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Research on Electronic Transformer Data Synchronization Based on Interpolation methods and Their Error Analysis

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Abstract: In this paper the origin problem of data synchronization is analyzed first, and then three common interpolation methods are introduced to solve the problem. Allowing for the most general situation, the paper divides the interpolation error into harmonic and transient interpolation error components, and the error expression of each method is derived and analyzed. Besides, the interpolation errors of linear, quadratic and cubic methods are computed at different sampling rates, harmonic orders and transient components. Further, the interpolation accuracy and calculation amount of each method are compared. The research results provide theoretical guidance for selecting the interpolation method in the data synchronization application of electronic transformer. *Copyright © 2015 IFSA Publishing, S. L.*

Keywords: Electronic transformer, Data synchronization, Interpolation method, Interpolation error.

1. Introduction

In relay protection and measurement control of power system, the synchronization of sampling data from electronic transformer is the key precondition and guarantee for protection device to measure and operate accurately. The output signal of traditional transformer is analog, which is directly sampled by the secondary electrical equipment and all channels are mainly synchronous [1]. With the continuous expanding promotion of digital substation technology, the primary and secondary equipment gradually develop toward small, intelligent and high steady. The electronic transformer, which has small volume, strong anti-saturation, excellent insulating property and vast dynamic range, meets the requirement of electrical engineering development and has been widely used in intelligent substation. When using electronic transformer, the primary electrical quantity is connected to the merging unit via the data sampling device, and then sent to the protecting and controlling devices in bay level. Without unified synchronizing mechanism to each sampling link of electronic transformer, the sampling data of each channel is nonsynchronous, thus resulting in the synchronization problem in the substation [2].

The IEC60044 standard has specified two methods for data synchronization: impulsive synchronization and interpolation methods [3-5]. The frontier method requires that each merging unit includes an external synchronous port for frequency doubling, which increases the cost and encounters the difficulties of real-time signal receiving and sending, thus the system is complicated and costs much. In engineering application, the interpolation method, which maintains synchronization by computing and reduces the cost effectively, is gradually widely adopted in power system to achieve data synchronization. The interpolation method abandons the requirement of synchronization of each sampling channel, and computes the sampling value of all channels at the same time with the prior knowledge of the time delay from data sampling to arriving at the merging unit. The mechanism of interpolation method is relative simple, and the cost of the system is very low by software computing to achieve electronic transformer data synchronization. Currently, there have been many researches on linear, quadratic and cubic interpolation methods and the analysis on the interpolating error of each method [6-14]. But these researches mainly focus on the interpolation error accuracy under certain application background, while the interpolation error varies greatly from each other and the calculation amounts are also different. In this paper, the most general composition form of the signal is considered, and the interpolation error expressions of linear, quadratic and cubic interpolation methods are deduced. Besides, the influencing factors of each method are analyzed and simulated, and the interpolation accuracies and amounts of all methods are compared.

2. Theoretical Analysis of Interpolation Methods

2.1. Introduction of the Problem

Without a centralized and accurate time standard to synchronize all the nodes in the substation, the sampling time of all channels are usually random. Therefore, the sampling data from all channels to the merging unit are generally asynchronous. As it is shown in Fig. 1, the $S_1(t)$, $S_2(t)$ are sampled signals from two independent channels. There is reset signal in the merging unit and when the number of sampling frames reaches to a certain number, the number will be reset to zero and begin to count in a new circle. Thus, combined with the internal crystal oscillator, the merging unit marks the arrival time of $S_1(t)$, $S_2(t)$. Assume that when the reset signal sets the count number to zero, t_{11}, t_{12}, t_{13}, t_{21}, t_{22}, t_{23} are the arrival time of signal $S_1(t)$, $S_2(t)$ from the sampling link to the merging unit, respectively. The interpolation method is to compute the sampling value of both channels at the time of interpolation time t_0, $2t_0$, $3t_0$ according to the sampling value at other times. At the sampling time t_0, the interpolation value of $S_1(t)$ is $y_1(t_0)$, the interpolation error can be calculated as follows:

$$\varepsilon = y_1(t_0) - S_1(t_0) \tag{1}$$

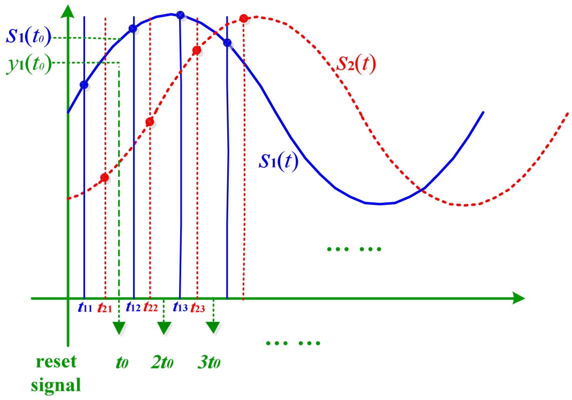


Fig. 1. Diagram of interpolation for data synchronization.

