{k-1|k-1}^2$  and  $\hat{U}_{k-1|k-1}^3$  are the optimal estimation values of capacitance voltage at time index  $k-1$ , namely the data of the state vector  $\hat{X}_{k-1|k-1}$  on the corresponding line.

4) Gain matrix of parameters filter

$$K_k^\theta = P_{k|k-1}^\theta H_{\theta,k}^T (H_{\theta,k} P_{k|k-1}^\theta H_{\theta,k}^T + R_k^\theta)^{-1} = G_{\theta,k} S_{\theta,k}^{-1}, \quad (24)$$

5) Parameter estimates

$$C_{\theta,k} = [-i_k \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1], \quad (25)$$

$$\hat{Y}_k^\theta = C_{\theta,k} \hat{\theta}_{k|k-1} - \hat{U}_{k|k-1}^1 - \hat{U}_{k|k-1}^2 - \hat{U}_{k|k-1}^3, \quad (26)$$

$$\hat{\theta}_{k|k} = \hat{\theta}_{k|k-1} + K_k^\theta (Y_{m|k} - \hat{Y}_k^\theta), \quad (27)$$

where  $Y_{m|k}$  is the measured value of the voltage across the battery at time index k,  $\hat{U}_{k|k-1}^1$ ,  $\hat{U}_{k|k-1}^2$ ,  $\hat{U}_{k|k-1}^3$  are the estimation value of terminal voltage across the capacitance at time index k, namely the data of the state vector  $\hat{X}_{k|k-1}$  on the corresponding line.

6) Parameter estimation error covariance matrix and its UD decomposition

$$\begin{aligned} P_{k|k}^\theta &= (I - K_k^\theta H_{\theta,k}) P_{k|k-1}^\theta \\ &= U_{\theta,k|k-1} (D_{\theta,k|k-1} - F_{\theta,k} S_{\theta,k}^{-1} F_{\theta,k}^T) U_{\theta,k|k-1}^T, \\ &= U_{\theta,k|k-1} \bar{U}_{\theta,k|k-1} \bar{D}_{\theta,k|k-1} \bar{U}_{\theta,k|k-1}^T U_{\theta,k|k-1}^T \\ &= U_{\theta,k|k} D_{\theta,k|k} U_{\theta,k|k}^T \end{aligned} \quad (28)$$

where  $\bar{U}_{\theta,k|k-1}$  and  $\bar{D}_{\theta,k|k-1}$  are the UD decomposition matrix of  $D_{\theta,k|k-1} - F_{\theta,k} S_{\theta,k}^{-1} F_{\theta,k}^T$ , derived from (28), which is written as:

$$\begin{cases} U_{\theta,k|k} = U_{\theta,k|k-1} \bar{U}_{\theta,k|k-1}, \\ D_{\theta,k|k} = \bar{D}_{\theta,k|k-1} \end{cases}, \quad (29)$$

7) State prediction error covariance matrix and its UD decomposition

$$\begin{aligned} P_{k|k-1} &= A_{k|k-1} P_{k-1|k-1} A_{k|k-1}^T + Q_{k-1}, \\ &= U_{k|k-1} D_{k|k-1} U_{k|k-1}^T \end{aligned} \quad (30)$$

$$H_k = [-1 \ -1 \ -1], \quad (31)$$

$$F_k = D_{k|k-1} U_{k|k-1}^T H_k^T \quad (32)$$

$$G_k = U_{k|k-1} F_k \quad (33)$$

$$S_k = H_k G_k + R_k \quad (34)$$

8) Gain matrix of state filter

$$K_k = P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_k)^{-1} = G_k S_k^{-1} \quad (35)$$

9) State estimation

$$\hat{Y}_k = H_k \hat{X}_{k|k-1} - i_k \hat{R}_{0,k} + \hat{E}_{k|k-1} \quad (36)$$

$$\hat{X}_{k|k} = \hat{X}_{k|k-1} + K_k (Y_{m|k} - \hat{Y}_k), \quad (37)$$

where  $\hat{E}_{k|k-1}$  is the estimation value of the open circuit voltage at time index k, namely the data of the parameter vector  $\hat{\theta}_{k|k-1}$  in the row 1, column 8.

