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Non-rigid Point Matching using Topology Preserving Constraints for Medical Computer Vision

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Abstract: This work presents a novel algorithm of finding correspondence using a relaxation labeling. For the variance experiments, the variance of all algorithms except the proposed algorithm is large. The largest variance of the proposed algorithm is +0.01 in the 0.08 deformation test of a character. Overall, the proposed algorithm outperforms compared to the rest of algorithms. Except the proposed algorithm, matching with neighborhood algorithm shows the best performance except an outlier to data ratio in a character test. The proposed algorithm shows the best performance as well as an outlier to data ratio in a character test.

Keywords: Non-rigid point matching, Image registration, Correspondence problem.

1. Introduction

The point matching is widely used in computer vision and pattern recognition since point representations are generally easy to extract. Point matching can be categorized as rigid and non-rigid based on the deformation of captured images [1-2]. Rigid point matching is relatively easy with a small number of transformation parameters. Non-rigid point matching is more complicated than rigid matching. There are two unknown parameters: correspondence and transformation. Most approaches to non-rigid point matching use an iterated estimation framework [3]. Iterated Closest Point (ICP) algorithm is the most well-known heuristic approach algorithm. It utilizes the relationship to assign the correspondence with binary values 0 and 1. However this assumption is no longer valid in the case of non-rigid transformations especially when the deformation is large. Thin Plate Spline Robust Point Matching (TPS-RPM) algorithm is EM-like algorithm to jointly solve for the feature correspondences as well as the geometric transformations [4]. The cost function that is being minimized is the sum of Euclidean distances between points [5-6]. The soft-assign and the deterministic annealing technique are used to search for an optimal solution. The algorithm is robust compare to the ICP in the non-rigid point matching problem, but the joint estimation of correspondences and transformation leads to a high complexity and calculation time. The searching time is approximately 3 minutes for 100 points. In addition, the distances make sense only when there is at least rough alignment of shape. Under large deformation or rotation, the algorithm is easily diverged. Recently, Shape Context algorithm is proposed. It is an object recognizer based on shape. For each point, the distribution of the length and the orientation for all lines is estimated through histogram [7]. This distribution is used as the shape context for the points. The correspondences can be decided by comparing each point's attributes in one set with the attributes in the other. Since attributes are compared, the search for correspondence can be conducted much more easily compared to ICP and TPS-RPM. The searching time is under 0.01 s for 100 points. However, original shape context is not rotation-invariant and the algorithm's ability to handle complex patterns and large deformation is not higher than TPS-RPM [8]. In addition, one point can be matched to two points which is far apart each other. Robust Point Matching by Preserving Local Neighborhood Structures algorithm introduces the notion of a neighborhood structure for the general point matching problem. Although the absolute distance between two points may change significantly under non-rigid deformation, the neighborhood structure of a point is generally well preserved due to physical constraints [9]. The author formulates an optimization problem to preserve local neighborhood structures. The perforation of an algorithm is higher than ICP, TPS-RPM, and shape context under various deformation and noise level. But under high occlusion and outlier rate, error is even higher than TPS-RPM. The searching time is under 0.01 s for 100 points.

This work presents a novel algorithm of finding correspondence using a relaxation labeling. This is done by means of a weighed graph matching algorithm. We observed that neighbors defined in the previous work keeps the structure more firmly if the distance between a point and a neighborhood becomes closer. This strength of relaxation becomes loose if a point and a neighborhood are far apart. We call these neighbors as adjacent points. To find the amount of strength between a point and an adjacent point, we propose a distance descriptor. Consider the set of vectors originating from a point to all adjacent points. For a point, we compute distance using bins that are uniform in log space making the descriptor more sensitive to positions of nearby adjacent points than to those of points far apart. The relaxation labeling combined with a strength coefficient is proposed to search the optimized correspondence. The searching time is under 0.01 s for 100 points. A new class of matching algorithm is proved to be stable and robust under all deformation, noise, and occlusion.

2. Problem Formulation

Let $S=\{s_1, s_2, ..., s_M, nil\}$ be a set of control points in a model image and $T=\{t_1, t_2, ..., t_K, nil\}$ be a set of control points in a target image. A model image S is composed of M points and a target image T is composed of K points. The dummy point nil is introduced for the outliers. Due to occlusion, number of M is not necessarily equal to K. For a given point $m \in S$, one can select an adjacent point set A^m, $A^m=\{a_1^m, a_2^m, ..., a_I^m\}$. Similarly, for a point $k \in T$, an adjacent point set is A^k, $A^k=\{a_1^k, a_2^k, ..., a_J^k\}$. The adjacent relationship is symmetric and it means if $i \in A^m$, then $m \in A^i$. A match between S and T is $f: S \Leftrightarrow T$ and common points match one to one and several points can be matched with the dummy point nil. Given two points $m \in S$ and $k \in T$ from two images, the optimal match f is

$$f = \arg\max C(S, T, f), \tag{1}$$

where

$$C(S,T,f)=\sum_{m=1}^{M}\sum_{i=1}^{I}\delta(f(m), f(i))+\sum_{k=1}^{K}\sum_{j=1}^{J}\delta(f^{-1}(n), f^{-1}(j)) \tag{2}$$

$$\delta(i,j)=\{\begin{matrix} 1 & j\in A^i \\ 0 & j\notin A^i \end{matrix} \tag{3}$$