2.2. The Common Interpolation Methods and their Interpolation Errors

Currently, there are three interpolation methods: linear, quadratic and cubic interpolation methods. The next paragraph will introduce the interpolating principle and error of each method taking the example of sampled signal $S_1(t)$.

a) Linear interpolation method.

The linear interpolation method is the most simple interpolation method which calculates the interpolating value at time t_0 with the prior knowledge of sampling points $(t_{11}, S_1(t_{11}))$, $(t_{12}, S_1(t_{12}))$ [15]:

$$y_{1\_l}(t) = \frac{t - t_{12}}{t_{11} - t_{12}} S_1(t_{11}) + \frac{t - t_{11}}{t_{12} - t_{11}} S_1(t_{12}) \tag{2}$$

The principle of liner interpolation is to consider the interpolation time t_0 to be one point on the line composed by $(t_{11}, S_1(t_{11}))$ and $(t_{12}, S_1(t_{12}))$. According to the Lagrange Interpolation Error Formula, the interpolation error of linear interpolation is as follows [16]:

$$R_l(t) = \left|S_1(t) - y_{1\_l}(t)\right| = \left|\frac{1}{2} S_1''(\xi)(t - t_{11})(t - t_{12})\right|, \tag{3}$$

where ξ is the time in interval $[t_{11}, t_{12}]$, $S_1''(\xi)$ is the second derivative of signal $S_1(t)$ at the time of ξ. Consider the most general situation and assume that $S(t)$ is composed by direct current, steady and transient components, whose form can be expressed as:

$$S_1(t) = I_0 + \sum_{n=1}^{\infty} I_n \sin(n\omega t + \varphi_n) + I_T e^{-\frac{t}{\tau}}, \tag{4}$$

where I_0 is the direct component, ω is the angular frequency of fundamental wave and $\omega = 2\pi f = 100\pi$; I_n and φ_n are the amplitude and initial phase of the *n*th harmonic wave and $n=1$ corresponds to those of the fundamental wave; I_T is the initial value of the transient wave and τ is the time constant. It can be derived form Equation (4) that the second derivative form of $S_1(t)$ is:

$$S_1''(t) = -\sum_{n=1}^{\infty} n^2\omega^2 I_n \sin(n\omega t+\varphi_n) + \frac{I_T e^{-\frac{t}{\tau}}}{\tau^2} \quad (5)$$

By introducing Equation (5) into Equation (3), the interpolation error of linear interpolation method can be obtained as follows:

$$R_l(t) = \left| \frac{1}{2}\left[-\sum_{n=1}^{\infty} n^2\omega^2 I_n \sin(n\omega t+\varphi_n) + \frac{I_T e^{-\frac{t}{\tau}}}{\tau^2} \right](t-t_{11})(t-t_{12}) \right| \le \left(\frac{\omega^2 \sum_{n=1}^{\infty} n^2 I_n}{2} + \frac{I_T}{2\tau^2} \right) \left|(t-t_{11})(t-t_{12})\right| \quad (6)$$

Obviously, Equation (6) gets the maximum value when $t = \frac{t_{11}+t_{12}}{2}$. Assume that there are *N* sampling points in each fundamental wave period, then $t_{12} - t_{11} = \frac{1}{50N}$, and Equation (6) is converted to:

$$R_l(t) \le \frac{1}{10000N^2}\left(\frac{\omega^2 \sum_{n=1}^{\infty} n^2 I_n}{2} + \frac{I_T}{2\tau^2} \right) = \frac{4.935\sum_{n=1}^{\infty} n^2 I_n}{N^2} + \frac{5\times10^{-5} I_T}{N^2\tau^2} = R_{H\_l}(t) + R_{T\_l}(t) \quad (7)$$

where $R_{H\_l}(t) = \frac{4.935\sum_{n=1}^{\infty} n^2 I_n}{N^2}$, $R_{T\_l}(t) = \frac{5\times10^{-5} I_T}{N^2\tau^2}$ are the maximum interpolation errors of harmonic and transient components of linear interpolation method. It can be seen from Equation (7) that $R_{H\_l}(t)$ is proportional to $\sum_{n=1}^{\infty} n^2 I_n$ and inversely proportional to N^2, $R_{T\_l}(t)$ is proportional to I_T and inversely proportional to $N^2\tau^2$.

b) Quadratic interpolation method.