10) State estimation error covariance matrix and its UD decomposition

$$\begin{aligned} P_{k|k} &= (I - K_k H_k) P_{k|k-1} \\ &= U_{k|k-1} (D_{k|k-1} - F_k S_k^{-1} F_k^T) U_{k|k-1}^T, \\ &= U_{k|k-1} \bar{U}_{k|k-1} \bar{D}_{k|k-1} \bar{U}_{k|k-1}^T U_{k|k-1}^T \\ &= U_{k|k} D_{k|k} U_{k|k}^T \end{aligned} \quad (38)$$

where  $\bar{U}_{k|k-1}$  and  $\bar{D}_{k|k-1}$  are the UD decomposition matrix of  $D_{k|k-1} - F_k S_k^{-1} F_k^T$ , computed from (38), which is written as:

$$\begin{cases} U_{k|k} = U_{k|k-1} \bar{U}_{k|k-1} \\ D_{k|k} = \bar{D}_{k|k-1} \end{cases} \quad (39)$$

The dual Kalman filtering algorithm runs basis on the algorithm initial obtaining from the above model preliminary parameter identification. By alternately using the dual Kalman filtering algorithm, the system state and parameter can be online estimated at the same time.

### 3.3. Parameter on-line Identification Result Contrast

For verifying the validity of the UD- DKF method, power lithium-ion battery, 20 Ah / 24 V, pack made by a certain manufacturer is selected to conduct experiments under the FUDS and the DST conditions, and the on-line identification results by the different identification methods as given in Table 1 and Table 2:

Table 1. FUDS working condition.

Identification method	Maximum error of Battery model voltage	Average error of Battery model voltage
UD-DKF in this paper	0.5551 V	0.1203 V
DEKF in references [14, 26]	1.6635 V	0.2789 V

Table 2. DST working condition.

Identification method	Maximum error of Battery model voltage	Average error of Battery model voltage
UD - DKF in this paper	0.4612 V	0.1639 V
DEKF in references [14, 26]	1.6924 V	0.2727 V

By resulting from Table 1 and Table 2, the on-line parameters identification method for the UD-DKF in this paper is better than the method of references [14, 26]. The UD decomposition method with the special structure of the matrix U and D matrix are employed in the process of filtering, the non-negative of the error covariance matrix  $P_{k|k-1}$  can be ensured, which enhancing the stability of algorithm, and reducing the computational complexity. The designed UD-DKF-based parameters on-line identification method obtains the complete parameters of the battery model, and the comprehensive information is provided for the estimation of SOC and SOH.

#### 4. SOC Estimation Strategy Based on the AEKF

In many practical systems, the system model parameters and noise are dramatically change, thus the classical EKF algorithm for estimating SOC cannot achieve satisfied effect or even filtering divergently. In this paper, on the basis of the EKF, the adaptive filter method is introduced, i.e. adaptive extended Kalman filtering (AEKF) [27, 28], on the one hand, actual measured values is applied to revise the forecast values, at the same time, the unknown or uncertainly known system model parameters and noise statistical parameters are estimated to inhibit the effect for the noise.

The SOC is estimated by the AEKF basis on the model parameters acquired from the UD-DKF, referred to as (UD-DKF-AEKF), the SOC and terminal voltages across the capacitor  $C_1, C_2, C_3$  are selected as state variables, namely  $X_k = [SOC_k \ U_k^1 \ U_k^2 \ U_k^3]^T$ , the system state equation and measurement equations are expressed as follows:

$$\begin{cases} X_k = \Psi_{k|k-1} X_{k-1} + \Gamma_{k-1} i_{k-1} + w_{k-1} \\ Y_k = E(SOC_k) - i_k R_0 - U_k^1 - U_k^2 - U_k^3 + v_k \end{cases} \quad (40)$$

where  $E(SOC_k)$  represents the nonlinear relationship between the battery open circuit voltage  $E$  and SOC, the mathematical equation is described as follows:

$$E(SOC_k) = k_1 SOC_k^8 + k_2 SOC_k^7 + k_3 SOC_k^6 + k_4 SOC_k^5 + k_5 SOC_k^4 + k_6 SOC_k^3 + k_7 SOC_k^2 + k_8 SOC_k + k_9 \quad (41)$$

where the open circuit voltage  $E$  is function of SOC.

The undetermined coefficients  $k_1 \sim k_9$  can be obtained by the least squares method.

