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Curvature as Obstacle to a Photo-Resistor Sensor of Illumining and Their Minimal Sensing Region

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Abstract: Through consider to the curvature as the deviation of any field interaction, even as obstruction to their flow, is designed and created an photo-resistor sensor to the control and optimization of illumination in a space M, with different gradients of illumination and luminous sources position. Then the sensing problem is a problem of free boundary conditions where is satisfied an energy functional of Dirichlet type with Sobolev norm $\|u\|_{1,2}$, to curvature functions κ, that satisfy in the luminous intensity change limit the condition $u|_{\partial\Omega} \leq \kappa$. This carries to that the illuminating sensor must be designed on a length gauge to measure the changes of illumination in the space. After this is generalized considering the stereographic projection of the light beam. Several applications derived of this research are mentioned.

Keywords: Basic guide of illuminating field, Curvature energy, Curvature Dirichlet energy, Energy spectrum of curvature, Illuminating field, Illuminating gradients, Illuminating sensor, Illuminating sensor stereographic component, Photo-Resistor sensor, Sensing obstacle problem.

1. Introduction

Let $M \subset \mathbb{R}^3$, a space that we want to illuminate through an artificial light source. As an artificial light source we call to the light source created by electronic technology and used to illuminate spaces with certain darkness or insufficient light.

Likewise, the illuminating problem is the control and optimizing of the light to proposes of improve the illuminating conditions and establish improvements in the light technology and their adequate positioning in the space such that is reduced the darkness under the minimal energy used to this goal.

From a topological point of view, the illumination is the action of illuminating field whose illuminating flow that impact in a surface determines differentiated regions of clear and dark illuminating fields which establish a measure of the illuminating gradient as phenomena of a space affected by a light source.

However, the boundary between both regions is not a well-delimitated line and their energy require a weak topology [1] defined by a norm or length $\|u\|_{s,p}$.

This norm or dipstick of measure illuminating energy will be used to design several sensors and their components to the sensing the illumination of a space and solve the wide form the illuminating problem defined [2].

2. Energy Functional, Sobolev Norm to Infrared Sensor, Illuminating Projection Components and Illuminating Gradients

Let $M \subset \mathbb{R}^3$, be a space that we want to illuminate through a light source of illumination intensity of illumination field $\aleph$ [1]. Let the domain $\mathcal{D}_{\mathfrak{T}}$, the sub-space of $L^2(\Omega)$, which is defined as[2]:

$$\mathcal{D}_{\mathfrak{T}} = \{u \in C^{\infty}(\Omega) \big| \Omega, \text{ bounded and } \ni \text{grad}u = \tilde{\aleph} \in L^2(\Omega) \times L^2(\Omega) \times L^2(\Omega)\},$$
$$\mathcal{D}_{\mathfrak{T}} = \{u \in C^{\infty}(\Omega) \big| \Omega, \text{ bounded and } \ni \text{grad}u = \tilde{\aleph} \in L^2(\Omega) \times L^2(\Omega) \times L^2(\Omega)\},$$
$$\mathcal{D}_{\mathfrak{T}} = \{u \in C^{\infty}(\Omega) \big| \Omega, \text{ bounded and } \ni \text{grad}u = \tilde{\aleph} \in L^2(\Omega) \times L^2(\Omega) \times L^2(\Omega)\}, \quad (1)$$

where $\mathfrak{T}$ is the microscopic region of the space with illuminating dark field [3] (in microscopy, this field is the result of to exclude the un-scattered beam from image), that is to say:

$$\mathcal{D}^*_{\mathfrak{T}} = \{T \in \mathcal{L}^2(H,H) = H^* \Big| Tu = \int_{\Omega} k(x,y)u(y)dy\}, ^3 \quad (2)$$

where the tube or the microscopic component to the illuminating dark field[4] (or basic guide of the illuminating field) is the space of points (see the Fig. 1)):

$$\mathfrak{T} = \{\tilde{\aleph} \in \mathcal{X}^{\infty}(\Omega) \big| \text{grad}\, u = \tilde{\aleph}\}, \quad (3)$$

The microscopic component to the dark field detects the dark and clear fields of an element "tube" whose diameter defines the quantity of light that affects on a photo-sensible device [4-5] located in the interior part and in an extreme. In the other extreme is introduced the existence light of the illuminated region.

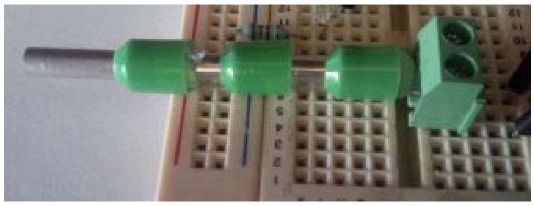


Fig. 1. Fine Device used to focus a specific zone of the illuminated plane composed for an electric resistance gauged in ($k\Omega$) , and a millimeter tube of cylindrical section of diameter (= 4 mm) of certain length 0.015 m.

Experiment 2.1. The realized experiment in typical operation conditions of illuminating zone throw the following useful data: exist a range 19.8 kΩ - 83.3 kΩ, with the specific sensing:

a) 19.8 kΩ, to the illuminated zone for the determined source.

b) 83.3 kΩ, to zone with darkness and other enclose sources.

The light decrease the resistance value of the LDR device. Likewise, the illumining flux stay described to this sensor devise as:

$$\Phi = kR, \quad (4)$$

where Φ is the illuminating flux, k is the proportionality constant, and R is the electrical resistance.

Table 1. Maximum and minimal illuminating flux measures.

No.	Area	Distance (=m)	Lx
1	2.5	25	8.956
2	1.70	32	4.67

The units of the luminous flux are Lumen (=Lm), then can be verified this flux given for (4), considering the photo-resistance element in $\mathfrak{T}$, which decrease when increase the incidence light intensity. Then exist a time which the photo-resistive element accumulates the charge created by light intensity per unit of resistance called RC-time constant[5] which helps to define the non-dimensional constant k, of (4). As the photo-resistive effect time is the sensing time, then

[1] We can consider from a point of view of the field theory two illuminating fields due the light scattering annulation or darkness concentration: The clear field illuminating, which consists of illumination with penetrant light for opposing to the dark field illuminating. In reciprocal way, we define the dark field illuminating. Then to the illumination theory that we can construct through to a sensor that detects the change of illumining yet considering the light beam scattering, will be the dark field illuminating designing a tube $\mathfrak{T}$, such that their functions *u*, inside the tube have a gradient equal to dark field of illuminating.

[2] Ω, is a minimal surface, which to our proposes will be the optimal illuminated surface.