The principle of quadratic interpolation method can be expressed as: $(t_{11}, S_1(t_{11}))$, $(t_{12}, S_1(t_{12}))$, $(t_{13}, S_1(t_{13}))$ are the sampling points of the same time interval, the interpolation expression of quadratic interpolation method is:

$$y_{1\_q}(t) = \frac{t-t_{12}}{t_{11}-t_{12}}\frac{t-t_{13}}{t_{11}-t_{13}}S_1(t_{11}) + \frac{t-t_{11}}{t_{12}-t_{11}}\frac{t-t_{13}}{t_{12}-t_{13}}S_1(t_{12}) + \frac{t-t_{11}}{t_{13}-t_{11}}\frac{t-t_{12}}{t_{13}-t_{12}}S_1(t_{13}), \quad (8)$$

where $T = t_{13} - t_{12} = t_{12} - t_{11} = 1/(50N)$ is the time interval of adjacent sampling points. The quadratic interpolation method is to consider the interpolation time t_0 to be one point on the parabola composed by $(t_{11}, S_1(t_{11}))$, $(t_{12}, S_1(t_{12}))$ and $(t_{13}, S_1(t_{13}))$ [17], and it is also called parabolic interpolation, whose interpolation error can be expressed as:

$$R_q(t) = \left| \frac{1}{3!} S_1'''(\xi)(t-t_{11})(t-t_{12})(t-t_{13}) \right| \le \left| \left(\frac{\omega^3 \sum_{n=1}^{\infty} n^3 I_n}{6} + \frac{I_T}{6\tau^3} \right)(t-t_{11})(t-t_{12})(t-t_{13}) \right| \quad (9)$$

By introducing $t = t_{11} + \mu$, the items in Equation (9) can be obtained as follows:

$$\left|(t-t_{11})(t-t_{12})(t-t_{13})\right| = \left|\mu(\mu-T)(\mu-2T)\right| = f(\mu) \quad (10)$$

The derivation of Equation (10) indicates that it gets the maximum value when $t = t_{11} + \frac{3\pm\sqrt{3}}{3}T$, then the maximum value of Equation (9) is:

$$R_q(t) \le \frac{15.192\sum_{n=1}^{\infty} n^3 I_n}{N^3} + \frac{5.132\times10^{-7} I_T}{N^3\tau^3} = R_{H\_q}(t) + R_{T\_q}(t), \quad (11)$$

where $R_{H\_q}(t)=\frac{15.192\sum_{n=1}^{\infty}n^3 I_n}{N^3}$ is the maximum interpolation error of harmonic components of quadratic interpolation method, and it is proportional to $\sum_{n=1}^{\infty}n^3 I_n$ and inversely proportional to N^3; $R_{T\_q}(t)=\frac{5.132\times10^{-7} I_T}{N^3\tau^3}$ is the maximum interpolation error of transient components of quadratic interpolation method, which is proportional to I_T and inversely proportional to $N^3\tau^3$.

c) Cubic interpolation method.

The idea of cubic interpolation method is: when the interpolation condition is satisfied, the interpolation interval is divided into several sections. Apart from the two boundary nodes, all other inside nodes have continuous first and second order derivative. From the viewpoint of geometric meaning, the cubic interpolation method enables the concave convex non-deformation of the curve, which avoids the Runge Phenomenon effectively. The cubic interpolation is the most simple spline interpolation. Take the signal $S_1(t)$ for example, when there are *n* sampling points $S_1(t_{11})$, $S_1(t_{12})$,…, $S_1(t_{1(n+1)})$ in interval $[t_{11},t_{1(n+1)}]$, and the sampling time for each point is $t_{11}<t_{12}<\cdots<t_{1(n+1)}$, respectively. The sampling intervals are T_1, T_2,…, T_n, then the interpolation expression of $S_1(t)$ in the i^{th} interval is approximated as $y_{1\_c}(t)$:

$$y_{1\_c}(t)=\frac{\left(t_{1(i+1)}-t\right)^3}{6T_i}M_i+\frac{\left(t-t_{1i}\right)^3}{6T_i}M_{i+1}+\left(S_1(t_{1i})-\frac{M_i}{6}T_i^2\right)\frac{t_{1(i+1)}-t}{T_i}+\left(S_1(t_{1(i+1)})-\frac{M_{i+1}}{6}T_i^2\right)\frac{t-t_{1i}}{T_i}$$
$$t\in\left[t_{1i},t_{1(i+1)}\right],\ i=1,2,\cdots n \tag{12}$$