The AEKF algorithm for estimating SOC process is summarized as follows:

1) State prediction

$$\hat{X}_{k|k-1} = \Psi_{k|k-1} \hat{X}_{k-1|k-1} + \Gamma_{k-1} i_{k-1} \quad (42)$$

$$\Psi_{k|k-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{-\frac{\Delta t}{\tau_1}} & 0 & 0 \\ 0 & 0 & e^{-\frac{\Delta t}{\tau_2}} & 0 \\ 0 & 0 & 0 & e^{-\frac{\Delta t}{\tau_3}} \end{pmatrix} \quad (43)$$

$$\Gamma_{k-1} = \begin{bmatrix} -\frac{\eta \Delta t}{C_n} R_1 (1 - e^{-\frac{\Delta t}{\tau_1}}) & R_2 (1 - e^{-\frac{\Delta t}{\tau_2}}) & R_3 (1 - e^{-\frac{\Delta t}{\tau_3}}) \end{bmatrix}^T \quad (44)$$

2) Prediction error variance matrix

$$P_{k|k-1} = \Psi_{k|k-1} P_{k-1|k-1} \Psi_{k|k-1}^T + Q_{k-1} \quad (45)$$

3) Filtering gain matrix

$$K_k = P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_k)^{-1} \quad (46)$$

$$H_k = \left[ \frac{dE(SOC_k)}{dSOC_k} \right]_{SOC_k} = \hat{SOC}_{k|k-1} - 1 - 1 - 1, \quad (47)$$

where  $\hat{SOC}_{k|k-1}$  is the SOC forecast value at time index k that state vector  $\hat{X}_{k|k-1}$  column 1 line 1.

4) State estimation

$$\hat{X}_{k|k} = \hat{X}_{k|k-1} + K_k (Y_{m|k} - \hat{Y}_k), \quad (48)$$

$$\hat{E}_{k|k-1} = k_1 \hat{SOC}_{k|k-1}^8 + k_2 \hat{SOC}_{k|k-1}^7 + k_3 \hat{SOC}_{k|k-1}^6 + k_4 \hat{SOC}_{k|k-1}^5 + k_5 \hat{SOC}_{k|k-1}^4 + k_6 \hat{SOC}_{k|k-1}^3 + k_7 \hat{SOC}_{k|k-1}^2 + k_8 \hat{SOC}_{k|k-1} + k_9 \quad (49)$$

$$U_k^1 = U_{k-1}^1 e^{-\frac{\Delta t}{\tau_1}} + R_1 (1 - e^{-\frac{\Delta t}{\tau_1}}) i_{k-1} \quad (50)$$

$$U_k^2 = U_{k-1}^2 e^{-\frac{\Delta t}{\tau_2}} + R_2 (1 - e^{-\frac{\Delta t}{\tau_2}}) i_{k-1} \quad (51)$$

$$U_k^3 = U_{k-1}^3 e^{-\frac{\Delta t}{\tau_3}} + R_3 (1 - e^{-\frac{\Delta t}{\tau_3}}) i_{k-1} \quad (52)$$

$$\hat{Y}_k = \hat{E}_{k|k-1} - i_k R_0 - U_k^1 - U_k^2 - U_k^3, \quad (53)$$

where  $Y_{m|k}$  is the measurement value of battery terminal voltage at time index k.

5) Estimate error variance matrix

$$P_{k|k} = (I - K_k H_k) P_{k|k-1}, \quad (54)$$

6) Update the noise covariance

$$\hat{E}_{k|k} = k_1 \hat{SOC}_{k|k}^8 + k_2 \hat{SOC}_{k|k}^7 + k_3 \hat{SOC}_{k|k}^6 + k_4 \hat{SOC}_{k|k}^5 + k_5 \hat{SOC}_{k|k}^4 + k_6 \hat{SOC}_{k|k}^3 + k_7 \hat{SOC}_{k|k}^2 + k_8 \hat{SOC}_{k|k} + k_9 \quad (55)$$

$$\tilde{Y}_k = \hat{E}_{k|k} - i_k R_0 - U_k^1 - U_k^2 - U_k^3, \quad (56)$$

$$\mu_k = Y_{m|k} - \tilde{Y}_k, \quad (57)$$

$$F_k = \sum_{n=k-L+1}^k \frac{\mu_n \mu_n^T}{L}, \quad (58)$$

$$R_k = F_k + H_k P_{k|k} H_k^T, \quad (59)$$

$$Q_k = K_k F_k K_k^T, \quad (60)$$

where  $\hat{SOC}_{k|k}$  is the optimum estimation value of SOC at time index k, namely the value of state vector  $\hat{X}_{k|k}$  in the row 1, column 1,  $L$  is the adaptive window, and let  $L = 10$  in this paper.