[3] $\mathcal{L}(H,H) = \mathcal{L}(H) = H^*$.

[4] Consists a tube section with microscopic diameter. To our research this tube section will be gauged in millimeters to the interest applications.

[5] The RC time constant, is the time constant (in seconds) of an RC circuit, which is equal to the product of the circuit resistance (in ohms) and the circuit capacitance (in farads):

$$\tau = RC.$$

$\tau \approx t$, where also we can consider with high precision that $C \approx 1$.

Likewise, we have [6]:

$$\Phi\left(=\frac{\sec\times Joule}{Ohms\times\sec}\times Oms\right)$$
$$\left(=\frac{\sec\times Joule}{Ohmx\times(Ohms\times Fd)}\times Ohm\right)$$
$$=\left(\frac{\sec\times Joule}{Ohms\times\sec}\right)\left(=\frac{\sec}{\sec}\times Joule\right)(=kR)(=Lm)$$
$$\Phi\left(=\frac{\sec\times Joule}{Ohms\times\sec}\times Oms\right)$$
$$\left(=\frac{\sec\times Joule}{Ohmx\times(Ohms\times Fd)}\times Ohm\right)$$
$$=\left(\frac{\sec\times Joule}{Ohms\times\sec}\right)\left(=\frac{\sec}{\sec}\times Joule\right)(=kR)(=Lm) \quad (5)$$

This component brings the following facts in the research:

1) The visible effect through tube inner is the difference:

$$\tilde{\aleph} = \aleph^{+} - \aleph^{-}, \quad (6)$$

Then in a tube extension, we have:

$$\aleph^{+} - \aleph^{-} = \nabla(\upsilon - u), \quad (7)$$

2) The dark illuminating field will does that the sensor perceives the illuminating difference.

3) The reason of the tube is to show the energy of the change of illuminating field. This carry us to weak topology to define the energy metric in the corresponding obstacle problem.

Theorem. 2.1.

1) The section $\tilde{\aleph}$, of the component $\mathfrak{T}$, is illuminated proportionally to the gradient $\nabla(\upsilon - u)$, $\forall u, \upsilon \in C^{\infty}(\Omega)$.

2) This is illuminated when their Dirichlet energy[6] is minimal.

Proof. In an illuminated enclosure can be differentiated different zones with different illumination. Likewise, if $\nabla w = \tilde{\aleph}$, is an illuminating field, the variation can be observed as the variation or oscillation:

$$\delta\aleph = \aleph^{+} - \aleph^{-}, \quad (8)$$

which in gradient terms $\forall u, \upsilon \in C^{\infty}(\Omega)$ is $\aleph^{+} - \aleph^{-} = \nabla\upsilon - \nabla u = \nabla(\upsilon - u)$, to two differentiated regions. Then i), is verified. Now ii), will be verified always that:

$$\int_{\Omega} < \nabla u, \nabla(\upsilon - u) > dx \geq 0, \quad (9)$$

Indeed, first we have that verify under a special metric of energy that the integral is the energy integral to our sensor system $\mathfrak{T}$, and after verify that this energy corresponds to the luminous flux given by (4). For last, we need verify that their energy is of Dirichlet [7].

We use the energy metric in the space $H^{1}(\Omega)$, given by the Hermitian product:

$$< f, \bar{f} >= \int_{\Omega} (|f|^{2} + |\nabla f|^{2}) d\omega, \quad (10)$$

since in this phenomena is required an energy topology more fine that usually given on Hilbert spaces, since the illuminating intensity decrease with the distance of the light source. Of fact, we need the Sobolev metric (10) to enclose the obstacle problem that require the Dirichlet energy.

Likewise in the first sensor device of $\mathcal{D}_{\mathfrak{T}}^{*}$, (Fig. 2), we consider the set of functions of $\upsilon \in C^{2}(\Omega)$, such that are annulled in the boundary $\partial\Omega$. Since this device only will sensing two states; the light existence or their absence of light through 1 or 0. Then we can establish the boundary condition to the space $\mathcal{D}_{\mathfrak{T}}$,

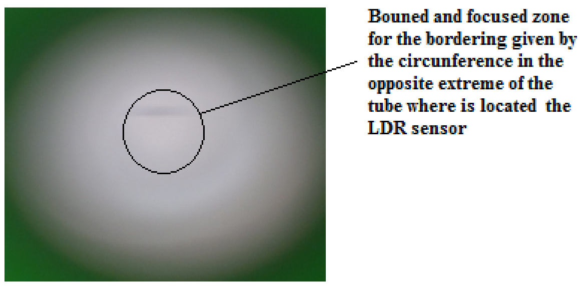


Fig. 2. Projection of light of a specific affecting zone through the tube element of diameter 4 mm.

$$u|_{\partial\Omega} = 0,$$

Likewise, we consider in particular $\forall u, \upsilon \in C^{2}(\Omega)$, an operator in $\mathcal{D}_{\mathfrak{T}}^{*}$, is defined by the mappings composition:

[6] The Dirichlet energy represents the measure of variation of the function u, in Ω. In an illuminating problem considering an illuminated enclosure, the sensing will be optimal if the Dirichlet energy is minimized. Since, if the variations are minimal then their Dirichlet energy will be minimal *(in the enclosure can be differentiated the different zones of different illuminating*), and the sensing of illumination will be more precise [7].

$$\mathcal{D}_{\mathfrak{T}} \xrightarrow{T} L^2(\Omega)^3 \xrightarrow{T^*} \mathcal{D}^*_{\mathfrak{T}}, \tag{11}$$

We consider $T = \nabla$, since in the mapping (11) we have considering (1) and (2) with correspondence rule[7]:

$$T \mapsto T^*, \tag{12}$$

where the energy integrals (inside the space $\mathcal{D}^*_{\mathfrak{T}} \subset H^*$) to the sensor are

$$\begin{aligned} < Tu, \upsilon >_1 &= \int_\Omega (\nabla u)\upsilon dx \\ &= \int_\Omega [\mathrm{div}(u\upsilon) - u\,\mathrm{div}\,\upsilon]dx \\ &= \int_\Omega u(-\mathrm{div}\,\upsilon)dx + \int_{\partial\Omega} n(u\upsilon)dx, \end{aligned} \tag{13}$$