From the Equation above, it can be obtained that the solution of cubic interpolation requires the value of $n+1$ unknown variables of M_1, M_2,…, M_{n+1}, which are determined by the matrix as follows:

$$\begin{bmatrix}2 & 1 & & & \\ \mu_1 & 2 & \lambda_1 & & \\ & \ddots & \ddots & & \\ & & \mu_{n-1} & 2 & \lambda_{n-1} \\ & & & 1 & 2\end{bmatrix}\begin{bmatrix}M_1\\ M_2\\ \vdots\\ \vdots\\ M_{n+1}\end{bmatrix}=\begin{bmatrix}g_1\\ g_2\\ \vdots\\ \vdots\\ g_{n+1}\end{bmatrix} \tag{13}$$

The unknown variables in the matrix above are as follows:

$$\begin{cases} g_1=\frac{6}{T_1}\left(\frac{S_1(t_{12})-S_1(t_{11})}{T_1}-S_1^{'}(t_{11})\right), \\ \left.\begin{aligned} &\mu_i=\frac{T_i}{T_i+T_{i+1}} \\ &\lambda_i=1-\mu_i=\frac{T_{i+1}}{T_i+T_{i+1}} \\ &g_{i+1}=\frac{6}{T_i+T_{i+1}}\left(S_1\left[t_{1i},t_{1(i+1)}\right]-S_1\left[t_{1(i-1)},t_{1i}\right]\right)\end{aligned}\right\} i=1,2,\cdots n-1, \\ g_{n+1}=\frac{6}{T_n}\left(S_1^{'}(t_{1(n+1)})-\frac{S_1(t_{1(n+1)})-S_1(t_{1n})}{T_1}\right)\end{cases} \tag{14}$$

where $S_1\left[t_{1i},t_{1(i+1)}\right]$ represents the first divided difference of $S_1(t)$ at the point of t_{1i} and $t_{1(i+1)}$. The interpolation error of cubic interpolation method is:

$$R_c(t)=\left\|S_1(t)-y_{1\_c}(t)\right\|_\infty\le\frac{5}{384}\left\|S_1^{(4)}(t)\right\|_\infty T^4$$
$$=\frac{20.294\sum_{n=1}^{\infty}n^4 I_n}{N^4}+\frac{2.08\times10^{-9} I_T}{N^4\tau^4}=R_{H\_c}(t)+R_{T\_c}(t), \tag{15}$$

where $R_{H\_c}(t)=\frac{20.294\sum_{n=1}^{\infty}n^4 I_n}{N^4}$ is the maximum interpolation error of harmonic components of cubic interpolation which is proportional to $\sum_{n=1}^{\infty}n^4 I_n$ and inversely proportional to N^4; $R_{T\_c}(t)=\frac{2.08\times10^{-9} I_T}{N^4\tau^4}$ is the maximum interpolation error of transient components of cubic interpolation which is proportional to I_T and inversely proportional to $N^4\tau^4$.

3. Comparison of Interpolation Error and Calculation Amounts of all Interpolation Methods

Aforementioned the principles and errors of different interpolation methods have been introduced and calculated. In practical application, due to the difference of the component of signal $S_1(t)$ and the sampling rates, the interpolation errors vary greatly when adopting different interpolation methods. To explore the differences between interpolation errors and calculation amount of all interpolation methods, further analysis should be carried out to show which method exhibit the highest interpolation accuracy and cost least calculation amount.

3.1. Interpolation Error of Harmonic Components

Suppose that the interpolation error relationship of harmonic components of three interpolation methods are $R_{H\_l}(t) > R_{H\_q}(t) > R_{H\_c}(t)$, which indicates that the interpolation error of linear, quadratic and cubic interpolation method declines in orders:

$$\frac{4.935\sum_{n=1}^{\infty} n^2 I_n}{N^2} > \frac{15.192\sum_{n=1}^{\infty} n^3 I_n}{N^3} > \frac{20.294\sum_{n=1}^{\infty} n^4 I_n}{N^4} \tag{16}$$

From Equation (16), it can be concluded as follows:

1) For given harmonic order and amplitude, the increasing of N will make the Equation (16) easier to be satisfied, which indicates that the rising of sampling rates will reduce the interpolation error of higher order interpolation method and promote the interpolating accuracy.
2) For given sampling rate, the in the increasing of n and I_n will make the Equation (16) more difficult to be satisfied, which indicates that the rising of harmonic order and amplitude will enlarge the interpolation error of higher order interpolation method.

The solution of Equation (16) can be calculated as follows:

$$N > \max\left\{\frac{3.078\sum_{n=1}^{\infty} n^3 I_n}{\sum_{n=1}^{\infty} n^2 I_n}, \frac{1.336\sum_{n=1}^{\infty} n^4 I_n}{\sum_{n=1}^{\infty} n^3 I_n}\right\}, \tag{17}$$

where $\max\{a,b\}$ represents the larger number of a, b. Especially, when there is only fundamental wave component in $S_1(t)$ and $n=1$, Equation (17) is simplified as:

$$N > 3.078 \tag{18}$$

From Equation (17) it can be derived that when there is only fundamental wave component in the signal, the interpolation error of linear, quadratic and cubic interpolation methods decline in orders.