Given the state estimation matrix  $\hat{X}_{k|k}$  and initial value of the estimation error matrix  $P_{k|k}$ , the optimal estimated value of SOC can be acquired through repeated recursive.

## 5. Simulation and Experiment

In order to verify the feasibility and accuracy of the SOC, power lithium-ion battery, 20 Ah / 24 V, pack made by a certain manufacturer is selected to conduct tests in two kinds of typical working conditions.

### 5.1. FUDS Working Condition

FUDS working condition lasted 1373 seconds [29], and the battery is fully charged (considered  $SOC=1$ ) and rested for a period of time before starting the experiment). After undergoing a total 27 cycles, the battery has been emptied. The current distribution of working condition is as shown in Fig. 4:

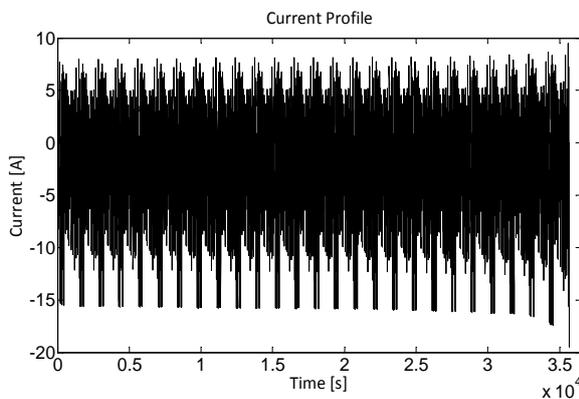


Fig. 4. Current distribution of FUDS.

The model simulation voltage and the measured voltage results are as shown in Fig. 5.

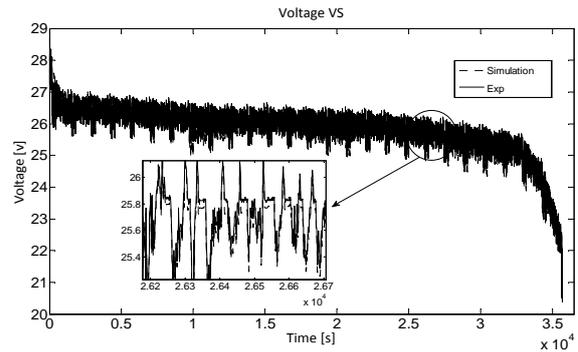


Fig. 5. Measuring voltage and simulation voltage of FUDS.

From Fig. 5, the average error between the model simulation voltage (dotted line) and the measure voltage (solid line) is about 0.1203 V, and the voltage error is small, which have verified the accuracy of the UD - DKF algorithm.

At the initial  $SOC = 0.8$ , namely initial SOC error is 20 % in the cases, the AEKF and EKF algorithm for SOC estimation results are as shown in Fig. 6.

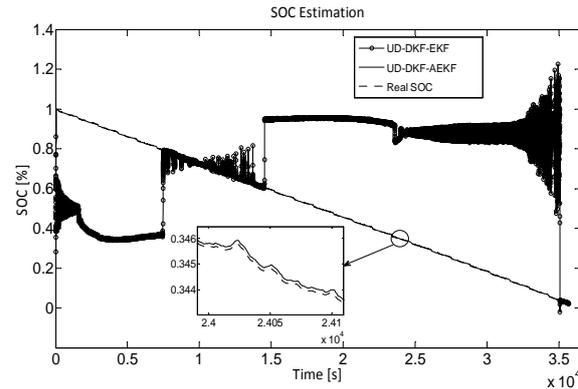


Fig. 6. SOC estimation results comparison.

From Fig. 6, it can be seen that AEKF algorithm (solid line) presents the fast convergence, where the SOC estimation maximum error is about 0.1 %, and the average error is about 0.0677 %. However, the SOC estimation maximum error and the average error of the EKF (circular line) algorithm are about 118.5 % and 44.2077 %, respectively.