But, we want the luminous flux, thus to the integrals (13) we take:

$$\nabla \mapsto \mathrm{div}(\mathrm{grad}u), \tag{14}$$

Then the integrals to the flux, considering the Gauss divergence theorem and the vector identity given by $\nabla u \nabla \upsilon = \nabla^2 u + \nabla(\nabla \upsilon)$, are:

$$\begin{aligned} < \nabla^2 u, \upsilon >_1 &= \int_\Omega \nabla u \nabla \upsilon dxdydz - \int_{\partial\Omega} (n\nabla u)\upsilon ds \\ &= \int_\Omega \left(\frac{\partial u}{\partial x}\frac{\partial \upsilon}{\partial x} + \frac{\partial u}{\partial y}\frac{\partial \upsilon}{\partial y} + \frac{\partial u}{\partial z}\frac{\partial \upsilon}{\partial z} \right) dxdydz - \\ &- \int_{\partial\Omega} \left(\frac{\partial u}{\partial x}u_x + \frac{\partial u}{\partial y}u_y + \frac{\partial u}{\partial z}u_z \right) \upsilon dxdy, \end{aligned} \tag{15}$$

But $\upsilon = 0$, in $\partial\Omega$, then the terms of boundary are annulled. Then only the integral on the space Ω, contributes to the flux of illuminating energy.

But $T = \nabla \in \mathcal{D}^*_{\mathfrak{T}}$, then

$$< \nabla u, \upsilon > = < u, \nabla \upsilon^* >, \tag{16}$$

which is positive defined. Indeed, if $< \nabla u, \upsilon > = 0$, then u=*cte* since $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} = \frac{\partial u}{\partial z} = 0$. As u=*0*, in $\partial\Omega$, to the device $\mathfrak{T}$, then the constant is zero. For other side, the constant is positive if in particular we consider the illuminating field $\nabla(\upsilon - u)$, which is true since exists a gradient non-null. Then of the last integral of (15) and the limit where $u = \upsilon$, (the Section or space $\tilde{\aleph}$) we have:

$$\begin{aligned} &\int_\Omega \left(\frac{\partial u}{\partial x}\frac{\partial \upsilon}{\partial x} + \frac{\partial u}{\partial y}\frac{\partial \upsilon}{\partial y} + \frac{\partial u}{\partial z}\frac{\partial \upsilon}{\partial z} \right) dxdydz \\ &= \int_\Omega \left(\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 \right) dxdydz \geq 0, \end{aligned} \tag{17}$$

But we need consider the Hermitian product (10) to energy metric in the space $H^1(\Omega)$, then

$$\begin{aligned} &< u, \nabla^* \upsilon >_1 = \int_\Omega u^2(x)dx \leq \int (|\nabla u|^2 + |u|^2) d\omega \\ &\leq C_1 \sum_{i=1}^{3} \int_\Omega \left(\frac{\partial u}{\partial x_i} \right)^2 dx + C_2 \int_{\partial\Omega} u^2(s)ds, \end{aligned} \tag{18}$$

$\forall C_1, C_2$, constants (no negative) depending of Ω, but no depends of u. But the second integral of (18) is zero, since u=*0* in $\partial\Omega$, then:

$$\int_\Omega u^2(x)dx = < \nabla u, \upsilon >_0 \geq C_1 \left(\sum_{i=1}^{3} \int_\Omega \left(\frac{\partial u}{\partial x_i} \right)^2 dx \right), \tag{19}$$

where

$$\frac{1}{C_1} \int_\Omega u^2 dx = \frac{1}{C} \|u\|_0, \tag{20}$$

Then is completed the demonstration of the Dirichlet energy to the corresponding Sobolev space $W^{1,2}(\Omega)$, of sensing.

From a sensor technology point of view, the energy metric in the obstacle problem in the incise iii), requires handle of a large wave to the perception of the change of the illuminating field (see the Fig. 3).

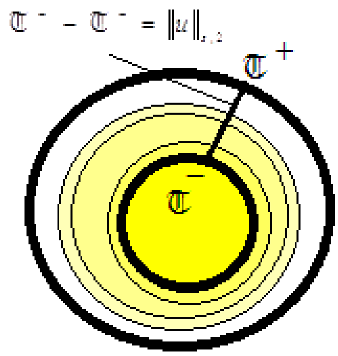


Fig. 3. Illuminating sensor topology. Maximum illuminating is given in the space $\mathfrak{T}^- \cap \mathfrak{T}^+$.

[7] We can define the commutative diagram:

$$\begin{array}{ccc} L^2(\Omega)^3 \times C^2(\Omega) & \xrightarrow{<,>} & H^* \\ Id \updownarrow & & T^* \uparrow\downarrow T \\ C^2(\Omega) \times L^2(\Omega)^3 & \xrightarrow{<,>} & H \end{array}$$

Then the application $\mathfrak{T} \to \mathcal{D}_{\mathfrak{T}}^{-}$, is the sensing technology, that is to say, the sensing classes are the given through the illuminating dark field.

We are interested in minimal surfaces Ω, of the space region M, illuminated by a light source such that

$$J(u) = \int_{\Omega} < \nabla u, \nabla(\upsilon - u) > dx \geq 0, \qquad (21)$$

$\forall u, \upsilon \in M$, being M, the space region, object of illumination analysis defined as points space:

$$M = \{u(x) \in H^1(\Omega) \big| u\big|_{\partial\Omega} \leq \kappa\}, \qquad (22)$$

where κ is the curvature or obstruction by the dark field flow.

But the device $\mathfrak{T}$, is only linear and defines the bounded condition between illuminating (clear and dark (see Fig. 4)) fields, as limit condition:

$$\lim_{u \to \upsilon} \nabla(\upsilon - u) = \lim_{u \to \upsilon} \aleph^{-} - \aleph^{+} = \delta\aleph.$$

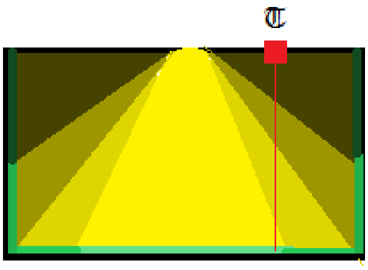


Fig. 4. Sensing through device $\mathfrak{T}$.

3. Stereo-graphic Component to Illuminating Sensor

We require a component to our sensor where we can to obtain a mean value of intensity of light that illuminates the space, with the goal of optimize the illuminating sources and with it the energy required in the illumination of a space and to do gathering of all light from all directions.

In this point, we require a reference surface that does gathering of the light in all directions, which is the sphere. Furthermore, the complex sphere have all harmonics, $\omega_0, \omega_1, \omega_2, \ldots \omega_N, \ldots$ over surface using the polar representation of a complex number (Fig. 5).