3.2. Interpolation Error of Transient Components

Assume that the interpolation error relationship of transient components of three interpolation methods is $R_{T\_l}(t) > R_{T\_q}(t) > R_{T\_c}(t)$, the interpolation error of linear, quadratic and cubic interpolation method declines in orders:

$$\frac{5\times 10^{-5} I_T}{N^2\tau^2} > \frac{5.132\times 10^{-7} I_T}{N^3\tau^3} > \frac{2.08\times 10^{-9} I_T}{N^4\tau^4} \tag{19}$$

Obviously, the interpolation error of each method is related with $N\tau$. The larger $N\tau$ is, the easier Equation (19) is satisfied. Therefore, the rising of sampling rate N or time constant τ will reduce the interpolation error of high order interpolation method. The solution of Equation (19) is calculated as follows:

$$N\tau > 1.026\times 10^{-2} \tag{20}$$

Equation (20) indicates that when the condition $N\tau > 1.026\times 10^{-2}$ is satisfied, the interpolation error of linear, quadratic and cubic interpolation error declines in orders.

3.3. Comparison of Calculation Amount

Due to the principle variations of all interpolation methods, the calculation amount of each method is different from each other. From Equation (2), (8) and (12) it can be concluded that for a interpolating point, the linear interpolation costs five times add operation, two times multiply operation; the quadratic interpolation costs fourteen times add operation, twelve times multiply operation; the cubic interpolation costs twenty-one times add operation, thirty times multiply operation, together with the solution of a interpolation polynomial matrix. Therefore, the calculation amount of cubic interpolation is larger than that of the quadratic interpolation, and the calculation amount of the latter is also larger than that of linear interpolation.

4. Calculating Simulation

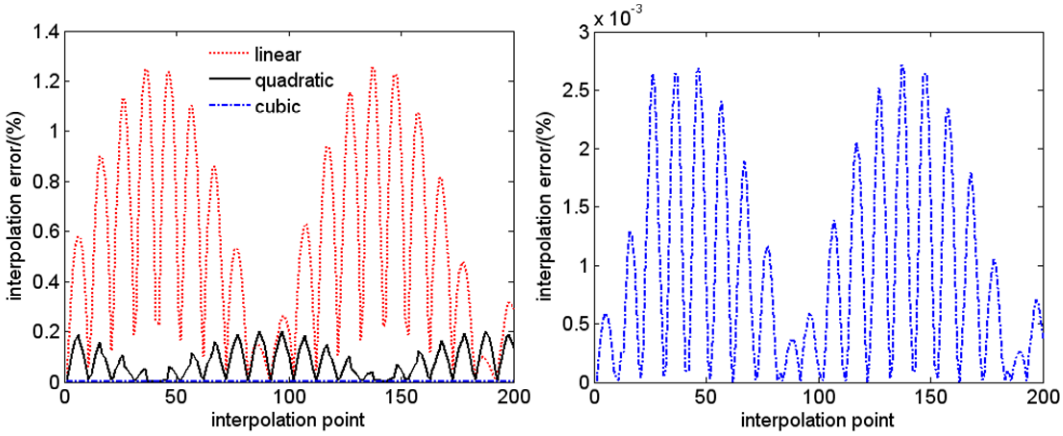
4.1. Calculating Model

The conclusion aforementioned has made it clear that the interpolation errors of all interpolation methods are no related with direct component. Assume that the expression of the signal is $S_1(t) = \sum_{n=1}^{\infty} I_n \sin(n\omega t + \varphi_n) + I_T e^{-\frac{t}{\tau}}$, and the initial phase of each harmonic wave is

$\varphi_n = 0,\ n = 1, 2, \cdots, \infty$, and there are N sampling points in the fundamental wave period. To fully exhibit the interpolation error of each method, two hundred interpolation points are calculated in one fundamental wave period to avoid the insufficience of calculating number. Utilizing MATLAB, the influencing factors of interpolation errors of linear, quadratic and cubic interpolation methods are calculated and the calculation amounts are also compared.