The optimal solution maximizes the number of matched edges of two graphs. If m in a model image is matched to point k in a target image, then the matching probability $P_{mk}=1$, otherwise, $P_{mk}=0$. Matrix P satisfies the following normalization conditions.

$$\sum_{k=1}^{K+1} P_{mk}=1 \text{ for } m=1,2,...,M, \tag{4}$$

$$\sum_{m=1}^{M+1} P_{mk}=1 \text{ for } k=1,2,...,K \tag{5}$$

with $P_{mk}\in[0,1]$.

Using matrix P, the object function (2) can be written as below.

$$C(S,T,P)=2\sum_{m=1}^{M}\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{j=1}^{J}P_{mk}P_{ij} \tag{6}$$

3. Point Correspondence Using Relaxation Labeling

We use relaxation labeling process to solve the optimization problem. In this section, we show how the relaxation labeling theory is applied to the point matching problem. It is widely known that the relaxation labeling process is greatly affected by the choice of the compatibility coefficient. Thus, compatibility coefficient for preserving physical constrains is also investigated.

3.1. Initialization

Generally, a relaxation labeling process finds a local maximum and the convergence can not always be guaranteed. Therefore, good initialization is very important for best matching result. We, therefore use shape context distance to initialize the matching probability matrix P . The shape context is a histogram which describes the relative position of the remaining points. Consider a point s in a model image. A point is selected as centers of K log-polar bin and counting points in each bin, providing a description of the entire shape relative to the center. The shape context of a point m is $h_m(d)$, $d=1,...D$. Let $C_{mk}=(m,k)$ denote the cost of matching two points m and k. As shape contexts are distributions represented as histograms, it is natural to use the χ^2 test statistic as follows,

$$C_{mk} \equiv C(m,k) = \frac{1}{2}\sum_{d=1}^{D}\frac{[h_m(d)-h_k(d)]^2}{h_m(d)+h_k(d)} \quad (7)$$

Taking cost C_{mk} as the energy of the state that points m and k are matched, Gibbs distribution is used to relate the energy of a state to its probability as follows,

$$P_{mk} \propto e^{-C_{mk}/T_{init}} \quad (8)$$

3.2. Relaxation Labeling

After the initial probability assignment, the relaxation labeling process updates the probability of each label iteratively. In a traditional relaxation labeling process, object refers to an entity whose characteristics are under investigation in the process and a label is the description of a characteristic that belongs to an object. In our problem, an object is a control point set S and T and a label is a possible matching probability P . Initially in the algorithm, each object is assigned with a set of labels based on the shape context distance. The purpose of the subsequent process is to assign one label to each object that maximize $C(S,T,P)$ under the relaxed condition as $P_{mk}\in[0,1]$. It is supposed that labels do not occur independently on each other. The ambiguity reduction process is accomplished iteratively by requiring the relationship adjacent objects' labels to the contextual constraints. To update P , the compatibility coefficient is necessary. In this original work, the compatibility coefficient is defined as the {0,1} binary value. As a key contribution, we propose a novel compatibility coefficient to relax this binary value into [0,1] continuous value. The compatibility coefficient quantifies the degree of agreement between the hypothesis that m matches k and i among A^m matches j among A^k . We measure the consistency between (m,i) and (k,j) by means of the compatibility coefficient. High values correspond to compatibility and low values correspond to incompatibility. From the knowledge of the deformation dynamics, we know the points and its adjacent points' distance before non-rigid deformation cannot be very different the distance after non-rigid deformation. This occurs stronger if the original distance between points is closer. Conversely, this strength of relationship becomes less if the points are originally far apart. It means the shorter the edge of m and i , the higher the probability to match with the edge of k and j . This is because the points tend to preserve the rough structure of the shape.

To make the strength between points, we use five bins that are uniform in log space making more sensitive to distances of nearby points than to those of points farther away. This structure based distance is defined as 0 in the origin and incremented by 1 toward outer bins. The set of vectors from originating from a point m to its A^m adjacent points are defined as the a distance set $D(m)=\{d_1(m),d_2(m),...,d_L(m)\}$. This choice corresponds to a linearly increasing positional uncertainty with distance from the origin. This idea is illustrated in Fig. 1.