To it, we use the mean curvature value originated by the minimal surface due to the stereographically projected Gauss mapping [8]

$$\mathcal{G} : M \to \mathbb{C} \cup \{\infty\}, \qquad (23)$$

which is meromorphic in the underlying Riemannian structure of the minimal surfaces[8].

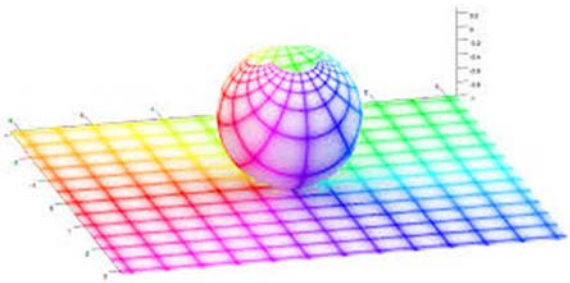


Fig. 5. Minimal surfaces generated in the stereographic projection of the Gauss mapping. Observe the colors. All color pieces of the projected surfaces in the plane are the space $\mathbb{C} \cup \{\infty\}$, and none is sphere piece.

Then is designed an opto-mechanical component which will be defined by the stereographic component of mean value to illuminating sensor (see Fig. 6).

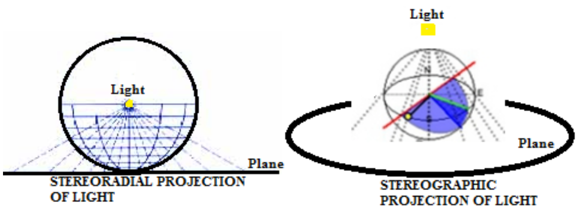


Fig. 6. Stereo-radial and stereographic projections of light.

In analogous way to the clear and dark fields of illumination, we need a field difference more fine, at geodesic level of light. This difference must be taken between the real light geodesic and geodesic of "infinitesimally close" class to the real geodesics[9]. This with the goal of define a tangent space to the geodesic in the space of all geodesics. In addition, use the curvature energy concept to our sensor design.

The rate of geodesic dispersion depends of Gaussian curvature. To positive curvature in a manifold[10] $K > 0$, the rate of dispersion is less that in a Euclidean space $\mathbb{E}^2$, as could be in the sphere Σ. Thus we have placed a sphere Σ, as surface or 2-dimensional manifold on which will be tangent some geodesics conforming the tangent bundle (see the Fig. 7 (a), (b) and (c)):

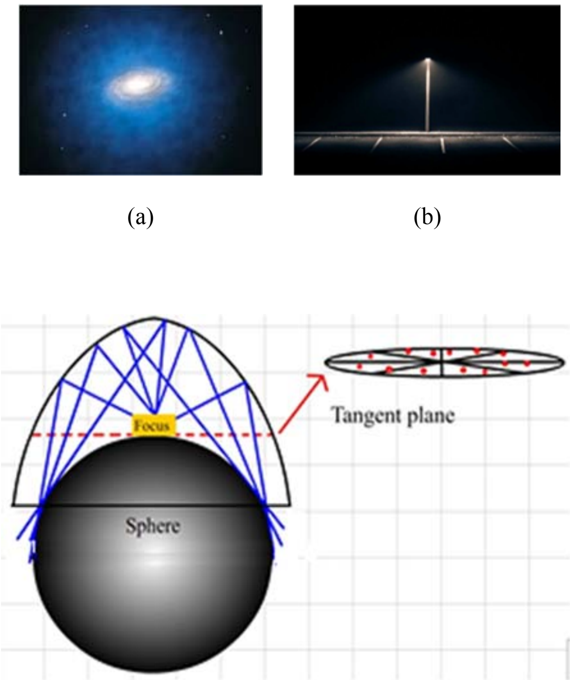
$$T\Sigma = \{p \in \Sigma \big| T_p \Sigma = \lambda_{R^2/\{0\}}\}, \qquad (24)$$

What is the form of these tangent geodesics of the space Σ?

[8] Indeed, we consider the space $\mathcal{G}(M)$, in pieces, which the holomorphicity is not satisfied [9].

[9] This defines a Jacobi field [10].

[10] Topological space.



(a) (b)

(c)

Fig. 7. (a) Example of stereo-radial illuminating in the Universe. The galaxy is the source of light; (b) The light source is azimuth. All stereographic projection is stereo-radial projection but not vice versa, and (c) Geometrical components of illuminating sensor.

The geodesics are of the form:

$$g(s) = r\sin\frac{s}{r}, \qquad (25)$$

to Σ, of radius r. Indeed, considering the Jacobi field equation we have that (25) satisfies $g''(s) + K(\gamma)g(s) = 0$, which:

$$K(\gamma) = -\frac{g''(s)}{g(s)} = -\left(-\frac{1}{r^2}\right) = \frac{1}{r^2}, \qquad (26)$$

which is the geodesic curvature of the sphere Σ. Now their energy spectra will be the spectral density [12][11]

$$G(\omega) = \frac{r\pi}{j}\left[\delta\left(\omega - \frac{1}{r}\right) - \delta\left(\omega + \frac{1}{r}\right)\right], \qquad (27)$$

If we consider the *luminosity energy density* ξ, because will be the concentration of this luminosity where will arrive all tangent light geodesics and will be detected by the sensor in their different variations of frequency. However, this density is due to illumining flux, which change a reflectance velocity [13, 14][12] (reflects inside the parabolic cavity of area (a). Then the analogous equation of (4) to this case will be:

$$\Phi = \upsilon\xi A \qquad (28)$$

Likewise, the illuminating flux contributes to the curvature energy measure as the variation of the intensity of the light field.

The curvature energy is equal to their luminance.