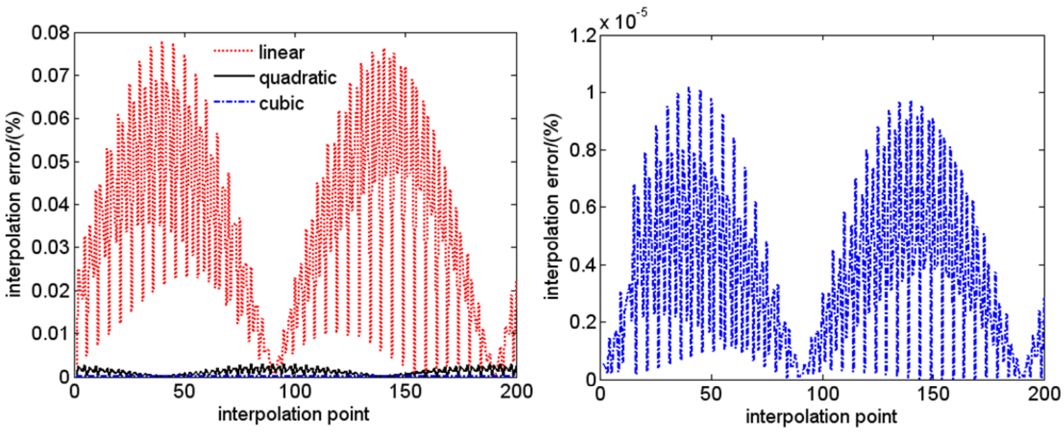
4.2. Influences of Sampling Rates

Ignore the transient component and assume that $I_T = 0$, when there is only fundamental wave in the signal, $I_1 = 1$, $I_n = 0(n \neq 1)$. The interpolation errors of linear, quadratic and cubic interpolation methods are calculated when $N = 20$ and $N = 80$, and the results are shown in Fig. 2 and Fig. 3.



(a) Comparison of interpolation errors of three methods **(b) interpolation error of cubic method**

Fig. 2. Interpolation accuracy of each method when $N = 20$.



(a) Comparison of interpolation errors of three methods **(b) interpolation error of cubic method**

Fig. 3. Interpolation accuracy of each method when $N = 80$.

When there are 20 sampling points in one fundamental wave period, the maximum interpolation error of linear interpolation method reaches to 1.2 %, while those of quadratic and cubic interpolation methods are 0.2 % and 0.0027 %, which indicates that the interpolation accuracy is higher for high order interpolation method. When there are 80 sampling points in one fundamental wave period, the maximum interpolation error of linear interpolation method has declined to 0.078 %, while those of quadratic and cubic interpolation methods are 0.003 % and 0.00001 %. The proportion decrease of three methods are 15.38, 66.67 and 270 respectively for two different sampling rates, which coincides with the proportion decrease results of 16, 64, and 256 shown in Equation (16).

To further analyze the influences of sampling rates on interpolation accuracy, the interpolation error of three methods are calculated when the sampling rates grows from 500 Hz ~ 10000 Hz, and the results are shown in Fig. 4. From the figure it can be seen that the interpolation accuracy is improved with the increasing of sampling rates; when the sampling rate is given, the interpolation error of linear, quadratic and cubic interpolation error declines in orders, which coincides with the conclusion in Paragraph 3.1 that when there are more than 4 sampling points in the fundamental wave period, the interpolation accuracy of three methods gets promoted in orders. With the increasing of sampling rates, the interpolation error of each interpolation method declines rapidly. Since there is only harmonic component error in the error composition, the interpolation errors of linear, quadratic and cubic interpolation method are inversely proportional to the square, cube and biquadratic of the sampling rates, and the declining slope the of interpolation error curve for three methods becomes steeper in orders.

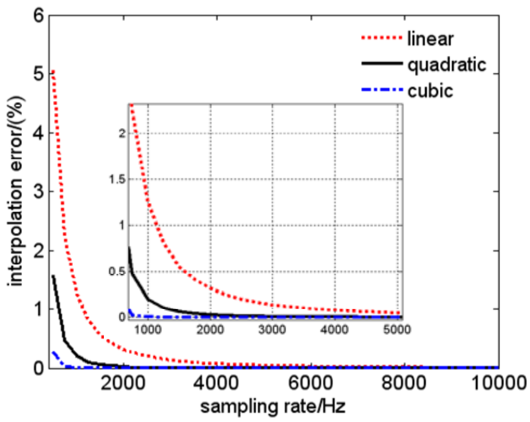


Fig. 4. Interpolation accuracy of three methods at different sampling rates when $N = 20$.

4.3. Influences of Harmonic Orders

The conclusions in Paragraph 3.1 indicate that the harmonic order has an effect on the interpolation error. To explore the influences of harmonic order on the interpolation accuracy, the interpolation errors of linear, quadratic and cubic interpolation method are calculated when the harmonic order is three ($I_3 = 1$, $I_n = 0 (n \neq 3)$), $N = 20$, and the results are shown in Fig. (5).