### 5.2. DST Working Condition

By the DST Working condition lasted 360 seconds [29], the battery is fully charged (considered  $SOC = 1$ ) before the experiment. Let the battery rest after a period of time, and thus beginning the experiment. After undergoing a total 102 cycles, the battery has been emptied. The current distribution of working condition is shown in Fig. 7.

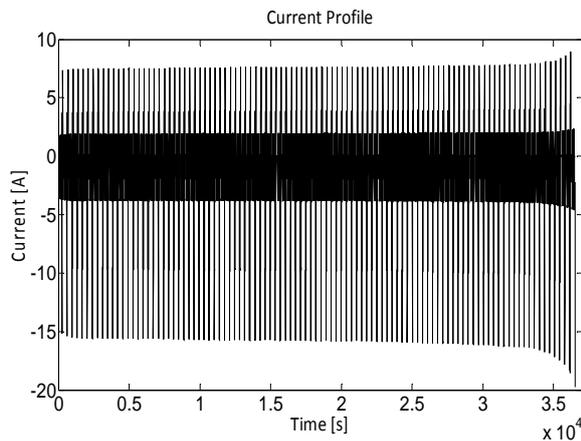


Fig. 7. Current distribution of DST.

The battery model simulation voltage and the measurement results are shown in Fig. 8.

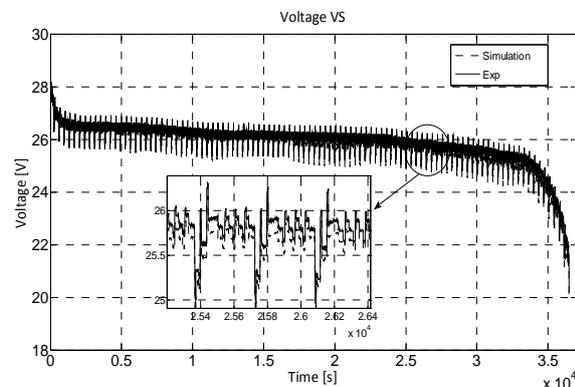


Fig. 8. Measuring voltage and simulation voltage of DST.

It can be seen from Fig. 8 that the average error between the model simulation voltage (dotted line) and the measure the voltage (solid line) is about 0.1639 V which voltage error is small, which have verified the accuracy of the UD - DKF algorithm.

At the initial  $SOC = 0.8$ , namely initial SOC error is 20% in the cases, AEKF and EKF algorithm for SOC estimation results are shown in Fig. 9.

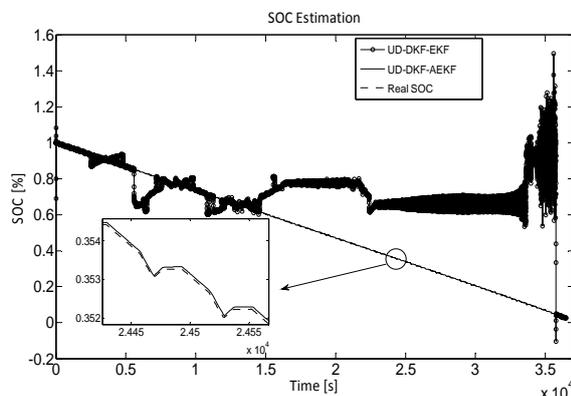


Fig. 9. SOC estimation results comparison.

From Fig. 9, it can be seen that the AEKF algorithm (solid line) presents the fast convergence, where the estimation maximum error is about 0.11 %, and the average error is about 0.0265 %. However, the SOC estimation maximum error and the average error of the EKF (circular line) algorithm are about 144.7 % and 23.8085 %, respectively.

## 6. Conclusion

In this paper, the dual Kalman filter (DKF) is applied to online identification for parameters in the model. In the process of filtering, the UD decomposition method is employed on time update and state update concurrently to enhance the stability of the algorithm and reduce the computational complexity. The experimental results verify that the model and on-line identification method for the parameters are effective under different working conditions.

Based on the designed UD-DKF, the classical EKF and the AEKF algorithm are used for the SOC estimation. The results indicate that the EKF algorithm cannot be accurate estimate SOC under the condition of the current dramatically change, however, the AEKF algorithm can accurately estimate the SOC and quickly converge to the true value, even under the different initial SOC, which has good robustness, convergence and stronger inhibitory effect on noise.

The factors impacting the battery performance, such as lifespan [19], temperature [30] and depth of discharge are not considered in this article, which will be researched in future for improving the estimate precision of the SOC.

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