Theorem (F. Bulnes) 3.1. We consider the geodesics of the space Σ, that is to say those whose curvature is $K = 1/r^2$. We consider the illuminating flux of the luminosity energy density ξ, which is proportional to this illuminating flux. Then their curvature energy is

$$\kappa(\omega_1, \omega_2) = \frac{\xi\upsilon^3}{r^2}\frac{1}{\omega_1\omega_2}, \qquad (29)$$

---

[11] $G(\omega) = \int_{-\infty}^{+\infty} g(s)e^{-j\omega s}ds.$

[12] This velocity is the rate of speed of the light geodesics that are not disperse and that meet in the tangent point between sphere Σ, and the polar plane. Indeed, if we consider a dynamical coordinates system given by the polar mapping, where u, υ, can be velocities in a generalized sense, we have the Jacobi field to one of these radial geodesics, written as:

$$g(\upsilon) = X(u, \upsilon),$$

But, as solution of Jacobi equation the function g, is differentiable in $\upsilon = 0$. Then their Taylor development around of this point is:

$$g(\upsilon) = g(0) + g'(0)\upsilon + g''(0)\frac{\upsilon^2}{2} + g'''(0)\frac{\upsilon^3}{6} + o(\upsilon^3)$$

But the initial conditions considered to geometry to tangent light geodesic around the sphere Σ, as the (25), that is to say, from the general solution $g(s) = (A\cos\frac{s}{r} + B\sin\frac{s}{r})$, the of conditions $g(0) = 0$, and $g'(0) = 1$, then from the Jacobi equation we have:

$$g'''(0) = 0,$$

Differentiating the Jacobi equation we have

$$g''' + K(\gamma)'g + K(\gamma)g' = 0,$$

where and using the last initial condition we have:

$$g'''(0) = K(\gamma(0)) = -K(p),$$

Substituting in the Taylor development we obtain:

$$g(\upsilon) = \upsilon - K(p)\frac{\upsilon^3}{6} + o(\upsilon^3),$$

where the term $o(\upsilon^3)$, is very little to contribute. Thus cannot to be taking in the expression to $g(\upsilon)$, [10].

Proof. Applying the curvature energy definition, we have:

$$\kappa(\omega_1,\omega_2) = -\xi\upsilon^3 \int_{\mathbb{E}^2} K(s,r)e^{-j(\omega_1 t_1+\omega_2 t_2)}ds_1 ds_2 = \frac{\xi\upsilon^3}{r^2}\frac{1}{\omega_1\omega_2},$$

■

The units of curvature energy in terms of illumination units (considering that in illumination theory $lumen = joule$) are:

$$\kappa\left(=\frac{lumen}{m^2}\right)\left(=1lumen\times\frac{1}{Coulumb}\times m^{-3}\right) \quad (30)$$

Indeed, we have in the spectra:

$$\frac{1}{r^2}\frac{1}{\omega_1\omega_2}\left(=m^{-2}\times\sec^2\right) \quad (31)$$

But the luminosity energy density has the units [6]:

$$\xi\left(=\frac{lumen\times\sec}{m^3}\right), \quad (32)$$

and by (28) we have:

$$\Phi\left(=\frac{lumen\times\sec}{m^3}\right)\left(=\frac{m}{\sec}\right)\left(=m^2\right) \quad (33)$$

Then [15]

$$\kappa(\omega_1,\omega_2)\left(=\frac{\frac{lumen\times\sec}{m^3}\times\frac{m^3}{\sec^3}}{m^2}\times\sec^2\right)$$

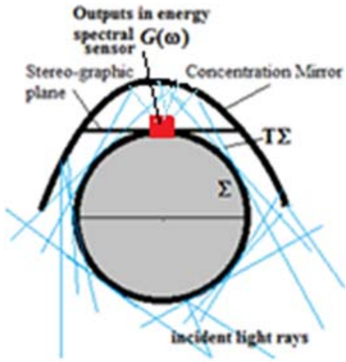
$$\left(=\frac{\frac{lumen\times\sec}{\sec^3}}{m^2}\times\sec^2\right)\left(=\frac{lumen}{\sec^2 m^2}\times\sec^2\right)\left(=\frac{lumen}{m^2}\right),$$

which proves the consistency of the curvature energy to the light sensor with the technique units.

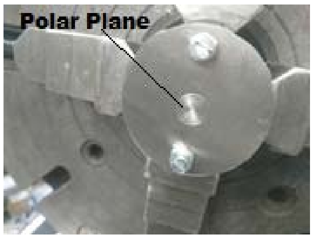
4. Laboratory Measurements and Rehearsals of Prototype

Remember that considering all before section the mean curvature energy in function of the luminance and illuminating flow we want to develop a mean light caption device of omnidirectional mode or way. This will do of such way that the different light sources, direct, indirect, fixed or mobile in a 3-dimensional media can be detected and measured. This permits that through an electronics system take control actions of the illuminating intensity of the light source with the goal of optimize the electric energy consumption in illuminating systems.

To use the combined effect of the equivalent photo-resistance effect is realized a distribution in a polar plane of the photo-resistance series given by (34). This will give the combined effect (see the Fig. 8). This hypothetical physical condition of identify each photo-resistance with a little hole corresponding to polar coordinate (see the Fig. 8) has for goal to determine the electric potential as field observable of the illumining intensity (lux) that have the illumining dark field generated for the little holes (see the experiment in the Section 2). This define a minimal surface of sensing which determine a minimal surface with maximum illumination (see Fig. 9).



(a)



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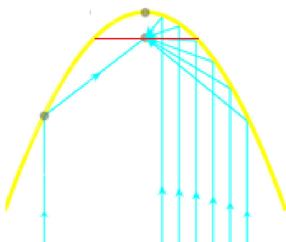


(c)

Fig. 8. (a) The sensor is located in the parabolic focus; (b) Recorded of the polar plane conventional lathe; (c) Concentration of the light rays in the reflecting parabolic section. The illuminating sensor is located in the circular paraboloid focus such that the light bundle across parallel of the cores distributed in the polar tangent plane to the sphere. This parallelism condition of the light bundles confirm the incidence by reflection in the focus of the sphere where is the sensor device [11]. The incidence condition of light in the focus does possible concentrate major illuminating flux of homogeneous way, which establish immediate and stable response.

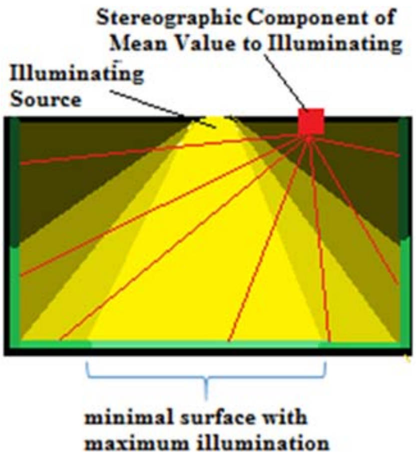
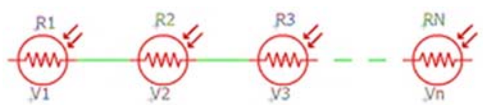


Fig 9. Sensing through $\Sigma - \mathfrak{T}$.

The equivalent circuit that describes the behavior of the opto-electronics system [3, 4] where each element is located in every little hole distributed in the polar plane, comes given by the series circuital coupling:



(34)

This brings the caption of the obscure field of the tangent region of the gauge sphere. Then the equivalent photo-resistance effect (34) will be the capitation of mean light in any 3-dimensional space. We analyze the electronic behavior of our sensor in the sensing moment (see Fig. 9 and Fig. 10).