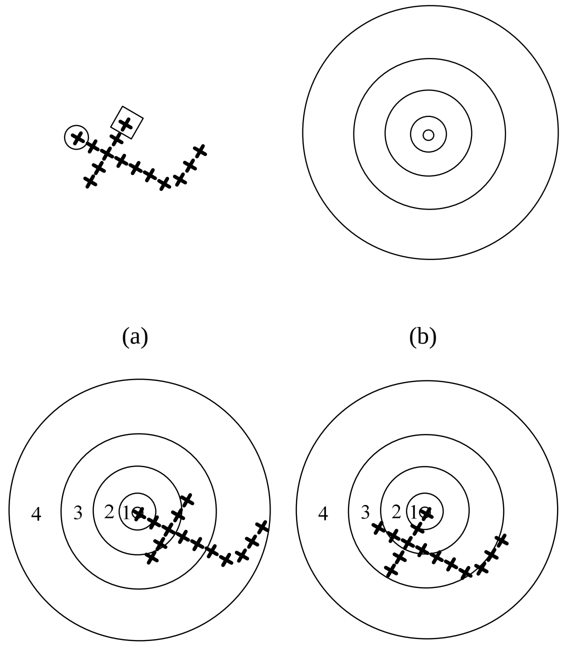


Fig. 1. Structure based distance computation. (a) Sample control points from the shape "t"; (b) Diagram of log bins used in computing the distance. We use five bins, meaning the distance 1 to 4 from the origin. (c) and (d): Example structure based distance marked by O and ϒ.

Referring to Fig. 2, the compatibility coefficient relating the adjacent distance is

$$\alpha(m,i;k,j) = (1 - \frac{d_i(m)}{\max(d_i(m))}) \quad (9)$$

Clearly, $\alpha(m,i;k,j)$ range from 0 to 1. The support for label p_{mk} in the n th iteration is given by

$$\begin{aligned} q_{mk}^n &= \sum_{i=1}^{I}\sum_{j=1}^{J} \alpha(m,i;k,j) p_{ij}^n \\ &= \sum_{i=1}^{I}\sum_{j=1}^{J} (1 - \frac{d_i(m)}{\max(d_i(m))}) p_{ij}^n \end{aligned} \quad (10)$$

Note that we use p_{ij}^n to weight $\alpha(m,i;k,j)$ because the support also depends on the likehood of neighboring matching probability. Finally, p_{mk}^n is updated according to

$$p_{mk}^{n+1} = \frac{p_{mk}^n q_{mk}^n}{\sum_{j=1}^{J} p_{mj}^n q_{mj}^n} \quad (11)$$

The iterative process can be summarized as follows. If a label, matching probability between points m and k, has relatively more support from adjacent points, its chance of being matched is increasing, i.e, its matching probability is increased. The probability is decreasing if the label is relatively less support. After predefined iteration, the estimated of probability is assigned to a point unless 0 and the probability less than 0.95 are labeled as an outlier. Finally, given a set of correspondences between points on model image and target image, the transformation is estimated using thin plate spline (TPS).

3.3. Adjacent Point Definition

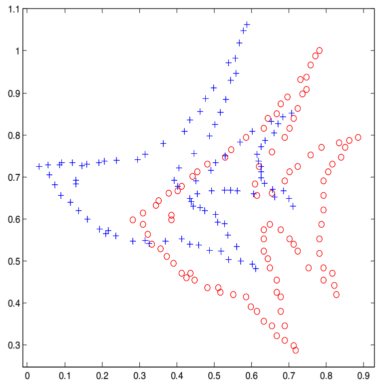
Defining adjacent point properly is very important to maximize the matching performance and reduce the time complexity. In this paper, we define an adjacent point searching boundary. Consider totally M points of a graph are fully connected. The length of an edge is d_e and the number of edges is $M(M-1)/2$. The searching boundary of adjacent point is the circle with a radius as an average length of total edges:

$$\frac{\sum_{e=1}^{M(M-1)/2} d_e}{M(M-1)/2} \quad (12)$$

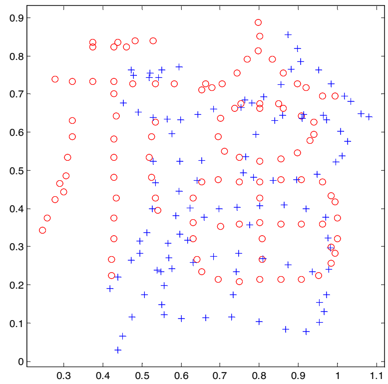
If a point i is reside within the circle with the center of a point m, it is considered as an adjacent point of m. From this definition, we can assign more edges to the crowed point's area and fewer edges to the sparse point's area.

4. Experiments Result

We have tested out algorithm on the same synthesized data as in [10]. There are three sets of data designed to measure the robustness of an algorithm under deformation, noise, and outliers. Two shapes, a fish and a character are generated, and 100 samples are generated for each degradation level. We then run our algorithm to find the correspondence between these two sets of points. The performance is compared with RPM, shape context, and the matching with neighborhood algorithm. Fig. 2 shows one example of template point sets and deformed target sets of a fish and a character.



(a)



(b)

Fig. 2. The model point sets (O) and the target point sets (+) of a fish and a character.

For the variance experiments, the variance of all algorithms except the proposed algorithm is large. The largest variance of the proposed algorithm is +0.01 in the 0.08 deformation test of a character. Overall, the proposed algorithm outperforms compared to the rest of algorithms. Except the proposed algorithm, matching with neighborhood algorithm shows the best performance except an outlier to data ratio in a character test. The proposed algorithm shows the best performance as well as an outlier to data ratio in a character test.

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