When there is only the third harmonic order wave, the maximum interpolation error of linear interpolation is 11 %, while those of quadratic and cubic interpolation are 5.2 % and 0.8 %, respectively. Compared with Fig. 2, when the harmonic order grows higher, the interpolation error of each method becomes larger and the interpolation accuracy has declined. Fig. 6 illustrates the interpolation errors of three methods when the harmonic order changes from 1 to 10. When the order is small, the interpolation errors of three methods are also small, and the error of linear, quadratic and cubic interpolation method declines in orders; with the increasing of harmonic order, the interpolation errors of three methods grows rapidly and the Equation (16) is no longer satisfied. The interpolation errors of quadratic, cubic interpolation method are higher than that of linear interpolation method, and there is severe error in interpolating results.

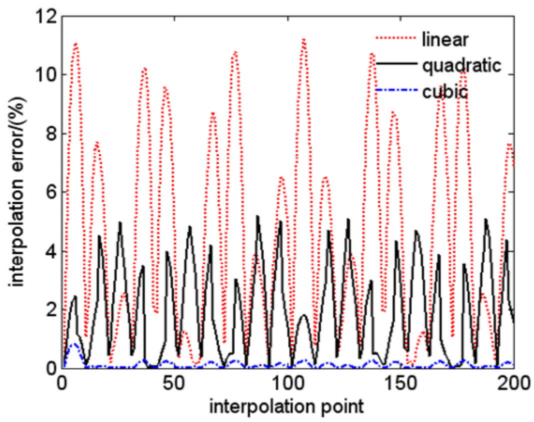


Fig. 5. Interpolation accuracy of each method for the third harmonic wave when $N = 20$.

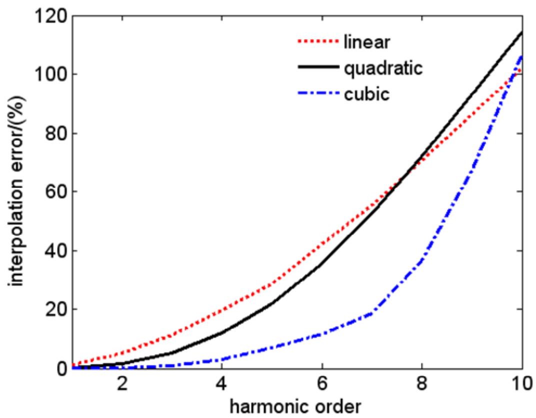
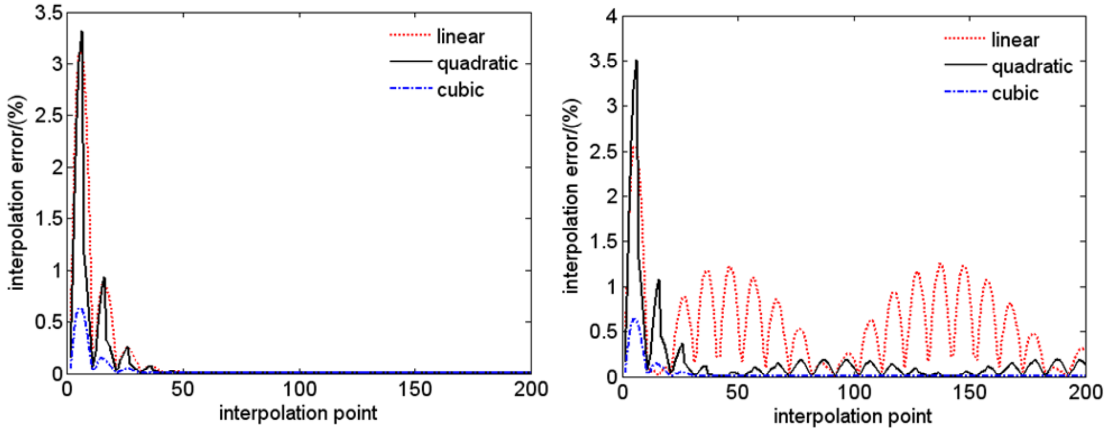


Fig. 6. Interpolation accuracy of each method for the different harmonic order waves.