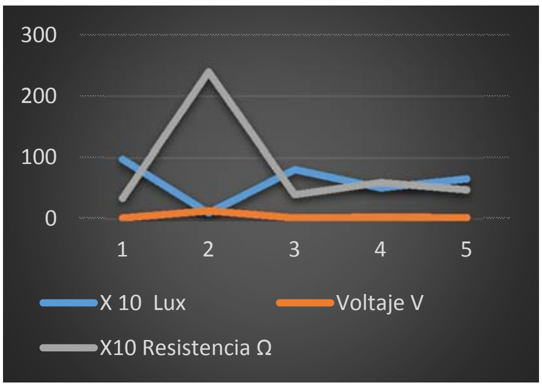


Fig. 10. Response comparing of question variables. See the Table 2.

Table 2. Obtained values during the Proteus simulation.

LDR	⁰L	Resis-tance (Ω)	I(t) Constant	Vn
1	980	343 Ω	0.005683	1.95 V1
2	102	2400 Ω	0.005683	13.6 V2
3	818	400 Ω	0.005683	2.28 V3
4	508	600 Ω	0.005683	3.43 V4
5	660	480 Ω	0.005683	2.74 V5

The illumining flow obeys to curvature radius R, due the spherical geometry after 2 LDR scale, where the basic dimension is the reflection velocity υ, and also the photo-resistance after 2 LDR scale has the similar behavior.

However, from a point of physical view, locate photo-resistance in all a polar plane (see the Fig. 11 and Fig. 12) results little practice. We need a sensor, which through an average of illuminating intensity of the illuminating flow that enter in the device obtain the same measure as the polar hypothesis explain. However, to this, we need demonstrate that the illuminating intensity of the light signals through the polar disk, which is tangent to the sphere Σ, is equal to their mean curvature. These are the curvature energy in function of the illuminating units, as was mentioned in the Section 3.

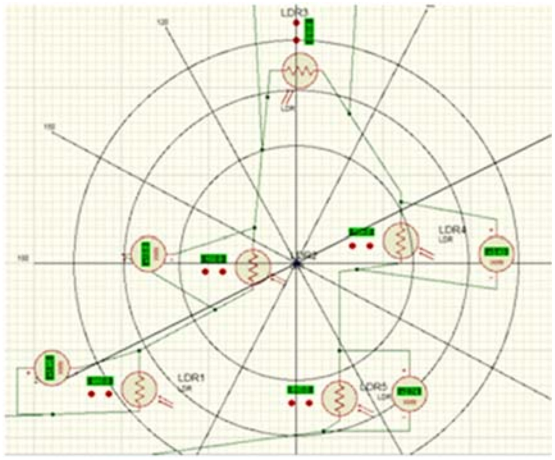


Fig. 11. Polar hypothetical distribution of photo-resistances.

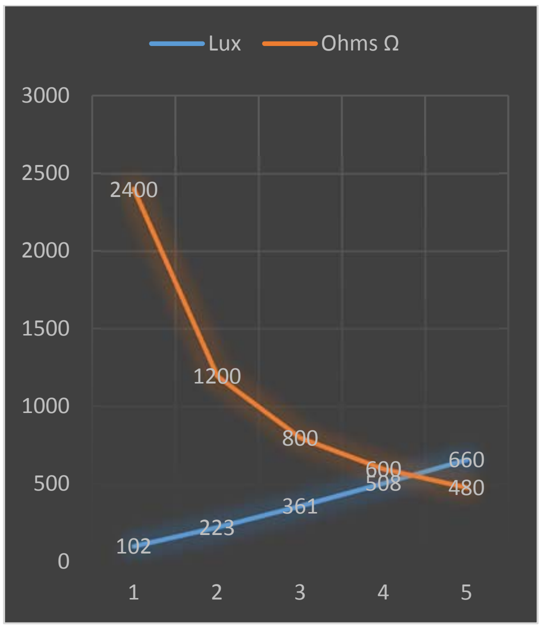


Fig. 12. Illumining intensity versus resistance. See the Table 3.

Table 3. Illumining intensity versus resistance.

Lux	Ohms Ω
102	2400 Ω
223	1200 Ω
361	800 Ω
508	600 Ω
660	480 Ω
818	400 Ω
980	343 Ω

We prove the following result that have that see with the design of our sensor as electronic device with a geometrical element captor of mean light in a 3-dimensional space. This element is stereographic thus must to detect and sensor the all light existing in a 3-dimensional space, concentrating this light in their average as mean curvature.

Proposition 4.1. The Radon Transform of the Gaussian curvature of light signals on the polar disk is their mean curvature.

Proof. Firstly, we have that understand that the geometrical property that we want show is the property of a geometrical element captor of mean light in the 3-dimensional space. Of fact, this is the space property generated in the light reflection of light geodesics in the proximity around of sphere Σ [13].
If we consider as characteristic function of curvature around the sphere as the function:

$$K(x,y)=\begin{cases}\frac{1}{r^2}, x^2+y^2+z^2=r^2\\ 0, x^2+y^2+z^2>r^2\end{cases}, \quad (35)$$

Then the Radon transform on the polar disk is:

$$\hat{K}(r,\theta)=\int_{<\xi,x>=r} K(x,y)ds=\frac{1}{r^2}\int_0^r ds=\frac{1}{r}, \quad (36)$$

For other side, the mean curvature around the sphere comes given by (considering the length of polar disk $2\pi r$):

$$H=\frac{1}{2\pi}\int_0^{2\pi} k(\theta)d\theta=\left(\frac{k_1+k_2}{2}\right)=\left(\frac{\frac{1}{r}+\frac{1}{r}}{2}\right)=\frac{2}{2r}=\frac{1}{r}, \quad (37)$$

joining (36) and (37) is proved the proposition. ■

But we require a field of light geodesics that include the tangent space of the change speed of direction of illumination. However, this change speed of directions is a curvature, which can be measured through stereographic projection on sphere and the localizing of an illuminating intensity sensor in ascendant straight line of a pole (see the second Fig. 13).

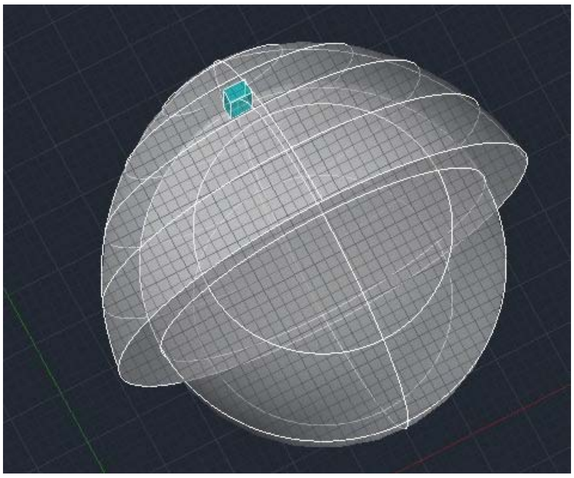


Fig. 13. This is located in the parabolic focus and tangent point between polar plane and sphere.

If we consider all field of the geodesics given by (25) these geodesics are of the tangent space. Thus satisfies the Jacobi equation and their energy spectra is:

$$X(\omega)=A(\cos K\omega+j\sin K\omega)+ B(\cos K\omega-j\sin K\omega), \quad (38)$$

which is accord to the parameterization in the polar plane of the integrals of energy to the flux given in the demonstration of curvature through light signals.

Remark: Applying the Fourier transform to the Jacobi equation on sphere (that is to say with the geodesics on the sphere), we have that solution of the Jacobi field of all light signals in the tangent space around of sphere, which is

$$\omega^2 X(\omega)+KX(\omega)=0,$$

Given that $X(\omega)\neq 0$, then

$$p(\omega)=(\omega^2+K^2)=0,$$

Then two roots are $\omega_1=jK$, $\omega_2=-jK$.

[13] There is a curvature radius due the paraboloid vault, which is defined in light energy terms as $\xi\upsilon^3/r^2$. Indeed, we consider the parabolic section of the circular paraboloid. Then considering a tangent point to the parabolic section given by $\Xi(x,y)$, we have that the curvature radius R, to the parabolic section is:

$$R=\frac{n^3}{p^2}\propto\frac{\upsilon^3}{r^2},$$

which is true for the time square. Indeed, the parabolic behavior can be due the double variation reflected light (acceleration) for time square.

The maximum curvature energy sensing zone[14] is given in the red stripe of paraboloid given in the following Fig. 14 [16].

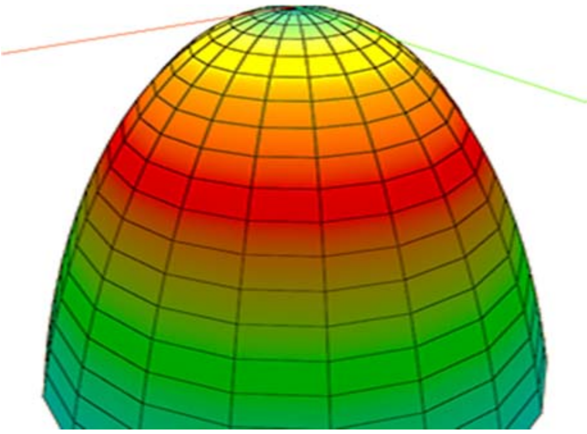


Fig. 14. The more intensity of curvature energy is the red stripe zone. In this zone is located the polar plane.

Newly considering the Gauss theorem to the illuminating flow on the differential operator:

$$\nabla u \nabla \upsilon = \nabla^2 u + \nabla(\nabla \upsilon),$$

and considering (22) that is to say the illuminating flow in the boundary limit, we have[15], (considering in particular the Euclidean measure of the light rays as electronic signals of constant voltage V, (see the Fig. 12))[16]:

$$\Phi = <\nabla^2 u, \upsilon>_1 = \int_{\Omega} \nabla u \nabla \upsilon dxdydz - \int_{\partial\Omega} (n\nabla u)\upsilon dxdy$$
$$= \iiint_{\Omega} \|\nabla_n \sigma(t) \bullet \nabla_n \sigma(t)\| dxdydz -$$
$$\iint_{\partial\Omega} \|\nabla_n \sigma(t) \bullet \sigma(t)\| dxdy \geq 0,$$

But the signals no will be detected beyond of certain voltage value C, such that $\|\nabla_n \sigma(t)\| \leq C/R$, [18] and by the Proposition 4.1,

$$\Phi = <\nabla^2 u, \upsilon>_1 = [C^2] \int_{\Omega} (kh) dxdydz -$$
$$2\int_{\partial\Omega} \|\nabla_n \sigma(t)\| dxdy \geq 0,$$

where

$$2H \int_{\partial\Omega} |u| dxdy \leq 2H \text{Supp} \|K(\zeta)\|, \tag{40}$$

Due to that we are interested in minimal surfaces Ω, of the space region M, illuminated by a light source such that (21) is satisfies, then the range of curvature energy stays as:

$$2H\text{Supp}\|K(\zeta)\| \geq [C^2] \int_{\Omega} (kh) dxdydz \geq 2H \int_{\partial\Omega} |u| dxdy,$$

We consider the response signal of the illuminating sensor with capacitance characterized for $1/R(\varphi)$, to the co-cycles of curvature energy [19]:

$$h(t) = V_{in} e^{-\left(\frac{\lambda}{R(\varphi)}\right)t}, \tag{41}$$

(41) is the signal detected by our illuminating sensor. Here φ, is the average flux value of the captor device, considering a regime of constant voltage. This average flux comes given by

$$\varphi = -\sum_{j=1}^{N} \varphi_j, \tag{42}$$

But, by the transitory analysis of response and considering the bordering condition

$$\lim_{\varphi \to 0} R(\varphi) = \infty, \tag{43}$$

we have that the resistance decrease when the flux increase (see the Fig. 12). In the reality the infinitum is re-interpreted as a non-measurable quantity. For other side, if $\varphi \to \infty$, then we have the limit

$$\lim_{R \to \infty} \left[-\frac{\lambda}{R(\varphi)} \right] = 0, \tag{44}$$

[14] We consider our reflecting paraboloid surface component of our electronic sensor as the equation

$$z(x, y) = -(x^2 + y^2),$$

with the restriction given by the plane that pass through the focus parallel to plane XY (that is to say, the polar plane that is tangent to the sphere in the focus (0, 0, -1/2)):

$$z + 1/2 = 0,$$

Then $z(x, y)$, is maximum in the points of the polar disk in the points: (0, $1/\sqrt{2}$, 0), (0, $1/\sqrt{2}$, 0), ($1/\sqrt{2}$, 0, 0) and ($1/\sqrt{2}$, 0, 0).