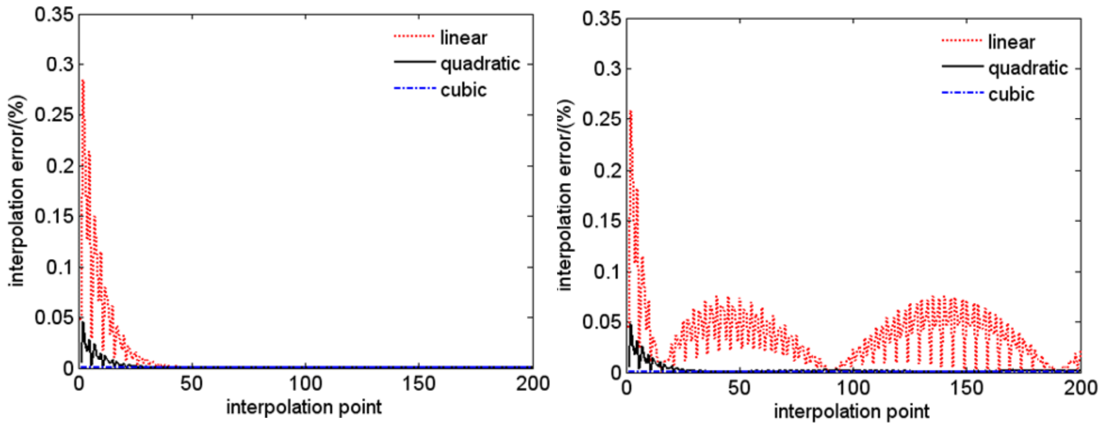
4.4. Influences of Transient Components

In order to explore the influences of transient components on interpolation accuracy, the interpolation accuracy is calculated when the expression of the signal is $S_1(t) = \sin 100\pi t + e^{-\frac{t}{0.0008}}$, $N = 20$ and $N = 80$, respectively. The interpolation error is divided into transient component error and total error(including harmonic component error), and the results are shown in Fig. 7 and Fig. 8.



(a) interpolation error of transient component **(b) total interpolation error(including harmonic component)**

Fig. 7. Interpolation accuracy of each method in two conditions when $N = 20$.



(a) interpolation error of transient component **(b) total interpolation error(including harmonic component)**

Fig. 8. Interpolation accuracy of each method in two conditions when $N = 20$.

When the time constant is 800 us and $N = 20$, the multiply of the both $N\tau$ satisfies the Equation (20) and the maximum interpolation error of quadratic interpolation method is 2.2 %, which is slightly smaller than the interpolation error of linear method. The cubic method has the least interpolation error and the maximum error is only 0.3 %. When considering the interpolation error of fundamental wave component, the transient component of the first twenty sampling points plays an important role in the interpolation error, and the error presents geometrical attenuation with the increasing of time and gradually the error of fundamental wave component is the major part. When $N = 80$, the interpolation errors of all methods become smaller than those of $N = 20$ when considering transient component error only. The maximum interpolation errors of linear, quadratic and cubic method are 0.28 %, 0.04 % and 0.001 %, respectively. When considering the total interpolation error, the first twenty sampling points are still the major part of the transient component error, and with the increasing of time the fundamental component wave plays an important role in the interpolation error. View the trend as a whole, the interpolation errors are prominently smaller than those when $N = 20$.

4.5. Comparison of the Calculation Amount

Although the interpolation errors of linear, quadratic and cubic interpolation method decline in orders when the sampling rate is large, the calculation is an important factor to evaluate whether the method is appropriate for the consideration of hardware realization. The operating time of MATLAB reflects the calculation amount. The operating time of the calculation in Paragraph 3.2 when $N = 20$ is counted 100 times and the average time of calculation is shown in Table 1.

Table 1. Computational complexity comparison between three methods.

Method	Linear	Quadratic	Cubic
Interpolation points	200	200	200
Operating time(ms)	5.748	10.34	78.647

From Table 1 it can be seen that the average operating time for linear method is 5.478 ms, for quadratic method is 10.34 ms, which is two times of the linear method; for cubic method, the operating time is 78.647 ms, which is fourteen times of the linear method. Therefore, the promotion of interpolating accuracy is gained by increasing the calculation amount.

5. Conclusion

In this paper, the original problem of the electronic transformer data synchronization is introduced first and the common interpolation methods are given. Based on the composition of the signal, the error expressions of linear, quadratic and cubic methods are deduced and the influences of sampling rate, harmonic order and transient component on the interpolation error are calculated and compared. The conclusions are as follows:

1. By increasing the sampling rate, reducing the harmonic order and amplitude, the harmonic component errors of the three methods can reduced;

2. By reducing the initial amplitude of transient wave and increasing the sampling rate and time constant, the transient component errors of the three methods can reduced;

3. When there are more than four sampling points in the fundamental wave period, the interpolation errors of harmonic component of linear, quadratic and cubic method decline in orders; when $N\tau > 1.026\times10^{-2}$, the interpolation errors of transient component of three methods decline in orders;

4. Although the increasing of sampling rate promotes the interpolation accuracy, the calculation amount of linear, quadratic and cubic method increases in orders.

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