[15] *Theorem (F. Bulnes) 1. 1.* [17, 18]. The Radon transform of the Gaussian curvature whose detection condition is the inequality (*censorship*[15]):

$$[\log \varphi(\xi(t))]^2 \left[\int \log \sigma(t)\right]^2 \geq \left(\int \Omega(1 - \nabla^2 \log \Omega)\right)^2 \geq 4\pi \int \Omega,$$

and using the signals the curvature measured by light beam, is:

$$\iint \left|K(\varphi, L_{\zeta}, \bullet)\right|_{L^1} = \frac{2}{R} \iint_{D^2} \left|K_h(\sigma(t))\right| dxdy,$$

Proof. [17, 18].

[16] The received signal after the reflection in the captor device on the photo-resistance is $\sigma(t) = e^{j(\omega_0 + kv)t}$. Their norm is 1.

Then the response signal is the proper voltage $h(t) = V_{in}$. But this only happens in a bandwidth characterized by the average curvature energy

$$|u|_{\partial\Omega} \leq \kappa$$

Such and as is established by the Hilbert inequality [20]:

$$[V]^2 \int_C hk^2 ds \geq (\int_C h^2 - k)^2 ds \geq \frac{1}{2} AV^2 \int_0^{2\pi} k(\theta) d\theta,$$

which is translated to our particular problem to:

$$\kappa_2 \geq H(\omega_1, \omega_2) \geq \kappa_1, \quad (45)$$

where ω_1 and ω_2 are the roots of the polynomial of the energy spectra of the Jacobi field.

Using the stable response signals in the sensor, is established a series circuit that permits operate in safe conditions given that the luminous flow, now incidences in an only photo-resistance, and exist the tendency of short-circuit (breakdown). But to it, is the resistance R_2 to ensure that this condition never will be given. Additionally in R_2 also is obtained the measurement of power as observable of luminous field.

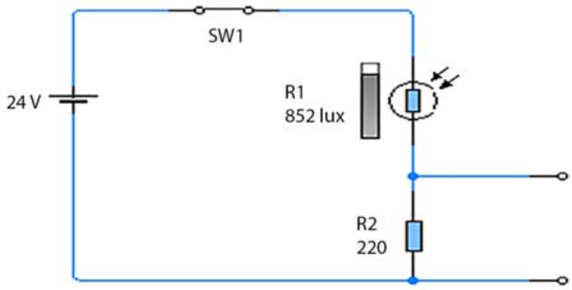


Fig. 15. Applicable circuit to geometrical captor device of average light of 3-dimensional space.

Likewise we consider the equations to circuit design accord with the mathematical physics realized in the before Section 2 and Section 3, [21, 22]

$$\Delta VR_2 = \left(\frac{24V}{220 + \Delta LDR}\right) 220 (= Volts), \quad (46)$$

$$I_{Total} = \sum_{N=1}^{10} \iota_N (= Lux), \quad (47)$$

$$\Delta LDR \approx -500 \left(\frac{1}{I_{Total}}\right) (= \Omega), \quad (48)$$

where I_{Total} is the luminous intensity in the focus of the circular paraboloid; ι_{Total} is the luminous intensity in the $nth-$core of the polar plane.

The Behavior is similar to the simulation 2 (Fig. 12), but with a major concentration due to the concentration and reflection properties in the focus of the circular paraboloid (see the Fig. 16).

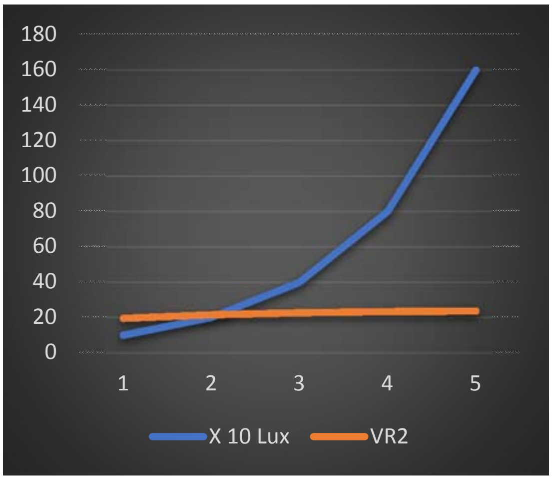


Fig. 16. The potential limit in R_2 is 24 V.

The feeding of the voltage is constant (see the voltage surface Fig. 17) while that the illuminating flow increase in the time in the curvature energy (see the units of curvature energy through illuminating units). The mean curvature is obtained as the spectra delimitated by the inequality (45), whose 2-dimensional waves are given for the Fig. 18.

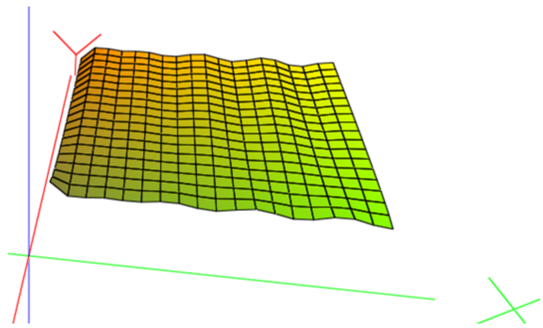


Fig. 17. Voltage surface applied.

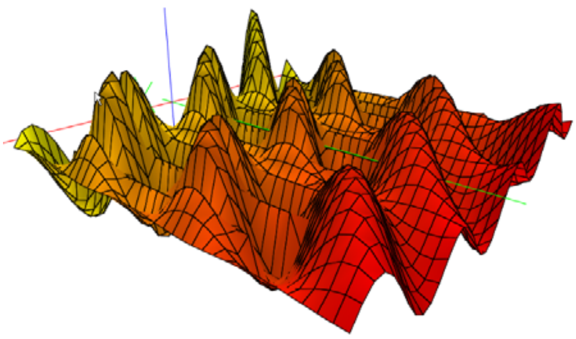


Fig. 18. Mean curvature spectra. The range (45) is taken on the Y-axis.

5. Conclusions

Finally, we can to conclude that we have the possibility to use a omnidirectional sensor with coherence and stability of the luminous intensity level (=Lux) in function of the time on the surface of the photo-sensor device as effect of produce of it a luminous flux (Lumen) through of the cores and rays reflected in the interior part to the circular paraboloid focus. The design obeys to a geometry considering the curvature of field as their obstruction. But, also, was necessary to consider their cycles in the space where happens this. On the device surface must to exist the boundary condition $u|_{\partial\Omega} \leq \kappa$, to use the curvature energy due the light field in tangent space defined inside the device. We have the incident light beam defined by the tangent space $T_q\Sigma$, where in p, is located our photo-resistive sensor (see the Fig. 19).

Fig. 19. Captor device of average light in a 3-dimensional space.

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